Model Based Adaptive Controller for Quadrotor UAV with Different Payload

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Abstract: - In this paper, an adaptive piecewise linear control scheme for Quadrotor Unman Autonomous Vehicle(UAV) with specifications on rise times is proposed. It is a gain stabilized control technique. Large gain is used for large tracking error to get fast response. Small gain is used between large and small tracking error for good performance. Large gain is used again for small tracking error to cope with disturbance. The switching point of the three segments piecewise linear controller is found by an automatic regulating time series which is function of output characteristics of the plant and reference model. Time series will be converged to steady values after the time response of the considered system matching that of the reference model. The proposed control scheme is illustrated by a second order altitude control example. It gives an almost command independent response. Time responses show that the proposed method gives significant improvements for response time and performance. The same designing procedures are applied to Quadrotor UAV for different pay load.

Key-Words: - Piecewise linear controller, Adaptive gain, Model based Controller, Quadrotor, UAV

1 Introduction

Gain and phase stabilized are two conventional design methods for feedback control systems. They can be analyzed and designed in gain-phase plots to get wanted gain and phase margins or gain and phase crossover frequencies [1, 2]. The gain crossover frequency is closely related to the system bandwidth(or response time). The phase margin is closely related to performance (or peak overshoot). In general, fast response time and good performance can not be obtained simultaneously for some feedback control systems. For example, the altitude control system of the airframe with altitude and altitude rate feedbacks needs large altitude loop gain for fast response time and low altitude loop gain for good robustness. It is conflict with another. A simple and effective way to solve this problem and can provide better results those of linear controllers are generally expected. This is the motivation of this paper.

Variable structure control is a switching control method for feedback control systems [3-7]. It gives good performance and robustness for coping with system uncertainty. But it is suffered from chattering problem and state measurements. In this paper, a fast response system and a good performance system are selected for switching. An adaptive switching algorithm is used. There is no discontinuous connection between two systems. Therefore, there is no chattering problem. Gain scheduling has been used successfully to control nonlinear systems for many decades and in many different applications, such as autopilots and chemical processes [8-10]. It is consisted of many linear controllers for operating points to cope with large parameter variations. This concept will be expanded for response time and performance. Operating points are replaced by fast response and good performance conditions and interpolation for gain evaluation is replaced by an adaptive switching point. It is determined by the filtered command tracking errors. Nonlinear controller's syntheses using inverse describing function for use with hard nonlinear system has developed for several researchers [11-13]. They are complicated but effective for nonlinear systems.

In this paper, a simple three segments piece- wise linear controller is proposed. It is symmetry and has two switching points. It is equivalent to two quasi-linear gains. There are two equivalent systems. One is the fast but bad performance system, and the other is the slow but good performance system. The switching point is used to select suitable gain and is the function of the dynamic of tracking error. It is easy to analyses and design. Based on the information of the reference model, the parameter of three segments piecewise linear controller can be tuned in the on-line manner. An automatic regulating time series [14] is used for tuning the parameter. It can be re-tuned again if the system is needed. Furthermore, it gives an almost reference input independent response. It implies that the controlled system is similar to a linear system.

The proposed control scheme will be applied to one illustrating 2nd order altitude control example and Quadrotor[17-22] those have been compensated by PI controllers. Time responses show that the proposed method gives exact model matching of the reference models. One can adjust the controlled system before mission operation.

2. The Model Based Control Scheme

Fig.1 shows the proposed model based control scheme in which reference model represents the wanted system responses, adaptive piecewise linear controller is a gain stabilized compensation and auto tuning process is used to find the switching point(D_1) of the adaptive piecewise linear controller. The piecewise linear controller can get fast response and performance simultaneously. It is a three- segment piecewise linear controller. The switching point (D_1) is dependent on the filtered dynamic of the tracking error (E). Parameters K_1, K_2, K_n and T_n are found by the auto tuning process for matching outputs of the reference model. Several identifying processes will be processed to find them. In general, the controlled system can be described by peak overshoot and rise time of step response. The concept of the proposed control scheme will be discussed detail by an illustrating example in next subsections

