Sliding Mode Control of Nonlinear Coupled Tank System

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Abstract: - Sliding mode control is a powerful approach for controlling nonlinear and uncertain systems. This paper presents the liquid level control of the nonlinear-coupled tank system using sliding mode control technique. At first, a static sliding mode control technique is proposed to control level height of the second tank. According to the chattering problem that happened with static sliding mode control, integral sliding mode controller is suggested to reduce the chattering problem, as well as guarantee the asymptotic stability of the feedback system. The closed loop system is simulated using MATLAB Simulink to demonstrate the characteristic performance of the developed control approaches. Therefore, the simulation results indicate that the proposed control schemes work very well. While, the integral sliding mode control has a better performance for the rejection of disturbance, as well as the system's parameters variation.

Key-Words: - Static sliding mode control, Integral sliding mode control, Coupled tank system, Level control, Nonlinear system, Modeling.

1 Introduction

The term control mean using methods to force parameters in the environment to have specific values. In general, all of the elements necessary to accomplish the control objective are described by the term control system. The basic strategy by which a control system operates is quite logical and natural. In most process industries many of the control applications deal with level, flow, pressure and temperature processes. The liquid level control is tremendous especially in process industries such as water treatment industries, boiler process plant, distillation column, paper-pulp industries, chemical and petrochemical industries. Therefore, the liquid level is very important process parameter, and to obtain better quality products, this parameter should always be maintained at the desired level.

An evaporator system is one example in which a liquid level control system is a part of the control loop. Evaporators are used in many chemical process industries for the purpose of separation of chemical products. For example, in a fertilizer process plant, the evaporator is used to transform a weak solution of chemicals to a more concentrated solution. The level of the solution and the pressure of the evaporator have to be controlled in order to get the required concentration.

The control of coupled tank liquid level system has attracted attention of many researchers around the world. It is one of the most challenging benchmark control problems due to its nonlinear characteristics. The control objective in a coupled tank system is that a desired liquid level in the tanks is to be maintained when there is an inflow and outflow of water out of the tank respectively [1].

The coupled-tank control apparatus CT-100 as shown in Figure 1 is a low-cost pilot plant designed for laboratory teaching of both introductory and advanced control systems theory. This apparatus can be used for teaching system modeling using static and transient measurements; steady state error analysis; transient response studies; and for evaluating the design, operation and application of common controllers as well as controller tuning methods. This apparatus also demonstrates fluid transportation and level controls; dynamic problems typical in the process control industry [2].

Fig.1: Coupled-Tank Control Apparatus CT-100

Sliding mode control technique is a type of variable structure control where the dynamics of a nonlinear system is changed by switching discontinuously on time on a predetermined sliding surface with a high speed, nonlinear feedback [3].

In general, SMC consists of two design steps, which are selecting of a sliding or switching surface in order to achieve the desired system dynamic characteristics, such as stability of the origin. The second step is to design a control law such that the closed loop system becomes stable on the sliding surface, system dynamic enters the sliding surface $S(t) = 0$ and remains on it for all time. Trajectories are forced to reach a sliding manifold in a finite time. State trajectories intersect the sliding surface as shown in Fig. 2. Sliding surface is usually chosen based on the error. In this project, the sliding surface is designed by taking the difference between desired level and actual level.

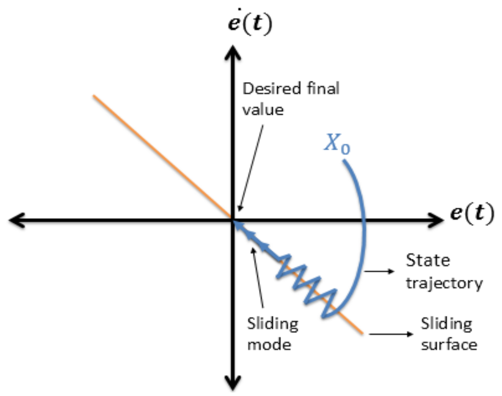


Fig. 2: Graphical representation of the sliding surface

Due to two design steps of the SMC process, the control has two parts, which are, continues part and discontinues part.

$$U = U_c + U_d$$

Because of the two control parts, SMC technique has two phases, namely a reaching phase and a sliding phase. In the reaching phase, the plant states are forced to reach the specified sliding surface in finite time, which is done by, discontinues control law. After the plant states reach the sliding surface, it is adaptively altered to a sliding mode such that the system dynamics slide toward the origin along the sliding surface. This duration is called the sliding phase which is done by continues parts of control law. In the sliding phase, the state trajectories correspond to the system response remains invariant, in the case of parametric and nonparametric uncertainties. However, among the reaching phase, the invariance of SMC is not guaranteed and the system remains sensitive to perturbations.

