

# A Multiple-Input Multiple-Output (MIMO) fuzzy nets approach for part quality prediction and planning

AHMED BUFARDI, OLCAY AKTEN and MUHAMMAD ARIF

Laboratory for Computer-Aided Design and Production  
Swiss Federal Institute of Technology Lausanne  
CH-1015 Lausanne  
SWITZERLAND

PAUL XIROUCHAKIS

Department of Design Manufacture and Engineering Management  
University of Strathclyde  
75 Montrose Street, Glasgow, G1 1XJ  
UNITED KINGDOM

ROBERTO PEREZ

GF Machining Solutions  
Agie Charmilles SA

Rue du Pré-de-la-Fontaine 8, 1217 Meyrin 1, Genève  
SWITZERLAND

[Paul.Xirouchakis@Strath.ac.uk](mailto:Paul.Xirouchakis@Strath.ac.uk) <https://www.strath.ac.uk/staff/xirouchakispaulprofessor/>

*Abstract:* In this paper, we present for the first time a general multiple-input multiple output (MIMO) fuzzy fuzzy-nets framework where the output parameters have different sets of inputs parameters and conflicting rules are generated. This is a significant departure from current fuzzy net approaches which are restricted to only one output. The proposed framework is based on a new model of the rule base architecture and of the predictive algorithm facilitating the resolution of conflicting rules. We demonstrate the application of the new framework for part quality prediction and input process parameter selection in Wire Electrical Discharge Machining (WEDM) with two part quality characteristics (surface roughness and average recast layer thickness) and three input process parameters (pulse-off time, feed rate and voltage); as a result a new technological paradigm for process planning in WEDM has been validated where not only surface roughness (the current situation) but also the average recast layer thickness is optimally planned.

*Key-Words:* Fuzzy nets, WEDM, surface roughness, recast layer thickness

## 1 Introduction

Manufacturing processes are not “perfect” in the sense that they very often produce parts with different quality defects. Because of the complexity of interaction between process parameters and part quality defects, the causes of these defects are not easy to find or to fix.

Chryssolouris [1] described manufacturing systems as open dynamic systems comprising several interacting components and having strong bi-directional relation with their environment, and where the main relations between performance measures and decision variables cannot always be expressed in an analytical way.

In this paper, we focus on wire electrical discharge machining (WEDM) which is a complex machining system that is difficult to fully understand due to the stochastic and nonlinear nature of the process and the multiple parameters it involves. Aspinwall et al. [2] reported that in highly complex modern EDM systems, there are up to 25 individual parameters (electrical/mechanical, etc.) influencing the cutting performance.

The main defects related to surface quality of WEDMed parts include: occurrence of ‘surface lines’, surface roughness, and recast layer (also called white layer) (Fig. 1). In this paper, we focus on two defects: surface roughness and recast layer. The occurrence of ‘surface lines’ which is a random

and localized phenomenon which requires an online prediction approach will be addressed in a separate publication.



Fig. 1: Main surface defects during surfacing in WEDM

To deal with surface roughness and recast layer thickness prediction in WEDM, we consider a fuzzy logic based approach called fuzzy-nets approach. This approach allows predicting the values of surface roughness and average recast layer thickness on the basis of the initial conditions represented by the values of the WEDM parameters. In the literature, the fuzzy-nets approach was applied to other machining processes such as milling and turning for one quality characteristic but not to WEDM and multiple quality characteristics.

In the fuzzy-nets approach, the relationships between the inputs and the outputs of the manufacturing process are described through a collection of fuzzy control rules involving linguistic variables rather than a complicated dynamic mathematical model (differential equations).

According to Hammell II and Sudkamp [3], the set of fuzzy rules allows representing the nonlinear behavior of the process and provides a functional approximation of the relationships of the underlying system.

Fuzzy models have been designed to represent approximate or imprecise relationships in complex systems and have been successfully employed in control systems, expert systems, and decision analysis [3].

Fuzzy logic (FL) is a kind of multi-valued logic derived from fuzzy set theory developed by Zadeh [4]. In FL, a statement has a degree of truth between 0 and 1 whereas in classical logic a statement is either true (degree of truth equals to 1) or false (degree of truth equals to 0). FL has the capability to deal with reasoning that is approximate/imprecise rather than precise. The “imprecision” as used in fuzzy set theory is meant as a sense of vagueness rather than lack of knowledge about the value of a parameter [5].