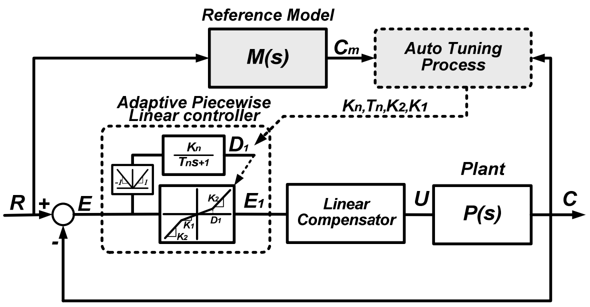

Fig.1. The proposed model based control scheme.

2.1 The adaptive piecewise linear controller

Fig.1 shows a three-segment piecewise linear nonlinearity. Now, the problem is to determine the values of switch points D_1 and gains (K_1, K_2) for the wanted responses time and performance. In this work, switching points $+D_1$ and $-D_1$ are not fixed and will be determined by the absolute value of the command tracking error for feedback control systems.

Considers a 2nd angular rate stabilizing altitude control example described by

$$G(s) = \frac{1}{s(s+2)} \qquad (1)$$

It is closed with a loop gain K. Then the closed-loop transfer function is

$$T(s) = \frac{K}{s^2 + 2s + K} \qquad (2)$$

Poles locations and natural frequency(ω_n) for two loop gains(K_1, K_2) are given as:

$$K_1 = 0.500; poles: -0.2929, -1.7071;$$

$$K_2 = 10.00; poles: -1.0 \pm j3.0; \omega_n = 3.1623;$$

They are an over-damped and under-damped systems. Time responses are shown in Fig.2 for $K = K_1$ (small-dot-line) and $K = K_2$ (large-dot- line) in which R represents the reference input altitude command and C represents the output. The strategy for gain switching is (1) large gain (K_2) for large tracking error to get fast response and (2) small gain (K_1) for small tracking error (E) to get good performance. It is a variable structure system and can be achieved by selecting a proper switching point D_1 of the piecewise linear controller shown in Fig.1.For example, the optimal switching point D_1 is selected as 0.525 for R=1 to get both fast response and good performance. Large gain (K_2) is used for $|E| > D_1$ and small gain (K_1) is used for $|E| \le D_1$. Step response is shown in Fig.2 (solid-line) also for R=1. It shows that adaptive gain can give a good result for fast response and good performance. However, it is not true for R is equal to 5, 10 and 50, respectively. Naturally, another switching point D_1 for R=5, 10 and 50 can be selected for getting good performance. They are 2.625, 5.250 and 26.250 for R=5, 10 and 50, respectively. They are true for step responses from zeros to 5, 10 and 50 only.

Another possible way for the switching point can be dependent on the tracking error (E). A possible switching rule for D_1 is found as $D_1 = 0.925|E|$ for good performance. Fig.3 shows time responses for R=1, 5, 10 and 50, respectively. It can be seen that the switching rule gives an input command (R) independent results. However, they are slower than results shown in Fig. 2. One possible way to speed up the time response is enlarging the large gain phase in the beginning. A low-pass filter $D(s) = K_n/(T_n s+1)$ is used for the absolute tracking error (E) to get D_1. Fig.4 shows faster response than that of shown in Fig.3 is got for $K_n = 1.0465$ and $T_n = 1/\omega_n$. The switching point D_1 is shaped for speed up the responses while keeping performance unchanged. Fig.4 shows that the rise time of the controlled system is quite close that of the controlled system using $K = K_2$; i.e., the rise time of the system is dependent on K_2. Note that the natural frequency (ω_n) for $K = K_2$ is used to find T_n also. Therefore, it is needed to find K_n only.

Fig.5. shows time responses for R=1, 5, 10 and 50, respectively. It shows that almost input independent responses are got. It implies that the controlled system is similar to the system controlled by linear controller. This is the major merit of the proposed method.

