Mixed Sensitivity Design of Discrete Time PID Controllers

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Abstract—The goal of this paper is to find three sets of algorithm for the coefficients of Discrete Time Proportional Integral Derivative (DT-PID) controllers that simultaneously stabilize the closed-loop system and satisfy a mixed sensitivity constraint. Additive uncertainty modeling describes the uncertainty of perturbed single input single output (SISO) system with an uncertain communication time delay. The DT-PID controllers' coefficients are defined based on the bilinear transformation technique in the frequency domain. The analysis of this procedure depends on the frequency response of discrete time modeling of the system. This methodology applies to an experimental data from a SRV-02 DC motor to regulate the shaft position of the motor.

Key-Words: - Mixed sensitivity, discrete time, PID controller, uncertain time delay

1 Introduction

Proportional integral derivative (PID) controller has an extensive use in many industrial applications. There is a significant attempt in the literatures to determine the set of PID controller parameters that meet certain design goals. Today, design methods that can be applied to an autonomous system are more in demand. In most of the autonomous system design methods that are implemented directly in the digital domain are more prevalent. One of the common challenges in tuning the sets of Discrete Time Proportional Integral Derivative (DT-PID) controller parameters is a communication time delay uncertainly. Time delays create more problem for both robustness and performance of the system response.

Most of the work in this area has concentrated on the design of continuous-time PID controllers [1-5], [7]-[18], [20]-[24] and [26]. In [1], Shafiei and Shenton found all PID controllers that placed the closed-loop poles in certain D-partitions. Bhattacharyya and colleagues determined the PID controller parameters where a rational transfer function model of the system was known [2].

In [3], [5], and [14] Saeki and colleagues looked at different methods for H_∞ design of PID controllers. Sujoldžić and Watkins in [15] and Saeki in [17] developed a method to determine the problem of stabilizing a system based on the frequency response of system. In [17], Saeki introduced a method for finding the number of unstable poles across the boundary of PID controllers.

In [18] and [22], the authors determined the parameters of PID controller by using a metaheuristic algorithm. In [20], Žáková developed constrained pole assignment for the design of PD controllers for a double integrator plant model with time delays or time constant.

In [23], the authors used a fractional PID controller to meet the performance requirement for an active magnetic bearing system. In this paper, an adaptive genetic algorithm was used to determine the PID controller parameters that optimized a multi-objective cost function.

Ho determined a generalization of the Hermite-Biehler theorem for H_∞ PID design [9]. Tantaris, Keel, and Bhattacharyya looked at a similar problem for first-order controllers [16].

In [13], Keel and Bhattacharyya developed PID design for a weighted sensitivity and weighted complementary sensitivity constraint for plants with no poles or zero on the $j\omega$ axis. In [7], Ho and Lin looked at PID controller design for robust performance for a plant that was described by a rational transfer function. Unfortunately, none of these methods that deal with robustness work directly with time-delays.

In [21], Keel and Bhattacharyya allowed for time delay in the nominal model when they investigated the weighted sensitivity and robust stability problems. However, they did not consider the mixed sensitivity problem. Sipahi and Mahmoodi Nia designed the parameters of single-delay system for marginally stable system for a known delay [26] but they did not consider the mixed sensitivity design.

Today, as most controllers are implemented in the digital domain, design methods that are formulated and implemented directly in the discrete time become more important. In [6], the authors used backward differences to design discrete time PID controllers that stabilize the Tchebysheve representation of a discrete time system.

In [19], the authors used delta operator to define a unified approach for the stability region of discrete-time or continuous-time PID controllers design. In [27], DT-PID controller parameters were designed that satisfied the robust stability constraint. In [27], this methodology was applied to an autonomous SailBot real data application.

The current paper is the extension of our previous techniques in [24] and [27]. The goal of current paper is to find all achievable DT-PID controllers that simultaneously stabilize the closed-loop system and satisfy a mixed sensitivity constraint for single input single output (SISO) system with an uncertain communication time delay. Additive uncertainty modeling describes the uncertainty of perturbed system. Additive uncertainty modeling is much easier and often allows for designs with reduced conservativeness in the uncertainty compare to the results for multiplicative uncertainty in [24]. The objective of current paper is to find the three sets of algorithm for the coefficients of DT-PID controllers that allow the closed-loop system to satisfy the mixed sensitivity constraint.