The paper is organized as follows. Section 2 is devoted to the survey of the related literature. The definition of the fuzzy-nets model is provided in Section 3. Section 4 is presenting the multiple-input multiple-output (MIMO) fuzzy-nets framework. The

case-study is given in Section 5. Finally, Section 6 presents concluding remarks about the presented approach.

## 2 Review of Related Literature

Wang and Mendel [6] reported that the fuzzy-nets approach is suitable for complex control systems for which no mathematical model exists, or, the mathematical model is strongly non-linear so that a design method does not exist. Manufacturing system is one of these systems.

The fuzzy-nets approach has the ability to account for vagueness/imprecision and nonlinear behavior which are common characteristics to manufacturing processes. Moreover the implementation of FL based approaches is relatively easy and computationally less demanding. Moreover, the neural network (NN) structure of the system provides the capability of training/learning the data from machining processes.

The fuzzy-nets approach is widely applied in the literature mainly for the prediction of machining quality characteristics and/or adjustment of process parameters in the case where the quality characteristics are not meeting the performance requirements (see e.g., Chen and Black [7], Chen and Lou [8], Chen and Savage [9], Kirby and Chen [10], Kirby et al. [11], Yang [12], Yang and Chen [13], Yang et al. [14], Zhu and Chen [15]).

Chen and Black [7] used the fuzzy-nets approach for in-process monitoring of tool breakage in end-milling operations.

Kirby and Chen [10] proposed a fuzzy-nets-based surface roughness prediction system in turning operations.

Kirby et al. [11] applied the fuzzy-nets approach to predict the surface roughness in turning and to adapt the feed rate of the turning operation in order to bring the surface roughness into the acceptable range in the case where the predicted value of the surface roughness is higher than the desired specification.

Yang [12] and [14] used the fuzzy-nets approach for online prediction of the surface roughness in end milling and adjustment of the feed rate in the case where the predicted value of the surface roughness is not meeting the desired level of quality.

The theoretical background of the fuzzy-nets approach is mainly based on the work of Wang and Mendel [6] who proposed a framework for:

- generating fuzzy rules from input-output data pairs,
- collecting the generated rules and linguistic rules into a common fuzzy rule base,

- constructing a final fuzzy logic system (FLS) based on the combined fuzzy rule base.

Generating fuzzy rules from input-output data pairs may have two problems: (i) conflicting rules with the same “if” part and different “then” parts and (ii) missing rules to cover all fuzzy regions of the input parameters. To deal with these two problems, Chen [16] integrated in the fuzzy-nets approach, procedures for resolving conflicts between fuzzy rules and multiple regression models for generating missing fuzzy rules.

Further improvements of the fuzzy-nets approach were introduced later in different publications: Yang [12], Kirby et al. [11] and Yang et al. [14]. The main improvement relates to the addition of an adaptive system to the fuzzy-nets approach in order to adjust the input parameters in the case where the predicted value of the output parameter is not meeting the required performance.

### 3 Defining the Fuzzy-Nets Model

#### 3.1 The main assumption

The fuzzy-nets approach is a rule-based approach. Hence, the main requirement for the application of the fuzzy-nets approach for surface roughness and average recast layer thickness prediction in WEDM is the existence of a set of input parameters which have a significant effect on surface roughness and recast layer thickness that can be captured by a collection of fuzzy rules.

The fuzzy rules can be generated from training data obtained from dedicated experiments.

#### 3.2 The main aspects involved in the fuzzy-nets model

The main aspects of the fuzzy-nets model are shown in Fig. 2.

The NN-like structure of the fuzzy-nets approach allows the use of training data from experiments to generate the predictive fuzzy rules. The Mamdani fuzzy inference system uses the fuzzy rules to determine the output value corresponding to a set of input values.

#### 3.3 Inputs and outputs of the fuzzy-nets prediction approach

The main inputs and outputs of the fuzzy-nets prediction approach are presented in Table 1.