In reality sampling noises, discretization, delays, and hysteresis effects usually provide oscillations in the state trajectories of the system. In sliding mode control, the high frequency oscillation that occurs in sliding phase as shown in Fig.2 commonly known as "chattering phenomenon" which needs to be reduced in all control applications.

2 Nonlinear Model of Coupled-Tank System

From the schematic diagram of CTS-100 shown in Fig. 3, the derivation of the nonlinear mathematical model will be done.

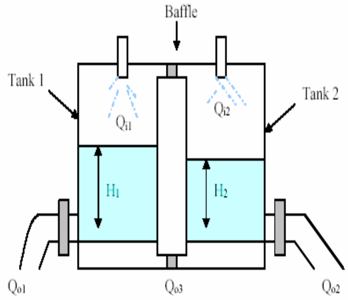


Fig. 3: Schematic diagram of coupled-tank system[4]

By simple mass balance equations rate of change of volume in each tank equals the net flow of liquid into each tank. Let H_1 and H_2 be the liquid level in each tank then the dynamic equation is derived as the following.

$$A_1 \frac{dH_1}{dt} = Q_{i1} - Q_{o1} - Q_{o3} \quad (1)$$

$$A_2 \frac{dH_2}{dt} = Q_{i2} - Q_{o2} + Q_{o3} \quad (2)$$

Where H_1, H_2 are heights of liquid in tank 1 and tank 2 respectively. A_1 and A_2 are cross-sectional areas of tank 1 and tank 2. Q_{o3} is the flow rate between tanks. Q_{i1}, Q_{i2} are pump flow rate into tank 1 and tank 2 respectively. Q_{o1} and Q_{o2} are the flow rate of liquid out of respective tanks.

From Bernoulli's equation for steady, non-viscous, incompressible for each orifice the outlet flow rate in each tank is proportional to the square root of the head of water in the tank; also, the flow between two tanks is proportional to the square root of the difference between two heights.

$$Q_{o1} = \alpha_1 \sqrt{H_1} \quad (3)$$

$$Q_{o2} = \alpha_2\sqrt{H_2} \quad (4)$$
$$Q_{o3} = \alpha_3\sqrt{H_1 - H_2} \quad (5)$$

Where α_1 , α_2 and α_3 are proportionality constant which depends on the coefficients of discharge; the cross-sectional area of each orifice and the gravitational constant.

Substitute equations (3), (4) and (5) into (1) and (2) the nonlinear state equations that describe the system dynamics are gotten as follows:

$$A_1\frac{dH_1}{dt} = Q_{i1} - \alpha_1\sqrt{H_1} - \alpha_3\sqrt{H_1 - H_2} \quad (6)$$

$$A_2\frac{dH_2}{dt} = Q_{i2} - \alpha_2\sqrt{H_2} + \alpha_3\sqrt{H_1 - H_2} \quad (7)$$

The valve or pump actuator dynamic considered as an important control element in the plant and can be modeled in the first order linear differential equation as following equation.

$$\tau\frac{dq_i(t)}{dt} + q_i(t) = Q(t)$$

Where τ is the time constant of the valve/pump (actuator), $q_i(t)$ is the time-varying input flow rate to the first tank, and $Q(t)$ is the computed flow rate. The first order transfer function of the pump actuator can be easily derived by taking the Laplace transform of equation (19). The maximum allowable flow rate is $300\ cm^3/sec$. Another parameters of the plant will be discussed in Table 1.

Table 1: Coupled tank system parameters

Name	Expression	Value
Cross Sectional Area of the coupled tank reservoir	A1, A2	32 cm^2
Proportionality constants that depend on discharge coefficients	á$_i$ Subscript i denotes which tank it refers	$\alpha_1 = 14.30\ cm^{5/2}/sec$
		á$_2 = 14.30\ cm^{5/2}/sec$
		á$_3 = 20.00\ cm^{5/2}/sec$
Pump gain	K_{pump}	13,571cm^3/sec/volt
Maximum allowable volumetric flow rate	Q_{imax}	300 cm^3/sec
Pump motor(valve) time constant	ô	1 sec

3 Sliding Mode Control Design for Coupled-Tank System

The sliding mode controller is designed based on the nonlinear dynamic model of the plant in equations (6) and (7). These mathematical descriptions are repeated in another compact form to simplify the SMC design procedure.

$$\frac{dH_1}{dt} = \frac{1}{A_1}U - \frac{á_1}{A_1}\sqrt{H_1} - \frac{á_3}{A_1}\sqrt{H_1 - H_2} \quad (8)$$

$$\frac{dH_2}{dt} = -\frac{á_2}{A_2}\sqrt{H_2} + \frac{á_3}{A_2}\sqrt{H_1 - H_2} \quad (10)$$

From Table 3.2 in chapter three, the cross-sectional area of two tanks are the same, so that assumed as one variable and equal to A. Inlet flow rate that supplies water to the first tank is the manipulated variable for the process so that $Q_{i1} = U$ in the last equations, and Q_{i2} is assumed to be zero.