The remainder of this paper is organized in four main sections. The design technique is presented in Section 2. This method is applied to an experimental data of a SRV-02 DC motor from Quanser Consulting, Incorporated to regulate the shaft position of the motor in Section 3. The conclusions of this paper are summarized in Section 4. Finally, the acknowledges are in Section 5.

2 Design Technique

This section introduces the design techniques in two subsections. First the fundamental of system network and equations are introduced. Next the mixed sensitivity design of DT-PID controller design is proposed in two main theorems.

2.1 Fundamental of system network

Consider a SISO system shown in Figure 1, where G_Δ presents the perturbed plant, G_p is the nominal plant, and G_c is the DP-PID controller. The reference input and the error signals are R and Z, respectively. W_P is the sensitivity function weight, W_A is the additive weight, and $|\Delta_A| \leq 1$ is the uncertain perturbation [12].

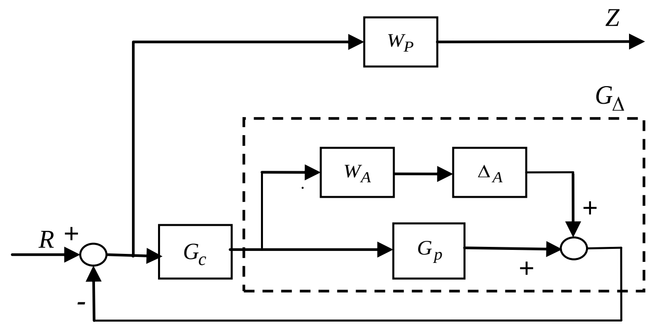

Fig. 1 Block diagram of the system with additive uncertainty

In this network, the nominal continuous time system can be written as:

$$G_p(s) = G_o(s)\, e^{-\tau s}, \tag{1}$$

where $G_o(s)$ is the system transfer function, and τ is an uncertain time-delay. The bilinear discrete-time transformation of the nominal system defines as:

$$G_p(z) = \Gamma\left(G_p(s), T_s\right), \tag{2}$$

where Γ and T_s are the bilinear transformation notation and the sampling period, respectively.

The bilinear transformation DT-PID controller parameter is defined as:

$$G_c(z) = K_p + \frac{K_i}{\frac{2}{T_s}\frac{z-1}{z+1}} + K_d \frac{2}{T_s}\frac{z-1}{z+1}, \tag{3}$$

where K_p, K_i, and K_d are the proportional, integral, and derivative of DT-PID parameters, respectively.

In this paper the discrete time bilinear frequency representation is:

$$\beta = \frac{2}{T_s}\frac{e^{j\omega T_s}-1}{e^{j\omega T_s}+1} =: \frac{2\left(\cos(\omega T_s)-1+j\sin(\omega T_s)\right)}{T_s\left(\cos(\omega T_s)+1+j\sin(\omega T_s)\right)}, \tag{4}$$

where ω is a positive, non zero frequency scale warping range between $0<\omega<\omega_s$, and ω_s is the Nyquist frequency.

The equivalent discrete time bilinear model of system in equation (2) defines in the frequency based on its real and imaginary parts defines as:

$$G_p(\beta) = R_p(\beta) + j\,I_p(\beta). \tag{5}$$

where $R_p(\beta)$, and $I_p(\beta)$ are real and imaginary parts, respectively. The DT-PID controller in equation (3) defines in the frequency domain such as:

$$G_c(\beta) = K_p + \frac{K_i}{\beta} + K_d\,\beta. \tag{6}$$

The bilinear transformation of additive weight, W_A, is and the bilinear transformation of sensitivity function weight, W_P, are defined in terms of their real, $A_A(\beta)$, $C_P(\beta)$, and imaginary $B_A(\beta)$, $D_P(\beta)$, parts such as:

$$W_A(\beta) = A_A(\beta) + jB_A(\beta), \tag{7}$$

and

$$W_P(\beta) = C_P(\beta) + jD_P(\beta). \tag{8}$$

The deterministic values of K_p, K_i, and K_d for which the closed-loop characteristic polynomial is stable has been found in [25] based on delta operator technique.