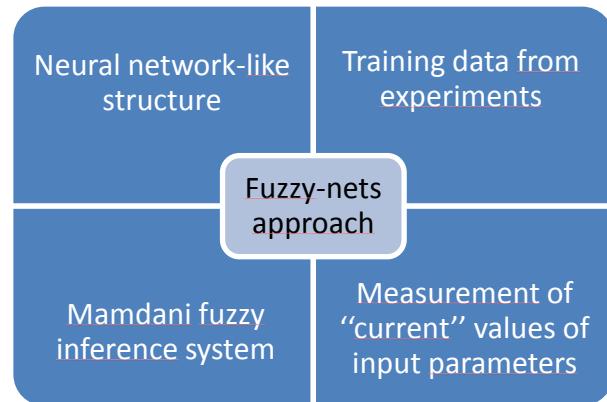


Fig. 2: The main aspects of the fuzzy-nets model.

Table 1: Inputs and output parameters of the fuzzy-nets predictive system

Parameter	Type of parameter	Definition	Characteristics
Vital quality parameter (VQP)	Output parameter	Part quality characteristic that shall be kept within specifications in order to assure zero-defect manufacturing	Parameters that need to be kept within certain range to ensure a desired quality level of the machined part
Vital process parameter (VPP)	Input parameter	Process parameter that affects identified VQCs of the product	These are the potential parameters to adjust in the case of deviation of VQC from the acceptable limits

#### 3.4 The general structure of the fuzzy-nets prediction approach

The main components of the fuzzy-nets prediction system are shown in Fig. 3.

The components of the fuzzy-nets prediction system in Fig. 3 are described in the following sub-subsections.

##### 3.4.1 Fuzzification

Fuzzification is the first component of the fuzzy-nets predictive system meaning that when an input parameter enters the fuzzy-nets predictive system the first operation it undergoes is fuzzification.

According to (Jantzen [17]) fuzzification converts the crisp values obtained from the input signals into degrees of membership by a lookup in one or

several membership functions (MFs). There is always a degree of membership which can be also 0. However, the MFs can be arranged in a way to always have at least one membership degree (MD) different from 0. In our case, the MFs are arranged in a way that the fuzzy regions of each input/output domain constitute a fuzzy partition. According to Hammell II and Sudkamp [3], the decomposition of a domain U into fuzzy sets  $A_1, A_2, \dots, A_n$  is a fuzzy partition if,  $\sum_{i=1}^n \mu_{A_i}(u) = 1, \forall u \in U$ .

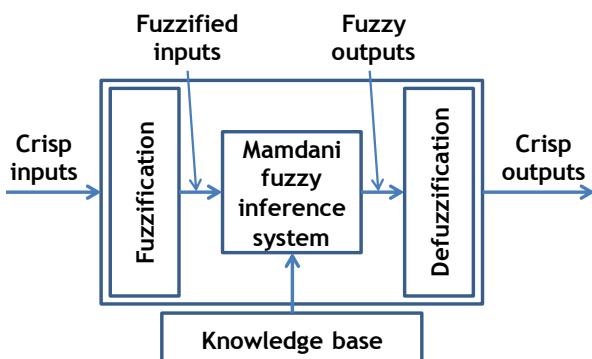


Fig. 3: Structure of fuzzy-nets prediction system  
(adapted from Jantzen [17])

### 3.4.2 Knowledge base

The knowledge base comprising the fuzzy rule base and the data base is used to support the fuzzy inference module in the predictive system. The data base contains mainly the definitions of the MFs of the fuzzy sets associated with the linguistic variables.

The number of fuzzy terms in the input space determines the maximum number of fuzzy rules which can be considered.

The triangular MFs are the most used for the fuzzy regions associated to the input and output variables. Here also we consider triangular MFs.

When partitioning a domain of an input or output parameter into fuzzy terms, there are two main conditions to be fulfilled:

- Each MF should overlap with only the closed MFs (one at the left-hand side when it exists and one at the right-hand side when it exists),
- For each input, the corresponding membership values in all fuzzy MFs should sum-up to 1.

In the database, are indicated for each input/output variable:

- Its fuzzy regions

- Their number
- Their localization within the domain interval
- The MF of each fuzzy region
  - The center point of the (triangular) MF
  - Its width

The fuzzy rule base can be developed in three different ways: (i) directly from operators' experience, (ii) simulated model, or (iii) experimental data.

