Multivariate Control Chart with Exponentially Weighted Moving Log-Likelihood for Monitoring Process Mean and Variability

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Abstract:**-** This study aims to detect various small changes in multivariate control charts. In previous studies, the MEWMA control chart was proposed as a detection of mean vector change, the MEWMC control chart was proposed as a detection of variance covariance matrix change, and the ELR control chart was proposed as a detection of the change of the mean vector and the variance covariance matrix. This study proposes two method using log-likelihood. The first method (MEWML control chart) uses the statistic obtained by directly weighting the log-likelihood. The second method (MEWMML control chart) uses obtained maximum likelihood estimate from log-likelihood using the maximum likelihood method. As a result of Monte Carlo simulations using the ARL evaluation index, the study shows that the MEWML control chart is useful for variance covariance matrix change, and the MEWMML control chart is the most useful for various patterns.

Key-words: - MEWMA, multivariate control chart, log-likelihood, maximum likelihood method, MEWMC, ARL

1 Introduction

In recent years, the complexity of data has been increasing. Thus, it is necessary to detect changes in plant processes and access logs, from multivariable, rather than single variable. Various change patterns—from large to small—exist, and useful methods for detecting them have already been proposed by many scholars. For example, Hotelling (1947) proposed the Hotelling T^2 control chart using the Mahalanobis distance. However, it can only detect large, but not small, changes.

Therefore, Woodall and Ncube (1985) proposed a multivariate cumulative sum control (MCUSUM) chart that accumulates the differences between previous and the current terms as a method for detecting small changes. Then, the theme related MCUSUM was studied by Woodall and Ncube (1985), Healy (1987), Crosier (1988), Pignatiello and Runger (1990), and Hawkins (1991).

Furthermore, Lowry et al. (1992) proposed a multivariate exponentially weighted moving average (MEWMA) control chart using statistic weighted by exponential parameters. The MEWMA control chart is still a topic of research due to its usefulness in studying minor changes. Zou and Qiu (2009) proposed a LEWMA (LASSO-based multivariate EWMA) control chart using LASSO, Jiang et al. (2012) proposed a VS-MEWMA control chart using variable selection, and Nishimura et al. (2015) proposed an AIC-MEWMA control chart that obtained the optimal combination of variables using AIC. In the MEWMA control chart, generally, the population mean vector and the population covariance matrix are assumed to be known. However, Zamba and Hawkins (2006, 2009) conducted numerical experiments assuming that they are unknown.

The MEWMA control chart is a method for detecting the change of the mean; it cannot detect the change of the variance covariance matrix. Therefore, Hawkins and Maboudou-Tchao (2008) proposed a MEWMC control chart to detect the change of the variance covariance matrix. Subsequently, Maboudou-Tchao and Diawara (2013) proposed a new accumulative method, based on penalized likelihood estimators, that is useful to detect small shifts of the variance in a process when sparsity is present. Although the MEWMA control chart is useful for detecting changes of the mean and the MEWMC control chart is useful for detecting changes of the variance covariance matrix, these methods can only be used for particular types of changes.

Therefore, Zhang et al. (2010) proposed an ELR control chart for detecting changes of the mean and the variance covariance matrix. The weighted mean and the variance covariance matrix are substituted into the log-likelihood to derive the ELR statistic. Then, Wang et al. (2014) proposed a new method, based on penalized likelihood estimators, which is useful to detect small shifts of the mean and the variance covariance matrix in a process when sparsity is present.

In the ELR control chart, the final statistic is log-likelihood, exponentially weighted, updating each of the mean vector and the variance covariance matrix. However, it is normal to add the log-likelihood; if the final decision statistic is log-likelihood, it is preferable to weight and update the log-likelihood as it is. Consequently, we propose a multivariate exponentially weighted moving likelihood (MEWML) control chart that weights the log-likelihood to be the statistic in order to detect more various changes. We also propose a multivariate exponentially weighted moving maximum likelihood (MEWMML) control chart that obtains the maximum likelihood estimate of the mean vector and the variance covariance matrix from the log-likelihood using the maximum likelihood method and substitutes it in the log-likelihood.

The rest of this paper is organized as follows. Section 2 introduces previous studies on this topic. Section 3 explains the proposed method. In Section 4, the proposed and the existing methods are tested with simulation experiments, and the accuracy is compared. In Section 5, we perform real data analysis and compare the accuracy of each method. Finally, Section 6 concludes the study.