2.2 Mixed sensitivity DT-PID controller design

In this paper, the problem is to find all achievable DT-PID controllers that satisfy the discrete time mixed constraint of the system in Figure 1 for all $|\Delta_A(j\omega)| \le 1$.

The mixed sensitivity constraint for the SISO system is defined as:

$$\left(|W_A(\beta)G_c(\beta)S(\beta)| + |W_P(\beta)S(\beta)|\right) \le \gamma_0 \qquad \forall\omega, \tag{9}$$

where $S(\beta) = \dfrac{1}{1+G_p(\beta)G_c(\beta)}$ is the discrete time sensitivity function, and γ_0 is the mixed sensitivity constraint and it is a positive scalar less than one. To find DT-PID controller parameters, the complex functions in equation (9) are written in terms of their magnitudes and phase angles as:

$$\left(\begin{array}{l}\left||W_A(\beta)G_c(\beta)S(\beta)|e^{j\left(\angle W_A(\beta)G_c(\beta)S(\beta)\right)}\right| + \\ \left||W_P(\beta)S(\beta)|e^{j\angle W_P(\beta)S(\beta)}\right|\end{array}\right) \le \gamma_0, \quad \forall\omega. \tag{10}$$

If (10) holds, then for each value of ω we write,

$$\left(W_A(\beta)G_c(\beta)S(\beta)e^{j\theta_A} + W_P(\beta)S(\beta)e^{j\theta_P}\right) \le \gamma_0 \qquad \forall\omega, \tag{11}$$

where $\theta_A = -\angle W_A(\beta)G_c(\beta)S(\beta)$ and $\theta_P = -\angle W_P(\beta)S(\beta)$ for some $\theta_A \in [0,2\pi)$ and $\theta_P \in [0,2\pi)$. Consequently, all DT-PID controllers that satisfy equation (9) must lie at the intersection of all controllers that satisfy equation (11) for some $\theta_A \in [0,2\pi)$ and $\theta_P \in [0,2\pi)$ [24].

To accomplish this region, for each value of $\theta_A \in [0,2\pi)$ and $\theta_P \in [0,2\pi)$ all DT-PID controllers on the boundary of equation (11) are found. It is easy to show from equation (11) that DT-PID controllers on the boundary must satisfy the following characteristic equation:

$$P(\beta,\theta_A,\theta_P,\gamma_0) = 0, \tag{12}$$

where,

$$P(\beta,\theta_A,\theta_P,\gamma_0) = 1 + G_p(\beta)G_c(\beta) - \frac{1}{\gamma_0}\left(\begin{array}{l}W_A(\beta)G_c(\beta)S(\beta)e^{j\theta_A} + \\ W_P(\beta)S(\beta)e^{j\theta_P}\end{array}\right).$$

Note that equation (12) reduces to the frequency response of the standard closed-loop characteristic polynomial as $\gamma_0 \to \infty$. Substituting equations (5), (6), (7), (8), and $e^{j\theta_A} = \cos\theta_A + j\sin\theta_A$, and $e^{j\theta_P} = \cos\theta_P + j\sin\theta_P$ into equation (12), the frequency response of this "modified" characteristic polynomial can be rewritten in terms of its real and imaginary functions such as:

$$X_{Rp}K_p + X_{Ri}K_i + X_{Rd}K_d = Y_R, \tag{13}$$

and

$$X_{Ip}K_p + X_{Ii}K_i + X_{Id}K_d = Y_I, \tag{14}$$

where,

$$X_{Rp} = \omega\left(R_p(\beta) - (\Phi)\right),$$

$$X_{Ri} = \frac{\cos(\omega T_s)+1}{2\mathrm{sinc}(\omega T_s)}\left(I_p(\beta) - (\Psi)\right),$$

$$X_{Rd} = \frac{2\omega^2 \mathrm{sinc}(\omega T_s)}{\cos(\omega T_s)+1}\left(-I_p(\beta) + (\Psi)\right),$$

$$Y_R = \omega\left(-1 + (\Omega)\right),$$

$$X_{Ip} = \omega\left(I_p(\beta) - (\Psi)\right),$$

$$X_{Ii} = \frac{\cos(\omega T_s)+1}{2\mathrm{sinc}(\omega T_s)}\left(-R_p(\beta) + (\Phi)\right),$$

$$X_{Id} = \frac{2\omega^2 \mathrm{sinc}(\omega T_s)}{\cos(\omega T_s)+1}\left(R_p(\beta) - (\Phi)\right),$$