In our case, the fuzzy rule base is generated from experimental data. Obtaining information from human experts (process designers, operators, etc.) in the required format is a difficult task and the use of simulation requires a well-defined model of the problem under consideration which is also difficult to obtain in the case of complex processes.

According to Li et al. [18] there are two main training schemes which can be used in fuzzy-nets systems: fuzzy logic membership schemes, and input/output data types. In our case, the training scheme considered is the fuzzy logic membership scheme (Kirby et al. [11]). After having generated the fuzzy rules from experimental data and after resolving the possible conflicts between certain rules, there may be missing rules in the rule base. In fuzzy-nets systems with fuzzy logic membership as training scheme, the multiple regression model (MRM) is the most used technique to generate the missing rules (see e.g., Yang and Chen [13], Yang et al. [14], Kirby et al. [11], Zhu and Chen [15], Kirby and Chen [10]).

Part of the fuzzy rule base is obtained from experimental data and the other part is obtained applying a rule base completion technique such as MRM (Multiple Regression Model) to the experimental data (Fig. 4).

### 3.4.3 Fuzzy inference system

The fuzzy inference system (FIS) is the core element of the fuzzy-nets prediction system. The inference technique determines the degree with which each fuzzy rule affects the outcome (Hammell II and Sudkamp [3]). It also handles the way in which the fuzzy rules are combined to predict the value of the VQC for a given set of values of VPPs. The recommendations of each rule are first considered independently, and then combined together. The output is obtained by summarizing the responses indicated by each individual rule (Hammell II and Sudkamp [3]).

The Mamdani FIS (Mamdani and Assilian [19]) will be used. It is a non-additive rule model that provides a fuzzified output. The application of Mamdami FIS

provides outputs in the format of fuzzy sets. However, usually what is needed for making decisions/taking actions is a crisp output value. That is why defuzzification is needed to extract a crisp value from the fuzzy sets produced by the inference mechanism.

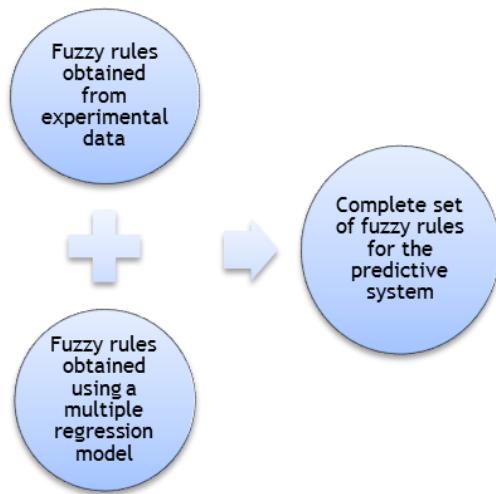


Fig. 4: Sources for generating the fuzzy rules for the predictive system

### 3.4.4 Defuzzification

According to Passino and Yurkovich [20], defuzzification operates on the implied fuzzy sets produced by the inference mechanism and combines their effects to provide the “most certain” value of the output.

It simply consists of extracting a crisp output value from a fuzzy output set (Kovačić and Bogdan [21]).

According to Lee [22], there is no systematic procedure for choosing a defuzzification strategy. However, the Center of Area (COA) also called the centroid method is the most commonly used defuzzification method. It is this method that we will use for the fuzzy-nets prediction system. It is worth to mention that defuzzification is not considered in the case of Takagi-Sugeno FIS (Takagi and Sugeno [23]).

### 3.5 The training scheme

The training scheme associated with the fuzzy-nets approach selected to use for surface roughness and average recast layer thickness prediction in WEDM is fuzzy logic membership scheme which requires experimental data to generate the fuzzy rule base of

the fuzzy-nets predictive system. Appropriate design of the experiments (e.g., following design of experiments (DOE) approach) is crucial for the success of the approach. That is why a special attention is given to the planning, design, realization and analysis of the experiments.

### 3.6 The general structure of the predictive fuzzy-nets approach

The technical specifications of the fuzzy-nets predictive system are summarized in Table 2.