The objective of controller development is that, to adjust the level of liquid in the second tank to the desired set point level H_{2d}. For the coupled tank system, the liquid flow rate to the tank 1 is always positive because the pump can only pump the liquid into the tank. Thus, the next two constraints must be satisfied [3].

$$Q_i = U \geq 0$$

$$H_1 \geq H_2$$

The dynamic model of the coupled tank system is highly nonlinear. Therefore, by considering the constraints and defining the variables;

$$z_1 = H_2 \quad , \quad z_2 = H_1 - H_2$$

The dynamic model becomes

$$\dot{z_1} = -\frac{á_2}{A}\sqrt{z_1} + \frac{á_3}{A}\sqrt{z_2} \quad (11)$$

$$\dot{z_2} = \frac{1}{A}U - \frac{á_1}{A}\sqrt{z_1 + z_2} - \frac{2á_3}{A}\sqrt{z_2} + \frac{á_2}{A}\sqrt{z_1} \quad (12)$$

The objective of the control scheme is to regulate the output to a desired value such as $y(t) = z_1(t) = H_2(t) = H_{2d}$.

Now the transformation will be defined so that, the dynamic model given in equations (11) and (12) can be transformed into a form that facilitates the control design.

$$x_1 = z_1 \quad (13)$$

$$x_2 = -\frac{á_2}{A}\sqrt{z_1} + \frac{á_3}{A}\sqrt{z_2} \qquad (14)$$

The inverse transformation of this relation can be found easily as $z = T^{-1}(x)$

$$\dot{x_1} = x_2 \qquad (15)$$

$$\dot{x_2} = \frac{á_3}{2A^2\sqrt{z_2}}U + \frac{á_2 á_3}{2A^2}\left(\frac{z_1 - z_2}{\sqrt{z_1}\sqrt{z_2}}\right) - \frac{á_1 á_3}{2A^2}\left(\frac{\sqrt{z_1 + z_2}}{\sqrt{z_2}}\right) + \left(\frac{{á_2}^2}{2A^2} - \frac{{á_3}^2}{A^2}\right) \qquad (16)$$

This dynamic model for level control in tank 2 can be written in simple form as follows

$$\dot{x_1} = x_2 \qquad (17)$$

$$\dot{x_2} = f(z) + \emptyset(z)U \qquad (18)$$

$$y = z_1 = x_1 \qquad (19)$$

Where

$$f(z) = \frac{á_2 á_3}{2A^2}\left(\frac{z_1 - z_2}{\sqrt{z_1}\sqrt{z_2}}\right) - \frac{á_1 á_3}{2A^2}\left(\frac{\sqrt{z_1 + z_2}}{\sqrt{z_2}}\right) + \left(\frac{{á_2}^2}{2A^2} - \frac{{á_3}^2}{A^2}\right) \qquad (20)$$

$$\emptyset(z) = \frac{á_3}{2A^2\sqrt{z_2}} \qquad (21)$$

The dynamic model in equation (17) and (18) is used to design control techniques for the coupled tanks system.

3.1 Static Sliding Mode Control

In this section, we will design a static sliding mode controller for the coupled tanks system based on the simplified dynamic model. Let H_{2d} be the desired output level of the system, thus x

$$S(t) = \left(\frac{d}{dt} + ë\right)^{n-1} (x_1 - H_{2d})$$

where n is the order of the system.

$$S(t) = \dot{x_1} + ë(x_1 - H_{2d}) = x_2 - ëe(t)$$

$$S(t) = -\frac{á_2}{A}\sqrt{z_1} + \frac{á_3}{A}\sqrt{z_2} - ëe(t)$$

Now by taking the derivative of sliding surface w.r.t. time, we get

$$\dot{S}(t) = \ddot{x_1} + ë\dot{x_1} = \dot{x_2} + ëx_2$$

$$\dot{S}(t) = \frac{á_3}{2A^2\sqrt{z_2}}U + \frac{á_2 á_3}{2A^2}\left(\frac{z_1 - z_2}{\sqrt{z_1}\sqrt{z_2}}\right) - \frac{á_1 á_3}{2A^2}\left(\frac{\sqrt{z_1 + z_2}}{\sqrt{z_2}}\right) + \left(\frac{{á_2}^2}{2A^2} - \frac{{á_3}^2}{A^2}\right) + ë\left(-\frac{á_2}{A}\sqrt{z_1} + \frac{á_3}{A}\sqrt{z_2}\right)$$

$$\dot{S}(t) = f(z) + \emptyset(z)U + ë\left(-\frac{á_2}{A}\sqrt{z_1} + \frac{á_3}{A}\sqrt{z_2}\right) \qquad (22)$$

In the reaching phase, the plant states are forced to reach the selected sliding surface in finite time. If the operating points of the system in the phase plane have negative values, the control law should force the state trajectories of the plant to be positive and if it has a positive sign, the control law will force it to be negative.