2.2 The automatic tuning process

In general, one can use rise time and peak overshoot of step response to describe the characteristics of the controlled system. They will be used as specifications to find parameters of the adaptive piecewise-linear controller; i.e., K_1, K_2, K_n and T_n.

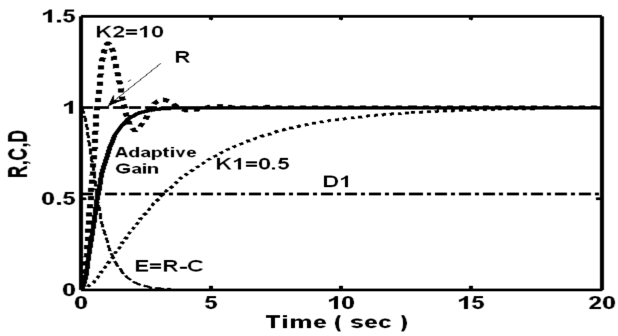

Fig.2. Time responses of the illustrating example for K=0.5, 10 and Adaptive gain.

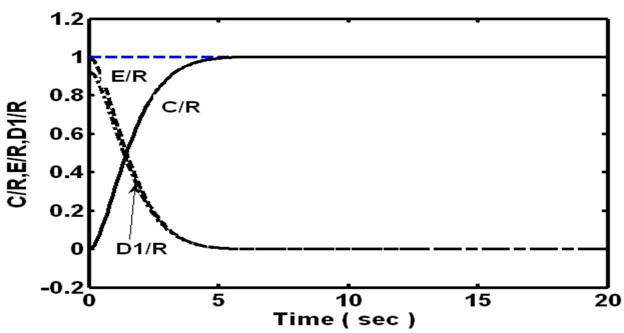

Fig.3 Time responses of the illustrating example for R=1,5,10,50 and $D_1 = 0.925|E|$.

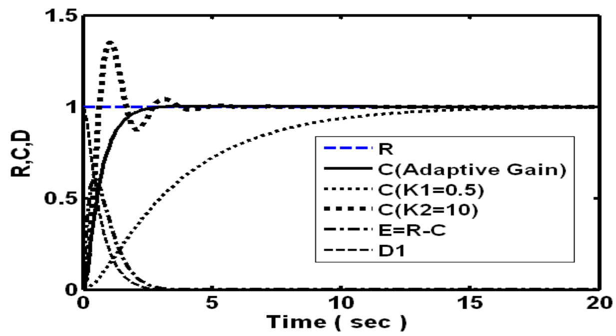

Fig.4 Time responses of the illustrating example for R=1 and $D(s) = K_n/(T_n s+1)$

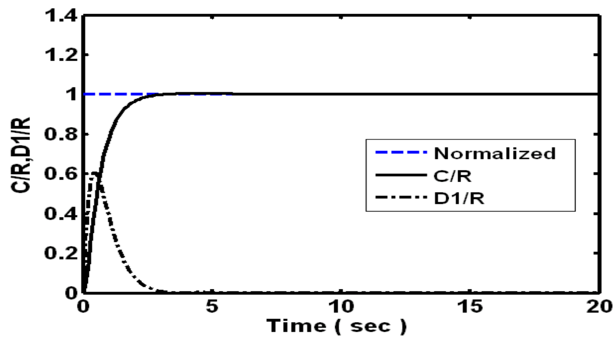

Fig.5 Time responses for of the illustrating example R=1, 5, 10 and 50.