2 Previous studies

2.1 MEWMA control chart

The MEWMA control chart proposed by Lowry et al. (1992) weights the latest and historical accumulated data by an exponential parameter. This method has excellent detection power for small changes. At this time, we assume a multivariate normal distribution with the situation, as shown in Eq. (1)

$$\begin{cases} \boldsymbol{x}_t \sim N_p(\boldsymbol{\mu}_0, \boldsymbol{\Sigma}_0) \ (t = 1,2,\ldots,\tau) \\ \boldsymbol{x}_t \sim N_p(\boldsymbol{\mu}_t, \boldsymbol{\Sigma}_t) \ (t = \tau+1, \tau+2, \ldots,) \end{cases} \tag{1}$$

We then weight the mean vector using exponential weighting based on $\lambda (0 \le \lambda \le 1)$:

$$\boldsymbol{w}_t = (1-\lambda)\boldsymbol{w}_{t-1} + \lambda(\boldsymbol{x}_t - \boldsymbol{\mu}_0), \ \boldsymbol{w}_0 = 0. \tag{2}$$

The $MEWMA_t$ statistic is expressed in Eq. (3)

$$MEWMA_t = \boldsymbol{w}_t' \boldsymbol{\Sigma}_{w_t}^{-1} \boldsymbol{w}_t. \tag{3}$$

The MEWMA control chart detects the change in the mean based on the statistic in Eq. (3). When the variable weights are all equal, we obtain the variance-covariance matrix in Eq. (4)

$$\boldsymbol{\Sigma}_{w_t} = \left[\frac{\lambda \left\{ 1 - (1-\lambda)^{2t} \right\}}{2-\lambda} \right] \boldsymbol{\Sigma}_0. \tag{4}$$

In general, Eq. (4) can be approximated by Eq. (5)

$$\boldsymbol{\Sigma}_{w_t} = \left[\frac{\lambda}{2-\lambda} \right] \boldsymbol{\Sigma}_0, \tag{5}$$

as $i \to \infty$.

The equation above is the outline of the MEWMA control chart. In $\lambda = 1$, this control chart is equal to the Hotelling T^2 control chart. In general, the MEWMA control chart is said to be able to detect very small changes compared to the Hotelling T^2 control chart (Hotelling, 1947).

2.2 MEWMC control chart

Hawkins and Maboudou-Tchao (2008) proposed the MEWMC control chart in order to detect changes in the variance covariance matrix.

We perform multivariate normalization on the multivariate normal distribution according to Eq. (1). Specifically, using **A,** which satisfies $\boldsymbol{A\Sigma_0 A'} = \boldsymbol{I}_p$, let $\boldsymbol{U}_t = \boldsymbol{A}(\boldsymbol{x}_t - \boldsymbol{\mu}_0)$. When $t < \tau$, we obtain $\boldsymbol{U}_t \sim N(\boldsymbol{0}, \boldsymbol{I}_p)$. Then, the statistic weighted by the exponential parameter λ is

$$\boldsymbol{S}_t = (1-\lambda)\boldsymbol{S}_{t-1} + \lambda \boldsymbol{U}_t \boldsymbol{U}_t', \tag{6}$$

where $\boldsymbol{S}_0 = \boldsymbol{I}_p$. The log-likelihood obtained by using $\boldsymbol{S}_t$ is expressed in Eq. (7)

$$MEWMC_t = tr(\boldsymbol{S}_t) - \log|\boldsymbol{S}_t| - p. \tag{7}$$

The MEWMC control chart detects the change in the variance covariance matrix based on the statistic of Eq. (7).

2.3 ELR control chart

Zhang et al. (2010) proposed the ELR control chart using the mean vector and the variance covariance matrix weighted by the exponential parameter λ. First, we simultaneously weight the mean vector and the variance covariance matrix by the exponential parameter λ:

$$\boldsymbol{w}_t = (1-\lambda)\boldsymbol{w}_{t-1} + \lambda \boldsymbol{U}_t, \quad (8)$$

$$\boldsymbol{S}_t = (1-\lambda)\boldsymbol{S}_{t-1} + \lambda \boldsymbol{V}_t. \quad (9)$$