$Y_I = (\Lambda)$, and

$$\Omega = \frac{1}{\gamma_0}\left(C_P(\beta)\cos\theta_P - D_P(\beta)\sin\theta_P\right),$$

$$\Lambda = \frac{1}{\gamma_0}\left(C_P(\beta)\sin\theta_P + D_P(\beta)\cos\theta_P\right),$$

$$\Phi = \frac{1}{\gamma_0}\left(A_A(\beta)\cos\theta_A - B_A(\beta)\sin\theta_A\right),$$

$$\Psi = \frac{1}{\gamma_0}\left(A_A(\beta)\sin\theta_A + B_A(\beta)\cos\theta_A\right).$$

This is a three-dimensional system in terms of the controller parameters K_p, K_i, and K_d. In this paper two theorems present to find all achievable DT-PID controller parameters.

Theorem 1: The mixed sensitivity region and stability boundaries in the (K_p, K_i) plane for a fixed value of derivative gain obtains the following curves for the proportional and integral coefficients of DT-PID controller parameters for some $0 < \omega < \omega_s$, $\theta_A \in [0, 2\pi)$, and $\theta_P \in [0, 2\pi)$:

The discrete time proportional:

$$K_p(\beta,\theta_A,\theta_P,\gamma_0) = \frac{\begin{pmatrix} -R_p(\beta)(1+\Omega) - I_p(\beta)(\Lambda) + (\Phi) - \\ \left(\Upsilon_1\left(\sin(\theta_P - \theta_A)\right) + \Upsilon_2\left(\cos(\theta_P - \theta_A)\right)\right)\end{pmatrix}}{(\Theta)}, \tag{15}$$

and the discrete-time integral:

$$K_i(\beta,\theta_A,\theta_P,\gamma_0) = \frac{4\omega^2 \mathrm{sinc}^2(\omega T_s)}{\left(\cos(\omega T_s)+1\right)^2}\tilde{K}_d + \frac{2\omega \mathrm{sinc}(\omega T_s)\begin{pmatrix} -R_p(\beta)(\Lambda) + I_p(\beta)(-1+\Omega) + (\Phi) + \\ \left(\Upsilon_1\cos(\theta_P - \theta_A) + \Upsilon_2\sin(\theta_P - \theta_A)\right)\end{pmatrix}}{\left(\cos(\omega T_s)+1\right)(\Theta)}, \tag{16}$$

where,

$$\Upsilon_1 = \frac{1}{{\gamma_0}^2}\left(A_A(\beta)D_P(\beta) - B_A(\beta)C_P(\beta)\right),$$

$$\Upsilon_2 = \frac{1}{{\gamma_0}^2}\left(A_A(\beta)C_P(\beta) + B_A(\beta)D_P(\beta)\right),$$

$$\Theta = \left|G_p(\beta)\right|^2 - \frac{2}{\gamma_0}\left(R_p(\beta)(\Psi) + I_p(\beta)(\Phi)\right) + \frac{1}{{\gamma_0}^2}\left|W_A(\beta)\right|^2,$$

and,

$$\left|G_p(\beta)\right|^2 = {R_p}^2(\beta) + {I_p}^2(\beta),$$

$$\left|W_A(\beta)\right|^2 = {A_A}^2(\beta) + {B_A}^2(\beta).$$

Proof: The boundary of characteristic equation in equation (12) can be found in the (K_p, K_i) plane for a fixed value of K_d. After setting K_d to the fixed value $\tilde{K}_d$, equations (13) and (14) can be rewritten as:

$$\begin{bmatrix} X_{Rp} & X_{Ri} \\ X_{Ip} & X_{Ii} \end{bmatrix}\begin{bmatrix} K_p \\ K_i \end{bmatrix} = \begin{bmatrix} Y_R - X_{Rd}\tilde{K}_d \\ Y_I - X_{Id}\tilde{K}_d \end{bmatrix}. \tag{17}$$