2.2.1 Automatic tuning process for matching peak point

Parameter $K_n = 1.0465$ of the low-pass filter $D(s) = K_n/(T_n s+1)$ of the adaptive piecewise linear controller used in Section 2.1 is found by use of the following equations:

$$G_n(kT+T) = G_n(kT)\times\left\{\alpha[Mp/Mps]^j + (1-\alpha)\right\};\ 0<\alpha<1; \quad (3)$$

$$K_n = G_n(kT) \quad (4)$$

where $G_n(kT)$ is an automatic regulating time series[14]; α and j are controlling parameters of the regulating property; Mps is the specification of the peak value; Mp is the peak point found by using $K_n = G_n(kT)$; T is simulation period of one step response; and k is the k^{th} step responses. Eq.(3) gives $G_n(kT)$ will be converged to a steady-state value after Mps being matched by Mp. Fig.6 shows the on-line tuning process for $T = 30\,\text{sec}; \alpha = 0.9;$ $Mps = 1.003;$ $j = 3$. The values of $G_n(kT)$ in each tuning interval are

$$G_n(0)=0.30; G_n(T)=0.6004; G_n(2T)=0.9580; G_n(3T)=1.0465;$$
$$G_n(0)=0.50; G_n(T)=0.8678; G_n(2T)=1.0484; G_n(3T)=1.0465;$$
$$G_n(0)=0.70; G_n(T)=1.0122; G_n(2T)=1.0453; G_n(3T)=1.0465;$$

for three initial guesses $G_n(0)$. They show $G_n(kT)$ are converged to 1.0465 quickly. Fig.7 shows also that the final system matches the specification of peak overshoot Mps.

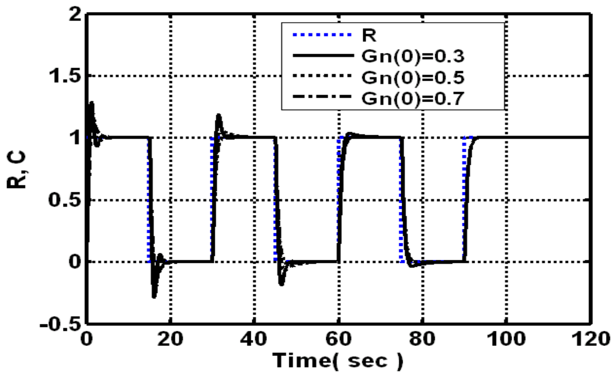

Fig.6. On-line automatic tuning process of the illustrating example with different initial $G_n(0)$.

2.2.2 Automatic tuning process for matching peak point and rise time

In above subsection, specification Mps is used to find parameter K_n. Another specification for rise time T_C can be used also to tune high gain K_2 and T_n. The automatic tuning equations are described below:

$$Q_n(kT+T)=Q_n(kT)\times\left\{\beta[T_c/T_{cs}]^l+(1-\beta)\right\}; \quad (5)$$

$$S_{CK2}=Q_n(kT) \quad (6)$$

where β is regulating parameter, K_2, K_1 and T_n will be replaced by $K_2S_{CK2}, K_1/S_{CK2}$ and T_n/S_{CK2}, respectively. Fig.7 shows the on-line automatic tuning process for matching specifications of peak overshoot and rise time simultaneously. Specifications used are $Mps=1.003$ and $T_{cs}=0.60\sec$. Parameters used in Eqs.(3) and (5) are $\alpha=0.6$, $\beta=0.5, j=5$, $T=30\sec$, $G_n(0)=0.7$ and $Q_n(0)=1.00$. Internal values of each tuning interval are given in Table 1. Table 1 gives system response is converged into wanted specification quickly. Note that system response shown in Fig.7 is faster than that of Fig.6.

Another method for finding parameters K_n, K_1, and K_2 can be used by performance index formulated by integration of the absolute error (IAE) and integration of the square error(ISE) or on-line parameterized method[15,16].

The proposed control scheme will be applied to altitude control of Quadrotor for coping with different pay load.

Table1. Internal values of each tuning interval.