Table 2: Technical specifications of the fuzzy-nets predictive system

Aspect of the predictive system	Technical specification
Fuzzy inference system (FIS)	Mamdani FIS
Type of decomposition of inputs and outputs domains into fuzzy regions	Fuzzy partition
Type of membership functions (MFs) for the fuzzy regions of input and output parameters	Triangular MFs with the same width for each parameter
Number of fuzzy regions for input parameters	Same for all input parameters; initially the number of regions is put to 3
Number of rules fired by a set of inputs values	Minimum: 1; maximum: $2^m$ where m is the number of input parameters relevant to the considered output parameter
Operator (t-norm) used for the membership degrees (MDs) of a fuzzy rule inputs to calculate the degree of the corresponding output	$T_{\min} (T_{\min}(x,y)=\min\{x,y\}, \forall x, y \in [0,1])$
Operator (t-conorm) for the aggregation of the fuzzy rules	$S_{\max} (S_{\max}(x,y) = \max\{x,y\}, \forall x, y \in [0,1])$
Defuzzification method	Centroid method
Decision making mechanism	Comparison of the predicted value of the VQC with a desired or reference value (or range of values)

### 3.7 The predictive procedure

We consider the most general case of fuzzy-nets system where multiple VQCs need to be predicted and for each VQC multiple VPPs need to be adjusted when the predicted value of the VQC is deviating from the desired value.

We assume that the MIMO predictive and adaptive fuzzy-nets system has the following characteristics:

- There are  $m$  output parameters:  $O_j, j = 1, 2, \dots, m$
- Each output parameter  $O_j, j=1,2, \dots, m$ , has  $n_j(j)$  input parameters:  $I_i, i = 1(j), 2(j), \dots, n_j(j)$

We have a MIMO predictive system that is decomposed into  $m$  MISO predictive systems MISO 1, MISO 2, ..., MISO  $m$ . Each MISO  $j, j = 1, 2, \dots, m$  is associated with the output parameter  $O_j, j = 1, 2, \dots, m$ .

The flowchart of the prediction procedure is shown in Fig. 5.

## 4 The MIMO fuzzy-nets framework

The different cases for the predictive fuzzy-nets approach are shown in Table 3.

Table 3: Different cases of the predictive fuzzy-nets approach

	Single output	Multiple outputs
Single input	SISO predictive system	SIMO predictive system
Multiple inputs	MISO predictive system	MIMO predictive system

It is worth to mention that a system with a single input parameter and a single output parameter is called a SISO (single input-single output) system and a system with a single input parameter and multiple output parameters is a called a SIMO (single input-multiple output) system. Since we focus on predictive systems with multiple inputs, then the SISO and SIMO predictive systems are not considered.

In general, it may be possible that an input parameter is significant for one output parameter but not for another. The simplest case of a MIMO (Multiple input-multiple output) predictive system is when all input parameters are significant for each output parameter.

In the case of a predictive system with multiple outputs having the same  $n$  input parameters, the minimum number of fired rules in this case is

$\underbrace{1 \times 1 \times \dots \times 1}_{n \text{ times}} = 1$  and the maximum number of fired rules is  $\underbrace{2 \times 2 \times \dots \times 2}_{n \text{ times}} = 2^n$ .

The total number of fuzzy rules fired by a data set is between 1 and  $2^n$ . The following cases can be distinguished:

- Each crisp value of an input parameter corresponds to the centre of a fuzzy region. In this case the total number of fired fuzzy rules is 1.
- None of the crisp values of input parameters corresponds to the centre of a fuzzy region. In this case the total number of fired fuzzy rules is  $2^n$ .