Solving equation (17) for some $0 < \omega < \omega_s$, $\theta_A \in [0, 2\pi)$, and $\theta_P \in [0, 2\pi)$, gives the equations (15) and (16) for the discrete time proportional and integral coefficients of DT-PID controller. This is completed the proof. ■

Theorem2: The mixed sensitivity region and stability boundaries in the (K_p, K_d) plane for a fixed value of integral term gives the following curves for the derivative coefficients of DT-PID controller for some $0 < \omega < \omega_s$, $\theta_A \in [0, 2\pi)$, and $\theta_P \in [0, 2\pi)$.

$$K_d(\beta, \theta_A, \theta_P, \gamma_0) = \frac{\left(\cos(\omega T_s)+1\right)^2}{4\omega^2 \mathrm{sinc}^2(\omega T_s)}\tilde{K}_i + \frac{\left(\cos(\omega T_s)+1\right)\begin{pmatrix} R_p(\beta)(\Lambda) - I_p(\beta)(-1+\Omega) - (\Phi) - \\ \left(\Upsilon_1 \cos(\theta_P - \theta_A) + \Upsilon_2 \sin(\theta_P - \theta_A)\right) \end{pmatrix}}{2\omega \mathrm{sinc}(\omega T_s)(\Theta)}. \tag{18}$$

Proof: The boundary of characteristic equation in equation (12) can be found in the (K_p, K_d) plane for a fixed value of K_i. After setting K_i to the fixed value $\tilde{K}_i$, equations (13) and (14) can be rewritten as:

$$\begin{bmatrix} \omega R_p(\omega) & -\omega^2 I_p(\omega) \\ \omega I_p(\omega) & \omega^2 R_p(\omega) \end{bmatrix}\begin{bmatrix} K_p \\ K_d \end{bmatrix} = \begin{bmatrix} Y_R - I_p(\omega)\tilde{K}_i \\ Y_I + R_p(\omega)\tilde{K}_i \end{bmatrix}. \tag{19}$$

Solving equation (19) for some $0 < \omega < \omega_s$, $\theta_A \in [0, 2\pi)$, and $\theta_P \in [0, 2\pi)$, gives the same expression as equation (15) for the proportional coefficients, and it gives equation (18) for the derivative coefficients of DT-PID controller. This is completed the proof. ■

3 Experimental Example

In this section, a DT-PID controller is designed to regulate the shaft position of a SRV-02 DC motor in Figure 2 from Quanser Consulting, Incorporated [28]. The feedback loop has an uncertain communication delay between $0 < \tau < 0.2$ seconds. The goal is to find all DT-PID controllers that stabilize the system and satisfy the mixed sensitivity constraint in equation(9), where the mixed sensitivity constraint is $\gamma_0 = 1$, and the sampling period is $T_s = 0.05$ seconds. The closed-loop step response is required to have an overshoot less than 5% and a settling time less than 5 seconds.

Fig. 2. SRV-02 DC motor in from Quanser Consulting, Incorporated

The continuous time, dash line, and discrete time bilinear transformation, star line, experimental magnitude frequency response of the SRV-02 DC motor with the mean value of communication delay of 0.1 seconds are shown in Figure 3.

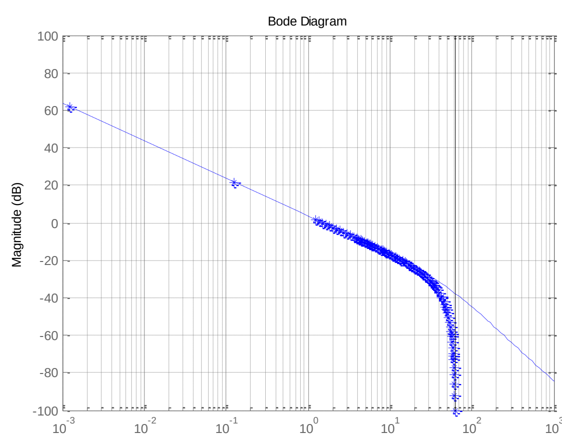

Fig. 3. Open loop frequency response of SRV-02 DC motor with a communication delay of 0.1 seconds.