To accomplish this design goal, we have the discontinues control law,

$$\dot{S}(t) = -w\ sgn(S) \qquad (23)$$

Where the sign function is defined as

$$sgn\,(S) = \begin{cases} +1 & if \quad S > 0 \\ 0 & if \quad S = 0 \\ -1 & if \quad S < 0 \end{cases}$$

Combining equations (22) and (23), and solving for U we get the standard control law

$$U = \frac{1}{\emptyset(z)}\left[-f(z) - ë\left(-\frac{á_2}{A}\sqrt{z_1} + \frac{á_3}{A}\sqrt{z_2}\right) - w\ sgn(S)\right]$$

$$U = \frac{2A^2\sqrt{z_2}}{á_3}\left[\frac{á_1á_3}{2A^2}\left(\frac{\sqrt{z_1+z_2}}{\sqrt{z_2}}\right) - \frac{á_2á_3}{2A^2}\left(\frac{z_1-z_2}{\sqrt{z_1}\sqrt{z_2}}\right) - \left(\frac{{á_2}^2}{2A^2} - \frac{{á_3}^2}{A^2}\right) - ë\left(-\frac{á_2}{A}\sqrt{z_1} + \frac{á_3}{A}\sqrt{z_2}\right) - w\, sgn(S)\right] \quad (24)$$

According to the standard sliding control law the output $y = z_1(t) = H_2(t)$ will asymptotically converge to its desired value H_{2d} as well as ë and w are positive scalars and by using the refine tune method, the good performance based on the minimizing the performance index ITAE will be achieved.

3.2 Integral sliding mode control

There are many different solutions for chattering reduction that can be presented. One approach places a boundary layer around the switching surface by applying the saturation function. Higher Order Sliding Mode Control can also carefully overcome the control disadvantages, which includes first and second dynamic sliding surfaces. The integral sliding mode technique is proposed in this section because of the decrease computationally extensive and chattering reduction capability.
The integral sliding surface proposed in [5] as follows

$$S(t) = \left(\frac{d}{dt} + ë\right)^n \int_0^t (x_1 - H_{2d})\, dt \quad (25)$$

According to this sliding surface the derived control law does not guarantee the asymptotical stability in terms of system perturbations. So, the integral sliding surface are modified to be

$$S(t) = \left(\frac{d}{dt} + ë\right)^{n+1} \iint_0^t (x_1 - H_{2d})\, dt^2 \quad (26)$$

This higher order sliding surface with integral control in equation (25) is modified to equation (26) by increasing an integral term to compensate for perturbation and at the same time, the sliding surface is increased. In the simple form, sliding surface becomes:

$$S(t) = \dot{x_1} + 3ë(x_1 - H_{2d}) + 3ë^2 \int_0^t (x_1 - H_{2d})dt + ë^3 \iint_0^t (x_1 - H_{2d})\, dt^2$$

Now by taking the derivative of sliding surface w.r.t. time, we get

$$\dot{S}(t) = \ddot{x_1} + 3ë\dot{x_1} + 3ë^2(x_1 - H_{2d}) + ë^3 \int_0^t (x_1 - H_{2d})dt$$

For using the error signal in the controller, sliding surface derivative can be written as:

$$\dot{S}(t) = \dot{x_2} + 3ëx_2 - 3ë^2 e(t) - ë^3 \int_0^t e(t)\, dt$$

$$\dot{S}(t) = \frac{á_3}{2A^2\sqrt{z_2}}U + \frac{á_2á_3}{2A^2}\left(\frac{z_1-z_2}{\sqrt{z_1}\sqrt{z_2}}\right) - \frac{á_1á_3}{2A^2}\left(\frac{\sqrt{z_1+z_2}}{\sqrt{z_2}}\right) + \left(\frac{{á_2}^2}{2A^2} - \frac{{á_3}^2}{A^2}\right) + 3ë\left(-\frac{á_2}{A}\sqrt{z_1} + \frac{á_3}{A}\sqrt{z_2}\right) - 3ë^2 e(t) - ë^3 \int_0^t e(t)\, dt$$