The MEWMC control chart uses the multivariate normalized vector $\boldsymbol{U}_t$. The variance covariance matrix $\boldsymbol{V}_t$, of each term in $\boldsymbol{S}_t$, is calculated using $\boldsymbol{w}_t$, weighting the mean vector

$$\boldsymbol{V}_t = (\boldsymbol{U}_t - \boldsymbol{w}_t)(\boldsymbol{U}_t - \boldsymbol{w}_t)', \quad (10)$$

where $\boldsymbol{w}_0 = \boldsymbol{0}_p$, $\boldsymbol{S}_0 = \boldsymbol{I}_p$. The final statistic is

$$ELR_t = \mathrm{tr}(\boldsymbol{S}_t) - \log|\boldsymbol{S}_t| + \|\boldsymbol{w}_t\|^2, \quad (11)$$

where, $\|\cdot\|$ represents the L2 norm.

The ELR control chart detects changes in the mean and the variance covariance matrix based on the statistic from Eq. (11).

3 Proposed Control Chart

3.1. Proposed method 1: the MEWML control chart

The MEWMA and MEWMC control charts can detect only specific changes, while the ELR control chart (Zhang et al, 2010), as per previous studies, is able to detect changes in the mean vector and variance covariance matrix.

However, because the log-likelihood, which is useful for measuring the degree of abnormality, must be the final statistic, it should be weighted.

Therefore, we propose a control chart for detection of small changes of various situations by weighting log-likelihood as it is.

We call this proposed method the MEWML control chart.

First, we define the negative log-likelihood in Eq. (12)

$$l(x_t) = p\log(2\pi) + \log|\boldsymbol{\Sigma}_0| + \boldsymbol{U}_t'\boldsymbol{\Sigma}_0^{-1}\boldsymbol{U}_t, \quad (12)$$

where $\boldsymbol{U}_t$ is a multivariate normalized vector of $\boldsymbol{x}_t$, and, $\boldsymbol{\Sigma}_0 = \boldsymbol{I}_p$. The weighted Eq. (12) is defined in Eq. (13)

$$L(\boldsymbol{x}_t) = (1-\lambda)L(\boldsymbol{x}_{t-1}) + \lambda l(\boldsymbol{x}_t), \quad (13)$$

where $L(\boldsymbol{x}_0) = l(\boldsymbol{U}_0 = \boldsymbol{0}_p, \boldsymbol{\Sigma}_0 = \boldsymbol{I}_p) = p\log(2\pi)$.

Then, the final statistic of the MEWML is calculated by the likelihood ratio, defined in Eq. (14)

$$MEWML_t = L(\boldsymbol{x}_t) - L(\boldsymbol{x}_0). \quad (14)$$

The MEWML control chart detects the change in the mean and the variance covariance matrix based on the statistic from Eq. (14).

3.2. Proposed method 2: the MEWMML control chart

The MEWML control chart, which is the proposed method 1, uses the log-likelihood as it is. However, we must originally substitute the maximum likelihood estimate into the log-likelihood.

Therefore, a method that estimates the maximum likelihood estimate using the maximum likelihood method for the log-likelihood is proposed as method 2, called the MEWMML control chart. In this method, we continuously update the log-likelihood weighted exponential parameter. Then, the log-likelihood at the *t* term is as shown in Eq. (15)

$$L(\boldsymbol{x}_t) = \lambda \sum_{k=1}^{t} (1-\lambda)^{t-k} l(\boldsymbol{x}_k). \quad (15)$$

Next, we obtain the mean vector $\mu_t{}^*$ and the variance covariance matrix $\boldsymbol{\Sigma}_t{}^*$ from Eq. (15) using the maximum likelihood method

$$\boldsymbol{\mu}_t{}^* = \frac{\sum_{k=0}^{t}(1-\lambda)^{t-k}\boldsymbol{U}_k}{\sum_{k=0}^{t}(1-\lambda)^{t-k}}, \quad (16)$$

$$\boldsymbol{\Sigma}_t{}^* = \frac{\sum_{k=0}^{t}(1-\lambda)^{t-k}\boldsymbol{V}_k}{\sum_{k=0}^{t}(1-\lambda)^{t-k}}, \quad (17)$$

where, $\boldsymbol{V}_k = \boldsymbol{U}_k\boldsymbol{U}'_k$, $\boldsymbol{U}_0 = \boldsymbol{0}_p$, $\boldsymbol{\Sigma}_0 = \boldsymbol{I}_p$.