This frequency response is corresponded to a continuous time such as:

$$G_p(s)=\frac{1.53}{s(0.024s+1)}e^{-0.1s}, \tag{20}$$

and discrete time bilinear transfer function of equation (20) is:

$$G_p(z)=\frac{0.01895z^2+0.0379z+0.01895}{z^2-1.008z+0.007937}z^{-2}. \tag{21}$$

The additive weight is chosen to bound the additive errors. Note, additive uncertainty modeling often allows for designs with reduced conservativeness in the time delay uncertainty. This will increase the size of the set of DT-PID controllers that robustly meet the performance requirements. The bilinear discrete time transformation of additive weight has been designed as:

$$W_A(z)=\frac{0.04068z^2+0.08136z+0.04068}{z^2-0.06536z-0.01961}. \tag{22}$$

The sensitivity weight is chosen to satisfy the performance requirement for the closed-loop system. The bilinear discrete time transformation of sensitivity weight has been designed such as:

$$W_P(z)=\frac{0.018828\,(z-0.9488)\,(z+1)}{(z-0.9598)\,(z-0.9213)}. \tag{23}$$

The procedures to design all PID controller parameters for a fixed value of derivative term using first theorem are as following steps:

1. Equations (5), (7), and (8) are used to find the real and imaginary parts of equations (21), (22), and (23), respectively.
2. Substitute the results of step 1 into equations (15) and (16) in the (K_p, K_i) plane for a fixed value of $\tilde{K}_d=0.02$.
3. All DT-PID controllers that satisfy the mixed sensitivity constraint in equation (9) are found by setting $\gamma_0=1$ into equations (15) and (16) for some interval of $\theta_A\in[0,2\pi)$, $\theta_P\in[0,2\pi)$, and the frequency range of $0<\omega<62.83$.
4. Find the intersection of all regions from step 3; as it shows in dark green area in Figure 4.
5. The DT-PID stability boundary of the nominal system can be found by setting $\gamma_0=\infty$ into equations (15) and (16) as it shows in bold red lines in Figure 4.
6. The region that satisfies both the mixed sensitivity constraint and the nominal stability boundary is shown in Figure 4.
7. The intersection of all regions inside the nominal stability boundary of the (K_p, K_i) plane is shown in the dark green is the mixed sensitivity constraint area.
8. All of the selected DT-PID controllers in the dark green area satisfy the mixed sensitivity constraint in equation (9).

To verify the results, an arbitrary controller from the dark green area is chosen that is giving the DT-PID controller such as:

$$G_c(z)=2.07+\frac{0.13}{\left(\frac{2}{0.05}\right)\left(\frac{z-1}{z+1}\right)}+0.02\left(\frac{2}{0.05}\right)\left(\frac{z-1}{z+1}\right). \tag{24}$$

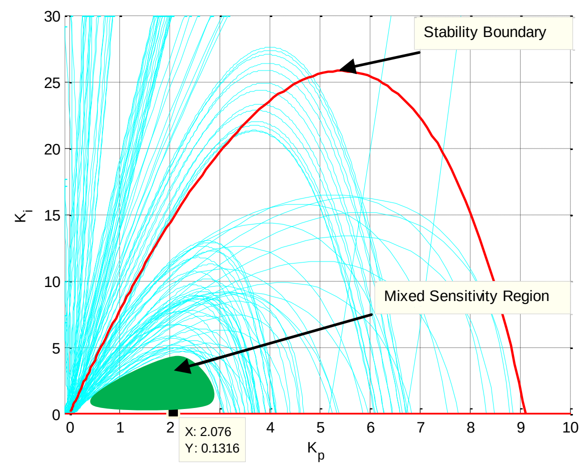

Fig. 4. Discrete-time stability boundary and mixed sensitivity region in the (K_p, K_i) plane

Substituting equations (21), (22), (23), and (24), into (9) gives $\left(|W_A(\beta)G_c(\beta)S(\beta)|+|W_P(\beta)S(\beta)|\right)\le 0.62$. As the magnitude of mixed sensitivity of the system is less than one the design goal is met. The closed loop step response with the DT-PID controller in (24) shown in Figure 5. As can be seen the closed loop step response has a setting time of 0.651seconds and an overshoot of 1.92%. The closed loop step response has met all the performances requirements.