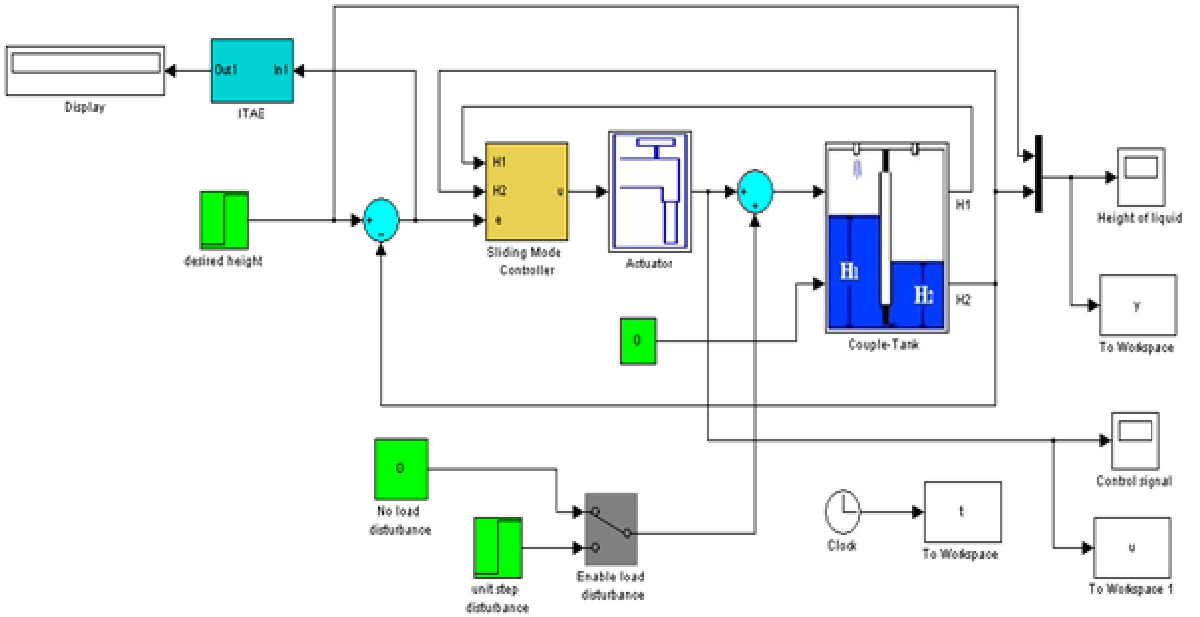
$$\dot{S}(t) = f(z) + \emptyset(z)U + 3ë\left(-\frac{á_2}{A}\sqrt{z_1} + \frac{á_3}{A}\sqrt{z_2}\right) - 3ë^2 e(t) - ë^3 \int_0^t e(t)\, dt \quad (27)$$

The discontinues control law are involved to ensure the reaching phase of sliding mode. Discontinues control law given in equation (23) is combined in equation (27), hence, the overall integral sliding mode control law is derived as

$$U = \frac{1}{\emptyset(z)}\left[-f(z) - 3ë\left(-\frac{á_2}{A}\sqrt{z_1} + \frac{á_3}{A}\sqrt{z_2}\right) + 3ë^2 e(t) + ë^3 \int_0^t e(t)\, dt - w\, sgn(S)\right] \quad (28)$$

The parameters of integral sliding control law ë and w are positive scalars, and will be tuned under the output $y = z_1(t) = H_2(t)$ converges to its desired value H_{2d} and it achieves a good performance based on the minimizing performance index ITAE.

Coupled-Tank system with two SMC control laws simulated in the MATLAB SIMULINK as shown in Fig. 4. The SSMC and ISMC are



implemented by using equations (24) and (28) are masked in a simple subsystem in the Simulink to make the simulation of the coupled tank in a simple form.

Fig. 4: Simulink Block to Control the Couple Tank using SMC

4 Results and Discussion of the

System performance has been studied in terms of step response in the MATLAB program. The liquid supplied to the first tank controlled in a proficient manner for the sake of achieving desired level in the second tank of coupled-tank system. The main objectives in this paper are to control the level of tank 2 by controlling the flow rate of liquid in tank 1. Where the desired performance specification for this system is listed as follows:

- The desired value of the liquid level height in tank 2 is taken to be $H_{2d} = 9\ cm$.
- Good transient response and zero steady state error should be observed.
- The designed controller should reject the disturbance and keep the output response in acceptable performance for the parameters change.

4.1 Static sliding mode controller results

The static sliding mode control law in equation (24) that derived in the previous section has been simulated by MATLAB program. The values of the controller parameters for the SSMC are taken to be ë = 0.36 , w = 0.7. Fig.5 shows the output response of the liquid level in the second tank.

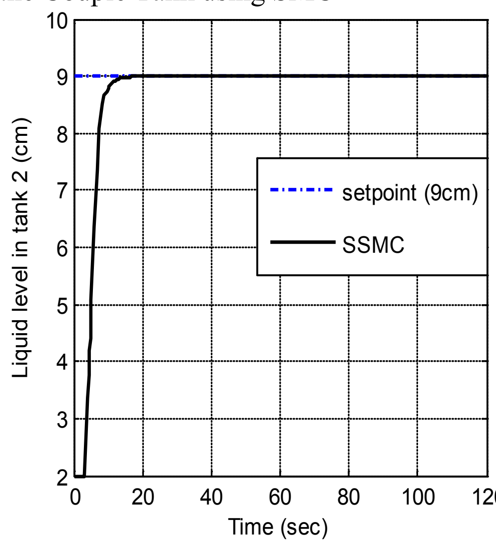


Fig. 5: Output response of couple tank system using SSMC control

According to the system response, it can be seen that the level of liquid requires around 10.38 sec to reach and settle within $\pm 2\%$ from the final value. It is clearly noticed that there is no overshoot where the output level does not shoot the desired value, it is proven that the static sliding mode controller results a fast rise time with 7.58 sec, and the ITAE is 155.04.