Time	***0***	***T***	***2T***	***3T***	***4T***
$G_n(kT)$	**0.70**	**1.101**	**1.091**	**1.093**	**1.091**
$Q_n(kT)$	**1.00**	**0.815**	**1.979**	**1.927**	**1.928**
Mps	**1.147**	**1.000**	**1.004**	**1.003**	**1.003**
T_C	**0.547**	**0.786**	**0.594**	**0.600**	**0.600**

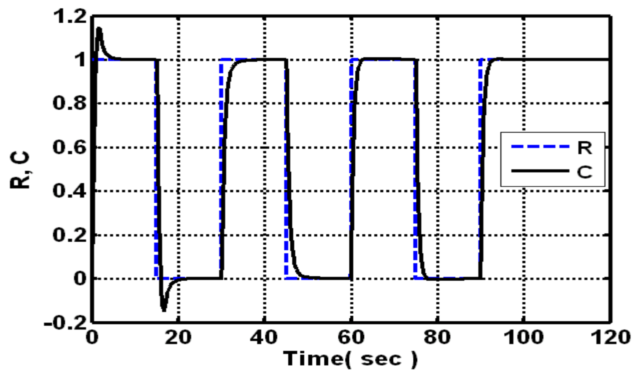

Fig.7. On-line automatic tuning process of the illustrating example for matching peak point and rise time specification.

3. Applications for Quadrotor UAV

3.1. Mathematical model of the Quadrotor

3.1 .1 Coordinate System Definition

The coordinate definition of the Quadrotor is shown in Fig.8 [17] in which shown rotating direction, speed(Ω), arm(l), lifting direction and thrust(T), attitude angle(ϕ,θ,ψ)..etc. one pair of propellers of Quadrotor rotate clockwise and the other pair rotate counter-clockwise.

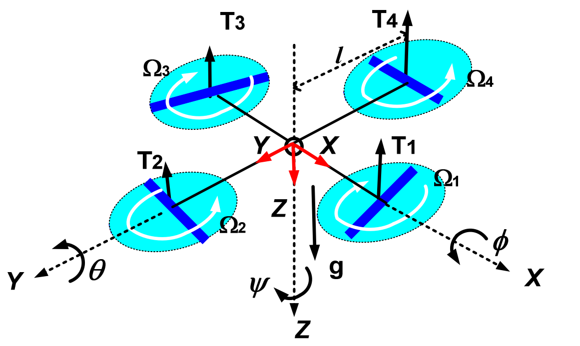

Fig.8. The coordinate definition of the considered system.

3.1.2 Kinematic of Quadrotor

The relationship between the rotating speed and the thrust of propeller [18-21] is given below:

$$T_i = b\Omega_i^2; \Omega_i \geq 0 \qquad \textbf{(7)}$$

Based upon the coordinate definition shown in Fig.8 Total thrust(u_1) of Z-axis, angular moments of rolling, pitching and yawing axes(u_2, u_3, u_4) are in the form of

$$\begin{aligned} u_1 &= -(T_1 + T_2 + T_3 + T_4); \\ u_2 &= l(T_4 - T_2); \\ u_3 &= l(T_1 - T_3) \\ u_4 &= d(-T_1 + T_2 - T_3 + T_4) \end{aligned} \qquad \textbf{(8)}$$

where d is the ratio of thrust to angular moment, l is the position of propeller from central gravity. Eq.(8) can be rewritten as

$$\begin{aligned} u_1 &= -b(\Omega_1 + \Omega_2 + \Omega_3 + \Omega_4); \\ u_2 &= lb(\Omega_4 - \Omega_2); \\ u_3 &= lb(\Omega_1 - \Omega_3) \\ u_4 &= db(-\Omega_1 + \Omega_2 - \Omega_3 + \Omega_4) \end{aligned} \qquad (9)$$

Using Eqs.(7), (8) and (9), the equation of motion of the Quadrotor can be represented as

$$\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} = \frac{1}{m}\begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} + g\begin{bmatrix} -\sin\theta \\ \cos\theta\sin\phi \\ \cos\theta\cos\phi \end{bmatrix} - \begin{bmatrix} qw - rv \\ ru - pw \\ pv - qu \end{bmatrix} \qquad (10)$$

where u,v,w are body-axis velocities ; p,q,r are angular rates ; ϕ,θ,ψ are attitude angles; g is gravity and m is the total mass. F_x, F_y, F_z are total three-axis forces

$$\begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ u_1 \end{bmatrix} + \begin{bmatrix} C_{fx} \\ C_{fy} \\ C_{fz} \end{bmatrix} \qquad (11)$$