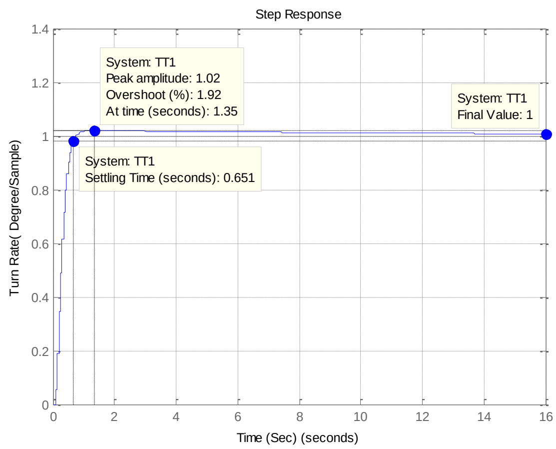

Fig. 5. The closed loop step response with DT-PID controller in (24).

The second theorem follows the same procedures as first theorem in steps 1-8th, but it uses equations (15) and (18) in the (K_p, K_d) plane for a fixed value of $\tilde{K}_i = 0.04$. As discussed previously, the DT-PID stability boundary of the nominal system can be found by setting $\gamma_0 = \infty$ in equations (15) and (18). In this plane, the DT-PID controller is designed to satisfy the mixed sensitivity constraint in equation (9). This objective is achieved by setting the constraint, $\gamma_0 = 1$, in equations (15) and (18) for some interval of $\theta_A \in [0, 2\pi)$, $\theta_P \in [0, 2\pi)$, $0 < \omega < 62.83$, and finding the intersection of all regions.

The region that satisfied the mixed sensitivity constraint and the nominal stability boundary are shown in Figure 6. The nominal stability boundary is shown in red-bold line. The mixed sensitivity area is the intersection of all regions inside the nominal stability boundary of the (K_p, K_d) plane as it shows in the dark green area.

All of the selected DT-PID controllers in the green area are satisfied the mixed sensitivity constraint in equation (9). To verify the results, an arbitrary controller from this region is chosen that is giving the DT-PID controller as:

$$G_c(z) = 2.01 + \frac{0.04}{\left(\frac{2}{0.05}\right)\left(\frac{z-1}{z+1}\right)} + 0.03\left(\frac{2}{0.05}\right)\left(\frac{z-1}{z+1}\right). \tag{25}$$

Substituting equations (21), (22), (23), and (25), into (9) gives $\left(\left|W_A(\beta)G_c(\beta)S(\beta)\right| + \left|W_P(\beta)S(\beta)\right|\right) \leq 0.61$. The mixed sensitivity magnitude frequency response with the DT-PID controller in the equation (25) is shown in Figure 7. As the magnitude of mixed sensitivity system is less than one, the design goal is met.

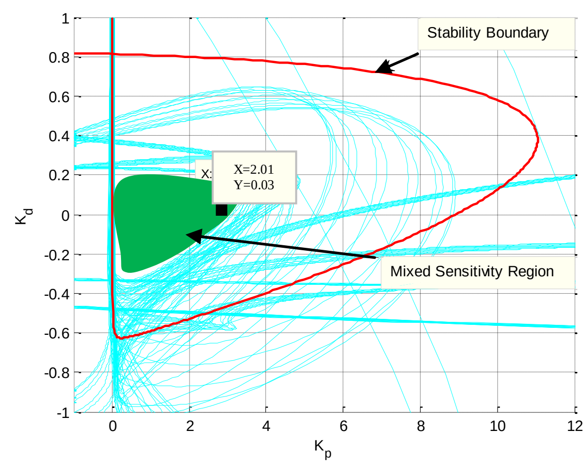

Fig. 6. Discrete-time stability boundary and mixed sensitivity region in the (K_p, K_d) plane