Eventhough, the output $H_2(t)$ has convergence the desired level $H_{2d}(t)$ the control effort clearly suffers from chattering phenomenon as shown in Fig. 6. In this project a higher order sliding mode control with integral is suggested, in order to minimize the effects of chattering in the static sliding mode control.

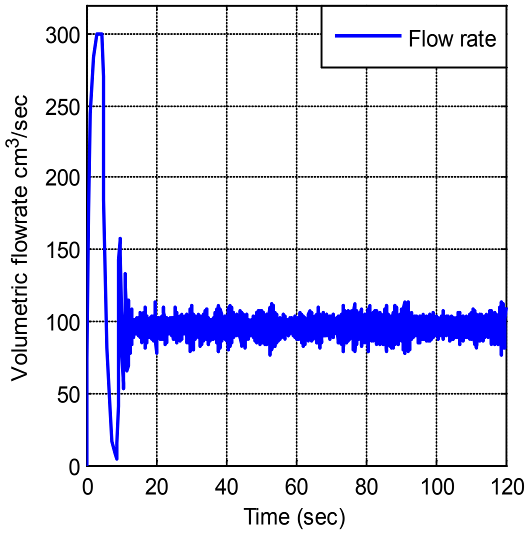


Fig. 6: Inlet flow rate into tank 1 using SSMC

An extra inlet flow rate with $12\ cm^3/sec$ is supplied into the manipulated variable, which acts as disturbance input to the system. This disturbance is applied, when the steady-state level is reached. Fig. 7 gives the output response of the system under effect of disturbance.

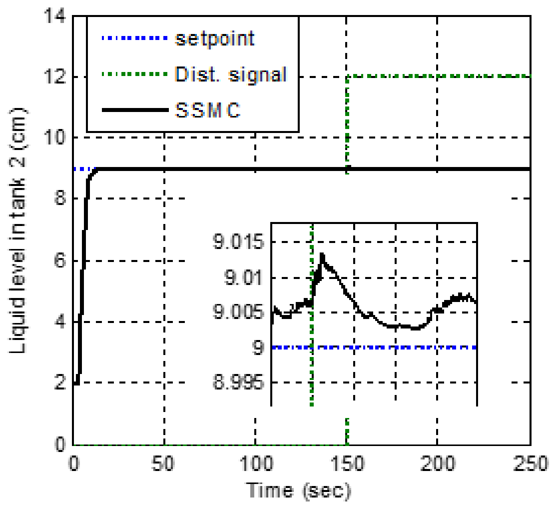


Fig. 7: SSMC response in the presence of load disturbance with $12\ cm^3/sec$ of flowrate

It could be seen that, the SSMC takes a minimum effort to reject the effect of disturbance and amount of time is taken to get back the liquid level to its desired level. In this case the ITAE is 261.01.

The system is tested for the parameters variations using SSMC, where the proportionality constant that depends on discharge coefficient, orifice cross sectional area and gravitational of each outlet flow rate is decreased by 25% from its original values. The behaviors of sliding mode control under the variations in the plant parameters are represented in Fig. 8.

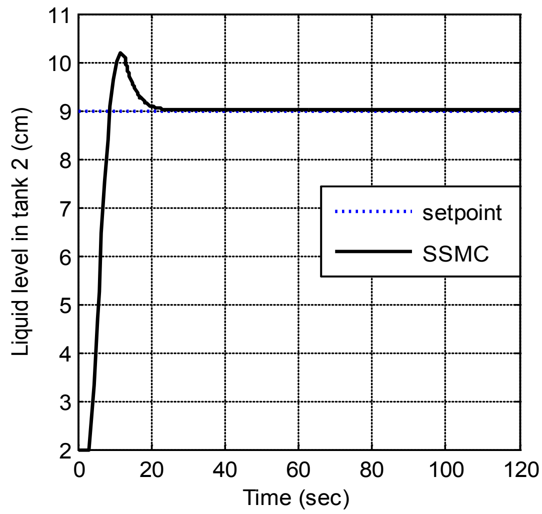


Fig. 8: SSMC response when the proportionality constantsare changed

Based on the Fig. 8, the level controlled is settled about 18.23 sec with very small steady state error, and produces around 13.22% percentage overshoot. The speed of response has a little change and measured by means of rise time as 7.86sec. The controller performance index is evaluated as 240.88.