where C_{fx}, C_{fy}, C_{fz} are three-axis aerodynamic forces. They can be neglected for low speed operations. In this work, they are neglected. The derivatives of angular rates(p,q,r) are given below:

$$\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} u_2 / I_{xx} + qr\dfrac{I_{yy} - I_{zz}}{I_{xx}} \\ u_3 / I_{yy} + pr\dfrac{I_{zz} - I_{xx}}{I_{yy}} \\ u_4 / I_{zz} + pq\dfrac{I_{xx} - I_{yy}}{I_{zz}} \end{bmatrix} \qquad \textbf{(12)}$$

where I_{xx}, I_{yy}, I_{zz} are moment inertia. Eqs.(10) and (11) are equations of six degree of freedom(6DOF). The derivation of attitude(ϕ,θ,ψ) and position(X,Y,Z) are given as in the form of

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \tan\theta\sin\phi & \tan\theta\cos\phi \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sec\theta\sin\phi & \sec\theta\cos\phi \end{bmatrix}\begin{bmatrix} p \\ q \\ r \end{bmatrix} \qquad \textbf{(13)}$$

$$\begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{bmatrix} = \begin{bmatrix} \cos\theta\cos\psi & \sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi & \cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi \\ \cos\theta\sin\psi & \cos\phi\cos\psi + \sin\phi\sin\theta\sin\psi & \cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi \\ -\sin\theta & \sin\phi\cos\theta & \cos\phi\cos\theta \end{bmatrix}\begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

(14)

3.2 Basic Control System Designs and Verification

3.2.1 Basic Control System Design

The Basic Control System is discussed and designed first. It will be used for guidance laws development. The relationship between thrust(T_i) and total thrust and angular momentum can be evaluated from Eq.(8) and represented as

$$\begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix} = \begin{bmatrix} -1/4 & 0 & +1/2 & -1/4 \\ -1/4 & -1/2 & 0 & +1/4 \\ -1/4 & 0 & -1/2 & -1/4 \\ -1/4 & +1/2 & 0 & +1/4 \end{bmatrix}\begin{bmatrix} -(T_1 + T_2 + T_3 + T_4) \\ T_4 - T_2 \\ T_1 - T_3 \\ -T_1 + T_2 - T_3 + T_4 \end{bmatrix} \qquad \textbf{(15)}$$

Eq.(15) is called as mixer. Using Eqs.(7) and (8), the Basic Control Configuration is designed and shown in Fig.9 in which the Quadrotor dynamic is shown in Fig.8. Eq.(15) gives the maneuverability of the Quadrotor is limited by maximal thrust of the propeller(T_i). Therefore, limitation for command ($u_{1c}, u_{2c}, u_{3c}, u_{4c}$) shown in Fig.9 must be added for preventing uncontrollable nonlinear dynamics.

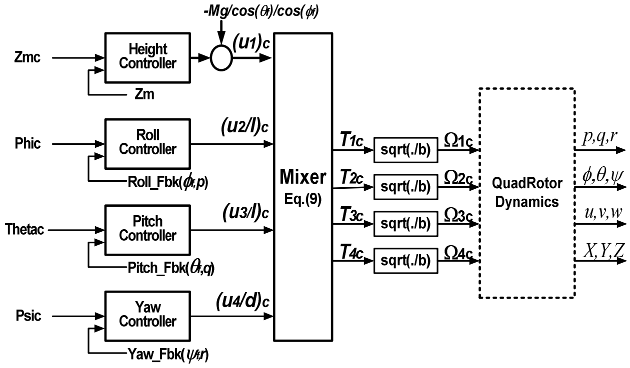

Fig.9 The Basic Control Configuration.