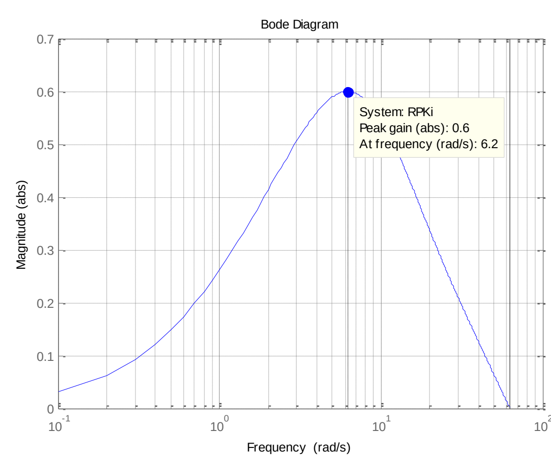

Fig. 7. The mixed sensitivity magnitude frequency response of the system with DT-PID controller in equation 25.

The closed loop step response with DT-PID controller in the equation (25) is shown in Figure 8. As can be seen the closed loop step response has a setting time of 0.76 seconds, no overshoot, and zero steady state error. The closed loop step response has met all the performances requirements.

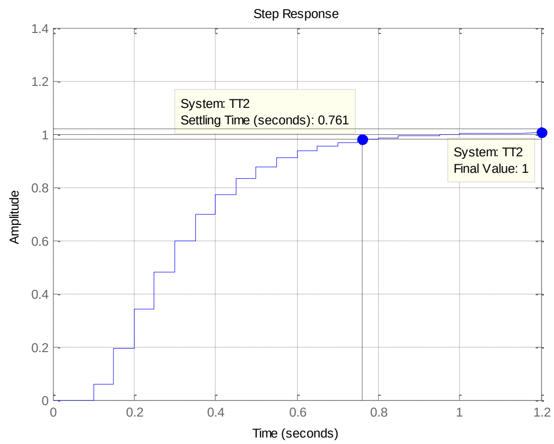

Fig. 8. The closed loop step response with DT-PID controller in (25).

The step responses of the closed-loop system with the DT-PID controller in equation (25) for various time delays in the interval of $(0.05, 0.2)$ seconds are shown in Figure 9.

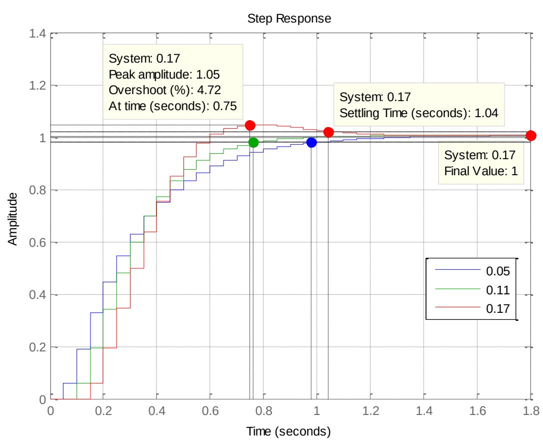

Fig. 9. The closed loop step response with DT-PID controller in (25) for various time delays

As can be seen, the closed-loop step responses all have an overshoot less than 5% and a setting time less than 5 seconds, and zero steady state errors. The maximum setting time is 1.04 seconds and the maximum percent overshoot is 4.72%, both are corresponded to the maximum time delay in the system.

4 Conclusions

In this paper three sets of algorithm were introduced for the coefficients of DT-PID controllers that stabilized and satisfy the mixed sensitivity constraint for the closed-loop system. Additive uncertainty modeling was used to describe the uncertainty of time delay of single-input-single-output (SISO) system with un uncertain time-delay. Additive uncertainty modeling often allowed for designs with reduced conservativeness. This modeling increased the size of all DT-PID controllers that met the mixed sensitivity constraint. The frequency domain bilinear transformation of DT-PID controllers was obtained here since the analysis of system depends on the frequency response of discrete time modeling of the system. An experimental data taken from a SRV-02 DC motor with an uncertain communication time-delay in the feedback path was used to demonstrate the application of this methodology.

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