The maximum likelihood estimate estimated from maximum likelihood method is substituted into the log-likelihood to be the final statistic

$$MEWMML_t = tr(\boldsymbol{\Sigma}_t{}^*) - \log|\boldsymbol{\Sigma}_t{}^*| + \|\boldsymbol{\mu}_t{}^*\|^2. \quad (18)$$

The MEWMML control chart detects the change in mean and variance-covariance matrix based on the statistic of Eq. (18).

4 Simulation

We conduct a simulation to compare the performance of the existing (the MEWMA, MEWMC, and ELR control charts) versus our proposed methods (the MEWML and MEWMML control charts). We assume Eq. (1) from Section 2 holds, we set τ=100, and state changes in the 101st term.

We set p=2, $\boldsymbol{\mu}_0 = \mathbf{0}_p$,$\boldsymbol{\Sigma}_\mathbf{0} = \boldsymbol{I}_p$.

Then, we also set Eq.(19) and (20)

$$\boldsymbol{\mu}_t = (\mu_1, \mu_2), \tag{19}$$

$$\boldsymbol{\Sigma}_t = \begin{pmatrix} \sigma_1{}^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2{}^2 \end{pmatrix}. \tag{20}$$

Here, we change $(\mu_1, \mu_2, \rho, \sigma_1, \sigma_2)$. Table 1 shows the setting of these parameters in the simulation.

Table 1 Simulation results of each method (ARL value)

$(\mu_1,\mu_2,\rho,\sigma_1,\sigma_2)$	*ELR*	*MEWMA*	*MEWMC*	*MEWML*	*MEWMML*
(0.0, 0.0, 0.0,1.0, 1.0)	201.18	200.62	199.24	200.56	200.70
1.(0.3, 0.3, 0.0, 1.0, 1.0)	44.59	34.67	126.09	99.48	45.24
(0.6, 0.6, 0.0, 1.0, 1.0)	14.81	12.11	34.39	28.91	13.94
(0.9, 0.9, 0.0, 1.0, 1.0)	8.26	7.13	12.35	11.31	7.33
2.(0.0, 0.0, 0.3, 1.0, 1.0)	78.68	173.91	67.63	168.34	85.11
(0.0, 0.0, 0.6, 1.0, 1.0)	24.81	126.02	21.65	119.03	28.11
(0.0, 0.0, 0.9, 1.0, 1.0)	11.97	91.68	10.71	83.13	12.88
3.(0.0, 0.0, 0.0, 1.2, 1.2)	55.66	63.41	34.60	22.87	34.37
(0.0, 0.0, 0.0, 1.3, 1.3)	28.95	43.48	19.08	13.73	19.74
(0.0, 0.0, 0.0, 1.4, 1.4)	17.94	31.79	12.54	9.50	12.99
4.(0.3, 0.3, 0.3, 1.0, 1.0)	32.11	33.60	45.74	86.05	31.48
(0.6, 0.6, 0.6, 1.0, 1.0)	10.96	12.51	12.84	25.91	10.55
(0.6, 0.6, 0.3, 1.0, 1.0)	13.54	12.40	20.72	27.60	12.60
(0.3, 0.3, 0.6, 1.0, 1.0)	18.16	32.67	18.64	67.87	19.00
5.(0.3, 0.3, 0.0, 1.2, 1.2)	27.77	25.00	26.55	18.40	20.43
(0.6, 0.6, 0.0, 1.3, 1.3)	10.91	10.65	11.08	8.69	8.82
(0.6, 0.6, 0.0, 1.2, 1.2)	12.60	11.09	15.14	11.61	10.38
(0.3, 0.3, 0.0, 1.3, 1.3)	19.44	21.49	16.17	25.55	7.63
6.(0.0, 0.0, 0.3, 1.2, 1.2)	36.82	60.44	25.55	22.49	26.85
(0.0, 0.0, 0.9, 1.3, 1.3)	8.81	33.18	7.63	13.62	8.54
(0.0, 0.0, 0.3, 1.3, 1.3)	23.57	42.37	16.81	13.88	17.58
(0.0, 0.0, 0.9, 1.2, 1.2)	9.88	42.68	8.60	19.93	9.85
7.(0.3, 0.3, 0.3, 1.2, 1.2)	22.96	25.68	20.60	18.46	18.02
(0.6, 0.6, 0.6, 1.3, 1.3)	8.88	11.30	8.31	9.05	7.82
(0.3, 0.3, 0.6, 1.3, 1.3)	12.53	22.11	10.70	12.17	10.97
(0.3, 0.3, 0.3, 1.3, 1.3)	17.64	22.45	14.48	12.17	13.77
(0.3, 0.3, 0.6, 1.2, 1.2)	14.73	25.07	13.07	17.86	13.28
(0.6, 0.6, 0.3, 1.2, 1.2)	11.86	11.50	12.76	11.76	9.95
(0.6, 0.6, 0.6, 1.2, 1.2)	9.84	11.90	9.71	11.86	8.82
(0.6, 0.6, 0.3, 1.3, 1.3)	10.40	11.05	9.92	8.80	8.58