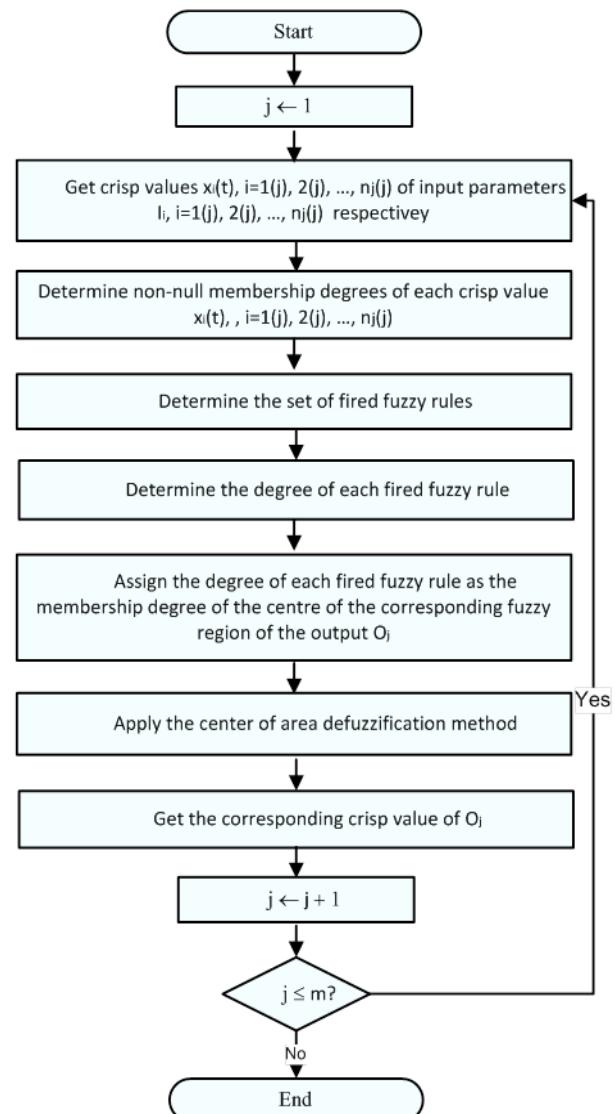


Fig. 5: Flowchart of the predictive procedure

In the other cases, the total number of fired fuzzy rules is strictly superior to 1 and strictly inferior to  $2^n$ .

Since the fired fuzzy rules have the same “if-part” and different “then-parts”, then the determination of the membership values of input parameters and the identification of fired rules is the same for all output parameters however the defuzzification procedure should be applied separately to each output parameter using the same information regarding the fired rules and the membership degrees of input parameters.

In the case where the output parameters have different sets of input parameters, an important issue to consider is how to organize the rule base and the predictive algorithm in order to benefit as much as possible from the information that is common to the output parameters.

## 5 The case study

### 5.1 Case study description

The proposed online prediction and adjustment system will be applied to ‘CUT 200 Sp’ from GF Machining Solutions.

Some characteristics of CUT 200Sp:

- Type of machining: Submerged
- Workpiece dimensions (W x D x H): 1000 x 550 x 220mm
- Max. workpiece weight: 750kg
- Available wire diameters (Standard): 0.33 - 0.15mm

GF Machining Solutions based in Switzerland is the world’s leading provider of machines, automation solutions and services to the tool and mold making industry and to manufacturers of precision components.

The main defects related to surface quality of WEDMed parts include: occurrence of ‘lines and marks’, surface roughness, and recast layer (also called white layer). In this paper we focus on surface roughness, and recast layer thickness.

GF Machining Solutions is interested in reducing the scrap rate of parts produced by their machines due to surface quality defects and in certifying that surface characteristics (roughness, recast layer) remains within the quality tolerance-band requirements of the customers.

### 5.2 Motivation and objectives

Based on past experiments different settings with the same surface roughness performance provide significantly different levels of performance for the recast layer thickness. That is why it is important to have technology tables providing indication of not only surface roughness performance but also the maximum recast layer thickness in addition to the possibility for the user to choose other settings for which there will be a need for the prediction of the corresponding surface roughness and recast layer thickness. To achieve this goal, an offline fuzzy-nets prediction and optimization system is proposed.

The overall objective of the offline prediction system is to account for the variation of the surface roughness, recast layer thickness and probability of occurrence of ‘lines and marks’ according to initial conditions represented by the values of voltage, feed rate and pulse off time.

### 5.3 Relevant parameters

The VQCs considered for the offline prediction system are surface roughness and average recast layer thickness.

There are two main important sources for the identification of potential parameters that influence the VQCs: (i) the explicit knowledge from the related literature and (ii) the implicit knowledge obtained from the WEDM experts.

The experimental investigation of the impact of the different parameters of WEDM process on the surface roughness revealed that pulse-off time is, by far, the most dominant factor influencing surface roughness followed by feedrate and voltage. In summary, the input and output parameters for this case study are shown in Table 4.