4.2 Integral Sliding Mode Controller Results

The Integral sliding mode control has been proposed as a solution to the chattering problems attended with standard sliding mode control technique. Fig. 9 shows the output response of the liquid level in the second tank by ISMC. The values of controller parameters are found to be ë = 0.4 , w = 2.85.

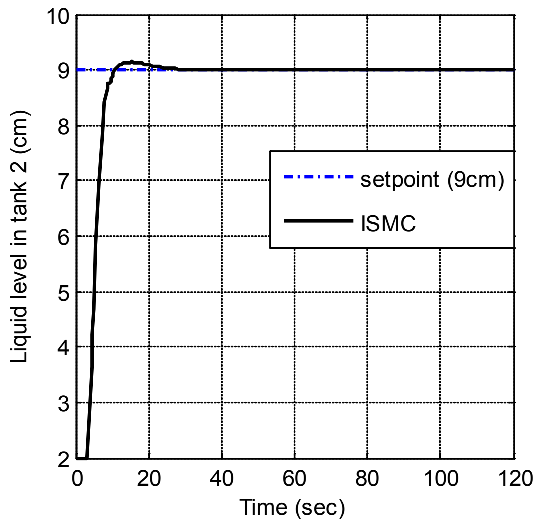


Fig.9: Output response of couple tank system using ISMC control

The controller gave a good response for a level height control with settling time of 9.63 sec and rise time of 4.7843sec. The transient response has a little overshoot about 1.55%. The controller has improvements in the transient and steady state regions of the response over the static sliding mode control with ITAE of 148.91.

The control signal in term of flow rate is represented in Fig. 10. It has been noticed that the control scheme overcome chattering problem, the applied solution gradually makes the control action smoother and limited.

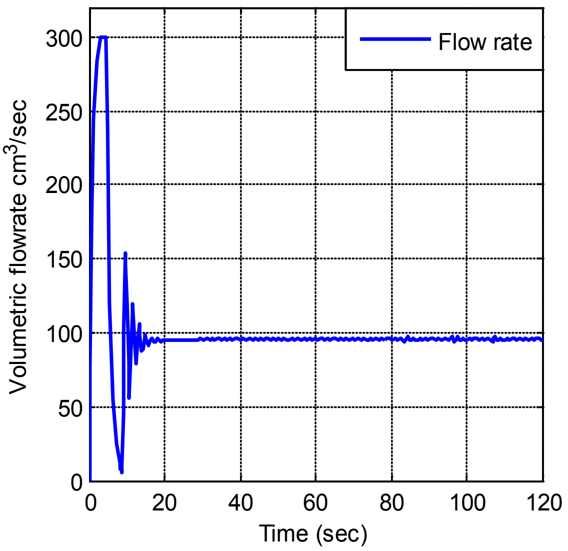


Fig.10: Inlet flow rate into tank 1 using ISMC

After the load disturbance is injected to the system, the simulation result is observed as shown in Fig. 11. According to the result, noticed that the controller work very well in cancellation of the unwanted disturbance, the output level takes around 20 seconds to settle back to its desired level.

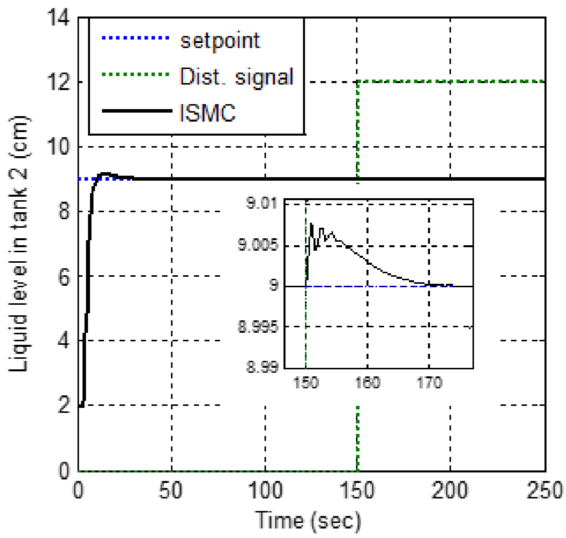


Fig. 11: ISMC response in the presence of load disturbance with 12 cm^3/sec of flowrate

The system outlet flow rates is changed by decreasing it with 25% from its original values. The output response of ISMC is obtained with plant parameter variations as shown in Fig. 12. The observation of the controller performance is evaluated by ITAE, which gave 298.06; where the percentage overshoot, rise time and settling time have a smaller change before applying the parameters variation.