We use the average run length (ARL) as the evaluation index because it is a well-established measure to evaluate the performance of control charts. It is obtained by the average lengths of runs beyond the control limit.

The ARL in the normal state is called IC-ARL (in-control ARL), while in the abnormal state it is called OC-ARL (out-of-control ARL). The former should be higher, and the latter, lower. Since, IC-ARL = 200, 370 is generally used for analysis, we define IC-ARL=200. The IC-ARL is set to 200 by 20,000 simulations, allowing it to be the control limit line. The OC-ARL that measures performance under abnormal conditions is used as an evaluation index. Note that the lower the OC-ARL, the better. The number of simulations performed is also 20,000.

Here, we consider the change of only the mean vector in the simulation pattern 1, the change of only the correlation in pattern 2, the change of only the variance in pattern 3, the change of the mean vector and the correlation in pattern 4, the change of the mean vector and the variance in pattern 5, the change of the correlation and the variance in pattern 6, and all parameters changes in pattern 7.

As shown in Table 1, the MEWMA control chart is useful when the mean vector changes, and the MEWMC control chart is useful when the correlation changes. The MEWML control chart, the proposed method 1, is useful when variance changes, but it cannot satisfactorily detect correlation change. Although the ELR and MEWMML control charts basically maintain good accuracy with any simulation pattern, the MEWMML control chart is better in many simulation patterns.

The mean vector is included in the calculation process of the variance-covariance matrix; it is impossible to calculate an optimum value if they are weighted, and update each other individually

On the other hand, in the MEWMML control chart, we update the log-likelihood itself to obtain the maximum likelihood estimate of the final log-likelihood. This way, we can obtain the optimum value of the mean vector and the variance covariance matrix.

5 Real data analysis

In this section, we verify the performance of the proposed methods using real data analysis. The data is obtained by continuously measuring the blood pressure and heart rate of the body, an approach used by Hawkins and Maboudou-Tchao (2008) and Zhang et al. (2010). Table 2 shows the data set.

Hawkins and Maboudou-Tchao (2008) describes as follows. “In this work subjects were equipped with instruments that measure and record physiological variables. The wearer’s blood pressure and heart rate were measured and recorded every 15 minutes for 6 years. Before analysis using statistical process control (SPC) methods, each week’s raw data are condensed into weekly summary numbers, which include mean systolic blood pressure (SBP), mean diastolic blood pressure (DBP), mean of heart rate (HR), and overall mean arterial pressure (MAP). High blood pressure is known to be an indicator for heart attacks and strokes. But it is not just the mean blood pressure that matters; an increase in the variance corresponds to short-term spikes in blood pressure, which can cause blood vessels to rupture, even if the mean is low.”

This particular person’s readings IC distribution is assumed to be known from his (long) historical sequence.

The readings have multivariate normal distribution with a mean vector, a covariance matrix, and a lower-triangular matrix A used to multi-standardize the data to $N(0, I_p)$

$$\boldsymbol{\mu}_0 = (126.61, 77.48, 80.95, 97.97) \quad (21)$$

$$\boldsymbol{\Sigma}_0 = \begin{pmatrix} 15.04 & 8.66 & 10.51 & 12.04 \\ 8.66 & 5.83 & 5.56 & 7.5 \\ 10.51 & 5.56 & 15.17 & 8.79 \\ 12.04 & 7.5 & 8.79 & 10.57 \end{pmatrix} \quad (22)$$

$$\boldsymbol{A} = \begin{pmatrix} 0.26 & 0 & 0 & 0 \\ -0.63 & 1.09 & 0 & 0 \\ -0.38 & 0.21 & 0.36 & 0 \\ -0.45 & -1.03 & -0.13 & 1.42 \end{pmatrix} \quad (23)$$

We set the smoothing constant λ to 0.1 and the IC ARL to 500.