Table 4: Input and output parameters for the case study

Parameter	Type of parameter	Unit
Voltage	Input	[V]
Pulse off time	Input	[mm]
Feed-rate	Input	[mm/min]
Surface roughness	Output	[ $\mu\text{m}$ ]
Average recast layer thickness	Output	[ $\mu\text{m}$ ]

Regarding the recast layer thickness the relevant parameters are voltage and feed rate (Morand [24]).

The corresponding fuzzy-nets prediction system is shown in Fig. 6.

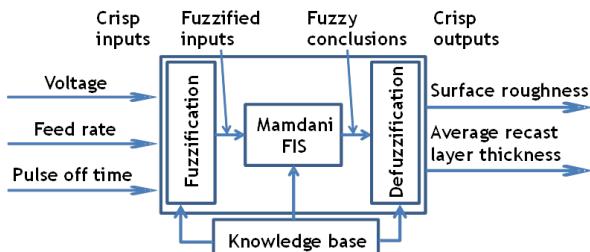


Fig. 6: Fuzzy-nets prediction system for WEDM

#### 5.4 Experimental plan

The main objective of this experimental campaign is to collect training data for the prediction of surface roughness and maximum recast layer thickness based on the initial conditions represented by machining voltage, pulse off time, and feed rate in addition to the type of wire.

For this experiment we consider:

- Material: steel and height: 20 mm
- Type of wire: **hard brass LT25A**

The levels of the input parameters in the experiment are shown in Table 5.

Table 5: Levels of input parameters in the experiment

Independent variables	Levels		
	Low (1)	Medium (2)	High (3)
Machining voltage ( $V$ ) [V]	110	150	190
Wire feed speed ( $f$ [mm/min])	6	11	16
Pulse off Time ( $T_{off}$ ) [ $\mu$ s]	1	6	11

#### 5.5 Experimental data

The surface roughness measurement was performed on finished surface by using surface profilometer ‘Mahr Perthometer PGK’.

The following steps are followed to measure the recast layer thickness:

- Surface of samples to polish is: **2mm x 7 mm**
  - The samples are mounted using resin which polymerizes at room temperature.
  - There is a need for polishing with 4 grain sizes (**15 $\mu$ m, 9 $\mu$ m, 6 $\mu$ m and 1 $\mu$ m**). 15 and 9 microns are realized with 1200 and 2400 SiC papers respectively and 6 and 1 microns are realized with diamond suspensions.
- Since the samples are in steel, **NITAL** is used for chemical attack of polished samples

- A microscope to measure the recast layer thickness
  - 20 pictures will be taken per surface from the beginning of the machining up to the end
  - On each picture the minimum and the maximum recast layer thickness will be measured

The obtained experimental data is shown in Table 6.

Table 6: Experimental data

Run No.	$V$ [V]	$f$ [mm/min]	$T_{off}$ [ $\mu$ s]	Surface roughness ( $R_a$ ) [ $\mu$ m]	Average recast layer thickness (RLT) [ $\mu$ m]
1	110	6	1	0.46	1.49
2	110	6	6	0.59	1.95
3	110	6	11	0.61	1.73
4	110	11	1	0.5	1.74
5	110	11	11	0.7	2.06
6	110	16	6	0.73	1.08
7	150	6	1	0.62	1.5
8	150	6	6	0.66	1.33
9	150	6	11	0.73	1.69
10	150	11	1	0.64	4.25
11	150	11	6	0.73	1.31
12	150	16	1	0.64	1
13	150	16	6	0.74	1.12
14	150	16	11	0.89	5.07
15	190	6	6	1.12	3.02
16	190	11	1	0.75	6.13
17	190	11	6	1.05	4.02
18	190	11	11	1.06	4.72
19	190	16	6	1.02	4.97
20	190	16	11	0.99	1.03

#### 5.6 Development of the prediction rule base

The input parameters such as voltage, pulse-off time and feed rate are divided into three regions and the output parameters, surface roughness and recast layer thickness are divided into 5 regions as shown in Fig. 7 (triangular membership function for all the parameters). The input parameters are defined with the help of linguistic terms such as high, medium, low and the output parameters are defined with the help of linguistic terms such as high2, high1, medium, low1 and low2. The output of the developed software, the generation of data base for the prediction system, is summarized in Table 7. The universe of discourse, along with the linguistic variables for each variable is shown in Fig. 7 in the appendix. From the triangular membership function, the degree of membership for each of the input and output process parameters can be determined from its crisp value.