Fig.9 shows that there are four input command and output controls. They are altitude control command (Z_{mc}) and three attitude control command (*Phic,Thetac,Psic*). Feedback datum for command tracking and stabilization are altitude measurement Z_{mf} , vertical speed w_f , angular rates(p_f, q_f, r_f) and attitudes(ϕ_f, θ_f, ψ_f). Outputs of the basic control ($\Omega_{1c}, \Omega_{2c}, \Omega_{3c}, \Omega_{4c}$) are rotating speed of propellers. Proportion plus Integration(PI) control laws are used for command tracking and disturbance rejection. The control laws are given below:

$$u_{1c} = K_{iz}[(K_{oz} + K_{ozi}/s)(Z_{mc} - Z_{mf}) - w_f] - mg/\cos\theta_f \cos\phi_f \quad (16)$$

$$u_{2c} = K_{ip}[(K_{op} + K_{opi}/s)(Phic - \phi_f) - p_f] \quad (17)$$

$$u_{3c} = K_{iq}[(K_{oq} + K_{oqi}/s)(Thetac - \theta_f) - q_f] \quad (18)$$

$$u_{4c} = K_{ir}[(K_{or} + K_{ori}/s)(Psic - \psi_f) - r_f] \quad (19)$$

where *K(*)* are loop gains will be selected. They are determined by bandwidth of command tracking and inner loop gain crossover frequencies of each channel. In this work, 1Hz for gain crossover frequency and 0.2Hz for bandwidth are used.

3.2.2 Verifications for the Basic Control System

In this subsection, the Basic Control System will be designed and verified by digital simulations. The system parameters used is given in Table 2[22]. Fig.10 shows performance of command tracking of the controlled system. Fig.10(a) shows that the Quadrotor flies to 100m height by vertical motion mode first. Fig.10(b) shows time responses of the rolling control; Fig.10(c) shows time responses of the pitching control; and Fig.10(d) shows time responses of the yawing control; Fig.10 shows that the controlled system gives good performance and the couplings between attitude and altitude are limited. This is the major merit of the Quadrotor.

Table 1: Parameters of the quadrotor[20].

Parameters	**Value**	**unit**
m	**4.34**	**Kg**
l	**0.315**	**m**
b	1.2953×10^{-5}	
d	**0.008**	
I_{xx}	**0.0820**	$Kg \cdot m^2$
I_{yy}	**0.0845**	$Kg \cdot m^2$
I_{zz}	**0.1377**	$Kg \cdot m^2$

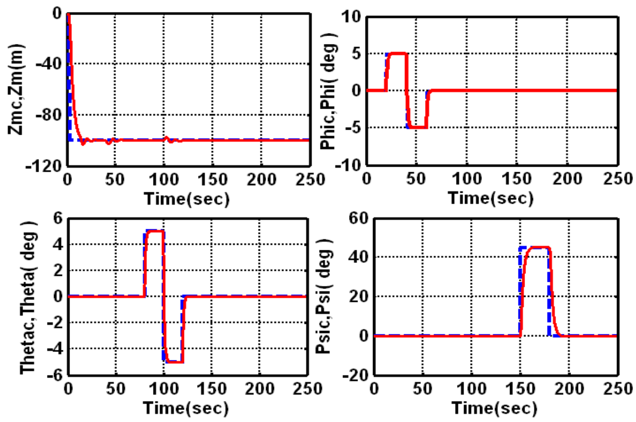

Fig.10. Command tracking performance of the Basic Control System;(a)Heigh control(Zmc, Zm); (b)Rolling control(Phic, Phi); (c)Pitch- ing control(Thetac, Theta); and (d) Yawing control (Psic,Psi).

4.Automatic Regulating for Tuning Control Parameters

In this Section, the automatic regulating process is illustrated by the altitude control system of Quadrotor. The reference model is

$$M(s) = \frac{1}{3s+1} \quad (20)$$

One parameter of the switching point filter $D(s) = K_n/(T_n s + 1)$ is set as $T_n = 3$ and the other parameter K_n will be found by auto regulating process represented as

$$G_n(kT+T) = G_n(kT) \times \{(1-\alpha)[T_S/T_{SM}]^\gamma + \alpha\}; \quad (21)$$