In Section 4, the control limit line is IC-ARL = 200. However, if we use 200 for the real data, the change is detected in any control chart and the difference is difficult to see. Thus we use 500 for the real data simulation.

Table 3 shows the control limit line for each method.

Table 2 Blood pressure data (Hawkins(2018))

	SBP	DBP	HR	MAP
1	128.538	78.357	79.02	99.318
2	130.691	79.283	81.73	99.204
3	128.591	80.756	81.746	100.226
4	132.362	81.412	87.121	103.54
5	133.066	81.294	83.401	103.531
6	127.667	79.634	80.547	100.19
7	130.427	81.06	81.975	102.411
8	124.869	77.676	81.642	98.458
9	129.391	78.729	82.293	99.308
10	127.812	78.731	80.9	98.961
11	129.975	78.267	81.876	100.033
12	128.298	76.632	83.393	98.916
13	124.357	75.246	80.747	96.772
14	131.203	78.153	82.212	100.993
15	127.08	76.428	80.523	97.17
16	123.843	75.742	79.443	94.554
17	125.493	75.994	81.222	96.127
18	121.078	73.581	79.061	92.319
19	121.464	73.946	76.604	91.393
20	128.09	79.588	84.957	98.551
21	127.761	77.35	83.823	98.464
22	125.201	75.729	82.672	96.087
23	128.209	77.828	82.948	101.14
24	125.803	76.389	78.917	95.087
25	105.416	66.306	82.763	86.447
26	124.888	75.474	78.272	96.47
27	130.844	78.732	82.46	101.401
28	128.62	78.548	79.441	100.133
29	126.323	76.791	78.255	96.987
30	126.196	76.908	80.004	97.226
31	123.909	76.408	79.724	95.12
32	128.435	79.45	80.939	98.93
33	127.534	78.408	79.468	98.439
34	127.828	78.941	80.948	98.968
35	125.425	76.901	79.839	96.452
36	129.294	79.797	81.258	99.888
37	127.634	80.445	83.881	100.648
38	128.572	78.868	81.45	99.208
39	124.712	76.758	80.306	95.135
40	126.367	77.354	80.277	96.166

Table 3 The control limit line of each method (IC-ARL=500)

	ELR	MEWMA	MEWMC	MEWML	MEWMML
IC-ARL=500	15.246	1.342	1.674	5.878	1.81

In this case, the change has started since the beginning, and the speed with which you can detect the change is important. The MEWMC, ELR, and MEWML control charts can gradually accumulate changes and detect them, as shown in Figures 2, 3, and 4.

In Figure 1, the MEWMA statistic does not change so much and the MEWMA has only one period beyond the control limit line.

Based on Figure 5, the accuracy of the MEWMML control chart appears to be the highest because the first term is estimated to be abnormal.