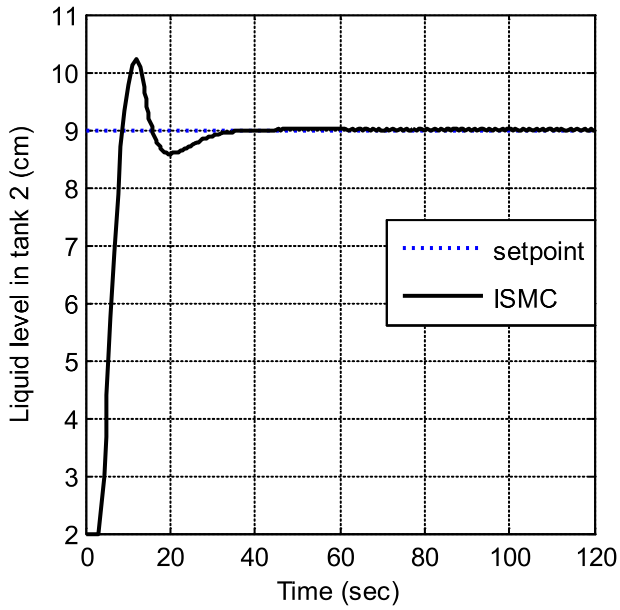


Fig. 12: ISMC response when the proportionality constants are changed

The function of controller in the control applications is to maintain the controlled variable at its desired level. While keeping the control signal in appropriate range to save the actuator from damages. To make comparison between the two sliding mode control methods, inlet flow rate and output responses are grouped and plotted using subplot command in MATLAB, as illustrated in Fig. 13. The output response for both SSMC and ISMC have good performance in terms of transient and steady state response. However, for the inlet flow rate, the ISMC has better performance compared with SSMC. The chattering problem in SSMC is solved by using ISMC where the inlet flow rate became smoother. Hence, the proficiency of ISMC over the SSMC is achieved.

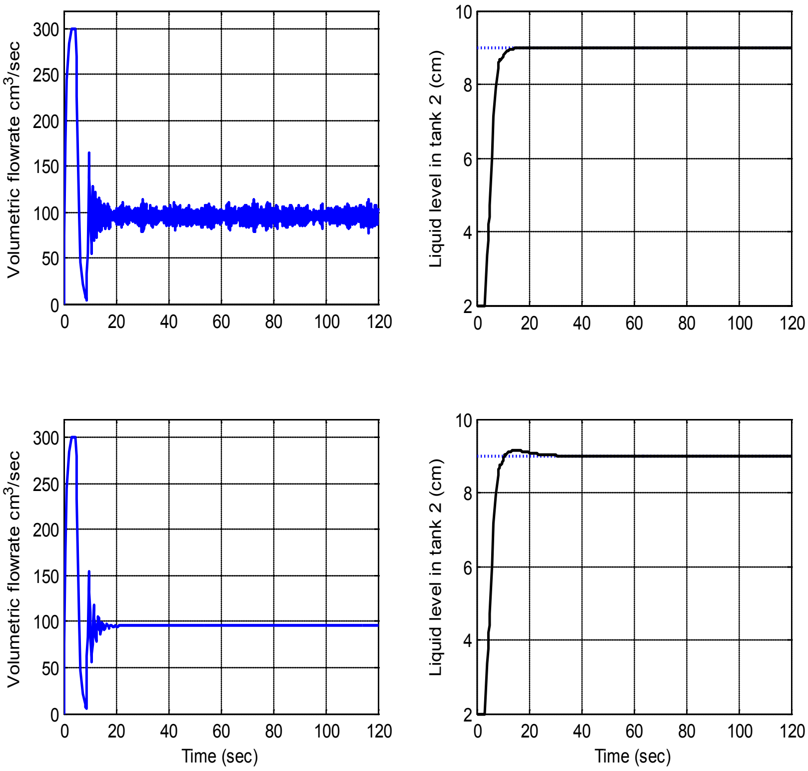


Fig. 13: Coupled tanks system response SSMC (upper pair) and ISMC (lower pair)

5 Conclusion

The main aims of this paper are to regulate the second tank liquid level height at the specific desired level, achieving this goal consists of the derivation dynamic model, which is firstly done. Followed by designing proposed control methods that having the ability to control the system in such different situations. Then, the output response of the coupled-tank system using MATLAB software program has been obtained, the performance of controlled system is analyzed and the comparison of control strategies is performed. Sliding mode control is designed, as it is an effective approach for controlling nonlinear and uncertain system in presence of model uncertainties and disturbances. The simulation result showed that integral sliding mode controller provides a better performance with respect to static SMC technique. Where the chattering as well as the control effort is greatly reduced compared with standard sliding mode controller.

In this research, the performance measure of ITAE is used to assist for choosing the best controller performance. However, SSMC controller has good performance in transient response, steady state response, disturbance rejection and parameters variation. The ISMC has smaller ITAE value Than SSMC, even though in presence of disturbance and parameters changes. Hence, it can be concluded that the robustness of the proposed ISMC is achieved.

References:

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