$$K_n = G_n(kT) \quad (22)$$

where T_{SM} is the time constant of the reference model and T_S the time constant of the altitude control system. Slopes of the piecewise linear segments are defined as

$$K_2 = 0.3 \times S_{CK2} \quad (23)$$

$$K_1 = K_2 / 5 \quad (24)$$

S_{CK2} will be found by the another auto regulating process represented as

$$Q_n(kT+T) = Q_n(kT) \times \{(1-\beta)[T_S / T_{SM}]^i + \beta\}; \quad (25)$$

$$S_{CK2} = Q_n(kT) \quad (26)$$

Parameters shown in Fig.2 and Eq.(21) and Eq.(25) are $\alpha = 0.3; \beta = 0.3$; i=7. Initial values are $G_n(0) = 1.0$ and . $Q_n(0) = 1.0$. Table 3 gives the automatic regulating processes. Fig.11 shows time responses of the process. Fig.12 shows automatic regulating processes for different pay load; and same responses will be get for total mass are 3.34, 4.34 and 5.34kg; respectively. It implies that one can adjust the system before mission operation.

Table 3, Fig.11 and Fig.12 show that the controlled system tracks the reference model M(s) quickly. Naturally, automatic regulating process can be applied for three attitude control systems to find the optimal values of loop gains.

Table 3. Time series and time constant of regulating processes.

Period	**0T**	**1T**	**2T**	**3T**	**4T**
$Q_n(*)$	**1.00**	**0.30**	**1.82**	**2.73**	**2.41**
$G_n(*)$	**1.00**	**60.0**	**23.1**	**16.3**	**18.6**
$T_S(s)$	**13. 6**	**2.21**	**2.78**	**3.09**	**3.01**

5. Conclusions

The proposed adaptive piecewise linear controller has been shown that it provides controlled systems are reference input independent and both good performance and fast response were obtained simultaneously. Three segments piecewise linear controller provided a switching algorithm for low gain and high systems; i.e., low gain for performance and high gain for response time. The switching points were dependent on the command tracking errors.

The proposed control scheme has been applied to one illustrating 2nd order altitude control example and Quadrotor. Time responses show that the proposed method gives exact model matching of the reference models after several auto regulating processes. This implied that one can adjust the controlled system before mission operation.

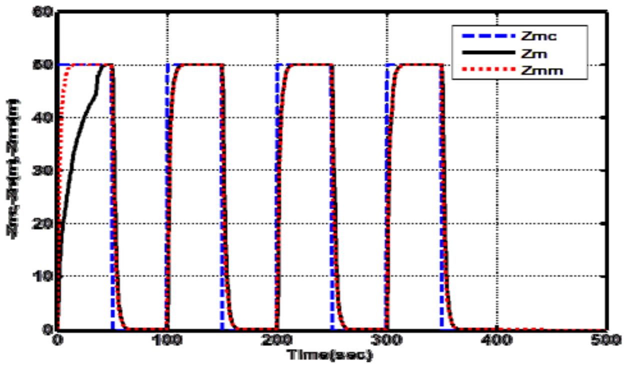

Fig.11. Model Based Automatic Regulation Process for altitude control.

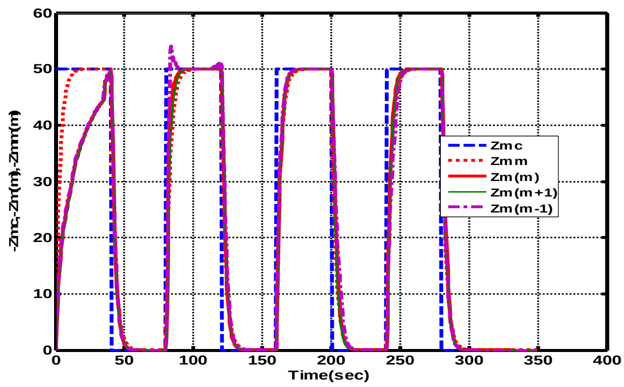

Fig.12. Model Based Automatic Regulation Process of altitude control for different pay loads.

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