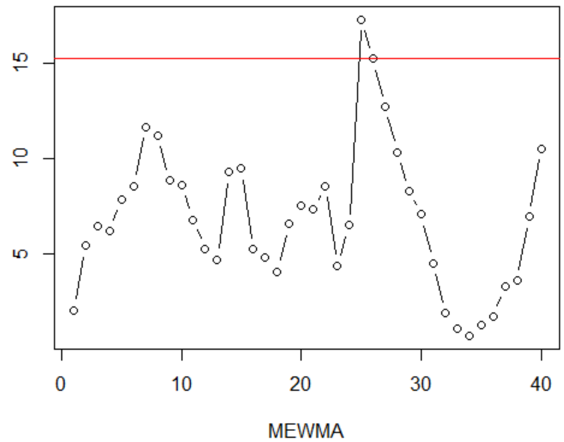

Figure 1 Real data results for MEWMA

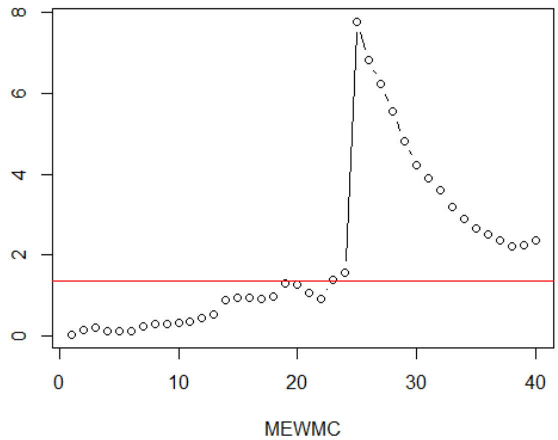

Figure 2 Real data results for MEWMC

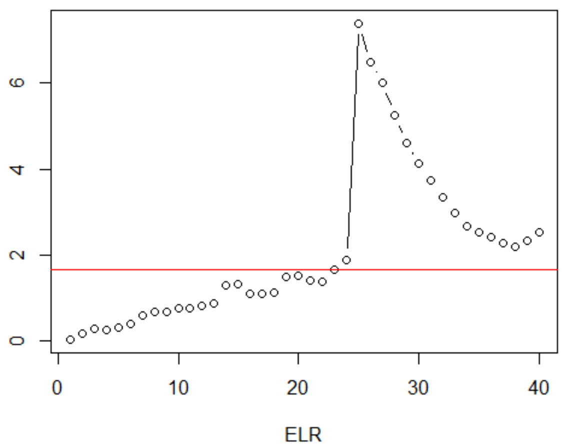

Figure 3 Real data results for ELR

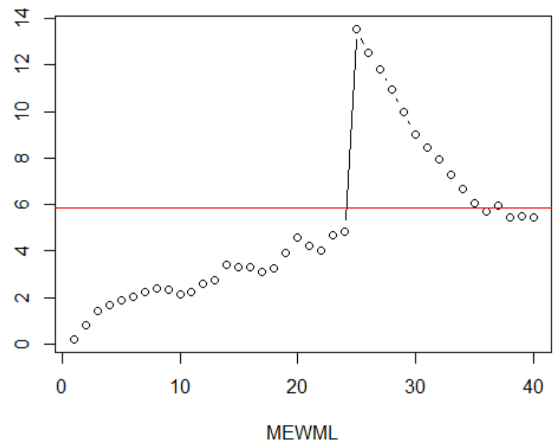

Figure 4 Real data results for MEWML

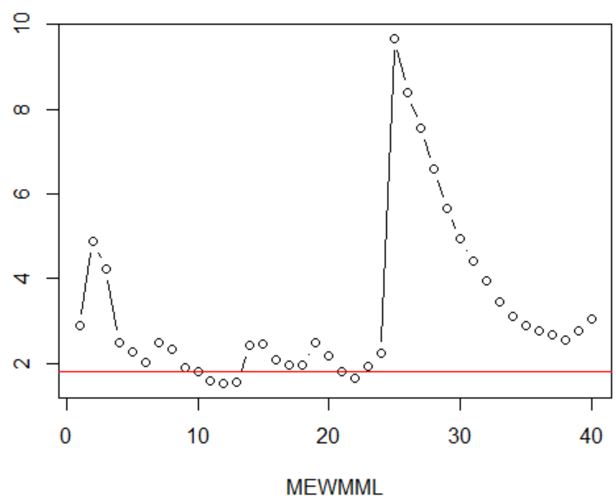

Figure 5 Real data results for MEWMML

6. Conclusion

The MEWMA control chart for mean vector changes, the MEWMC control chart for variance covariance matrix changes, and the ELR control chart for changes in either the mean vector or the variance covariance matrix had already been proposed.

In this paper, we proposed new MEWML and MEWMML control charts to detect changes in either the mean vector or the variance covariance matrix, and we compared their accuracy.

Both proposed methods weight the log-likelihood exponentially. The MEWML control chart uses the weighted log-likelihood as the final statistic. On the other hand, the MEWMML control chart estimates the mean vector and the variance covariance matrix of the weighted log-likelihood, using the maximum likelihood method, and then substitutes them in the log-likelihood. Finally, it is used as the final statistic.

As a result of simulation using the ARL evaluation index, we found that the MEWML control chart is useful for variance changes, while the MEWMML control chart is the most comprehensively useful method. Our proposed method, the MEWMML control chart, is superior in most simulations compared to the ELR control chart from existing studies.

It is possible to detect small changes in various situations promptly by weighting the log-likelihood itself, calculating the maximum likelihood estimate, assigning it to the log-likelihood, and then using it as a statistic.

To conclude, in real-life situations, when a pattern of abnormality cannot be predicted or when various pattern changes are expected, the MEWMML control chart should be the preferred tool. The MEWMA control chart should be used for changes in the mean vector. The MEWMC control chart should be used for changes in the correlation of the variance covariance matrix, while the MEWML control chart should be deployed for changes in the variance of the variance covariance matrix.

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