The generated fuzzy rule base is shown in Table 8.

Table 7: Range of values and regions of input and output parameters

Parameter	Min value	Max value	# Regions	Width	Centers
<b>V</b>	110 [V]	190 [V]	3	40	110, 150, 190
<b>f</b>	6 [mm/min]	16 [mm/min]	3	5	6, 11, 16
<b>T<sub>off</sub></b>	1 [μs]	11 [μs]	3	5	1, 6, 11
<b>RLT</b>	3.55 [μm]	41.15 [μm]	5	9.4	1.1, 86, 2.71, 3.57, 4.42, 5.27, 6.13
<b>R<sub>a</sub></b>	0.46 [μm]	1.12 [μm]	5	0.17	0.46, 0.57, 0.68, 0.79, 0.9, 1.01, 1.12

Table 8: Generated fuzzy rule base

No. of rule	
1	IF voltage IS low AND feed rate IS low AND pulse-off time IS low THEN recast layer thickness is low2 AND surface roughness is low3
2	IF voltage IS low AND feed rate IS low AND pulse-off time IS medium THEN recast layer thickness is low2 AND surface roughness is low2
3	IF voltage IS low AND feed rate IS low AND pulse-off time IS high THEN recast layer thickness is low2 AND surface roughness is low2
4	IF voltage IS low AND feed rate IS medium AND pulse-off time IS low THEN recast layer thickness is low2 AND surface roughness is low3
5	IF voltage IS low AND feed rate IS medium AND pulse-off time IS high THEN recast layer thickness is low2 AND surface roughness is low1
6	IF voltage IS low AND feed rate IS medium AND pulse-off time IS medium THEN recast layer thickness is low2 AND surface roughness is low1
7	IF voltage IS low AND feed rate IS high AND pulse-off time IS low THEN recast layer thickness is low3 AND surface roughness is low2
8	IF voltage IS low AND feed rate IS high AND pulse-off time IS medium THEN recast layer thickness is low3 AND surface roughness is low1
9	IF voltage IS low AND feed rate IS high AND pulse-off time IS high THEN recast layer thickness is low1 AND surface roughness is medium
10	IF voltage IS medium AND feed rate IS low AND pulse-off time IS low THEN recast layer thickness is low2 AND surface roughness is low2
11	IF voltage IS medium AND feed rate IS low AND pulse-off time IS medium THEN recast layer thickness is low3 AND surface roughness is low1
12	IF voltage IS medium AND feed rate IS low AND pulse-off time IS high THEN recast layer thickness is low2 AND surface roughness is low1
13	IF voltage IS medium AND feed rate IS medium AND pulse-off time IS low THEN recast layer thickness is high1 AND surface roughness is low1
14	IF voltage IS medium AND feed rate IS medium AND pulse-off time IS medium THEN recast layer thickness is low3 AND surface roughness is low1
15	IF voltage IS medium AND feed rate IS high AND pulse-off time IS low THEN recast layer thickness is low3 AND surface roughness is low1
16	IF voltage IS medium AND feed rate IS high AND

	pulse-off time IS medium THEN recast layer thickness is low3 AND surface roughness is medium
17	IF voltage IS medium AND feed rate IS high AND pulse-off time IS high THEN recast layer thickness is high2 AND surface roughness is high1
18	IF voltage IS medium AND feed rate IS medium AND pulse-off time IS high THEN recast layer thickness is low1 AND surface roughness is medium
19	IF voltage IS high AND feed rate IS low AND pulse-off time IS low THEN recast layer thickness is high2 AND surface roughness is high1
20	IF voltage IS high AND feed rate IS low AND pulse-off time IS medium THEN recast layer thickness is low1 AND surface roughness is high3
21	IF voltage IS high AND feed rate IS low AND pulse-off time IS high THEN recast layer thickness is low2 AND surface roughness is high3
22	IF voltage IS high AND feed rate IS medium AND pulse-off time IS low THEN recast layer thickness is high3 AND surface roughness is high2
23	IF voltage IS high AND feed rate IS medium AND pulse-off time IS medium THEN recast layer thickness is high1 AND surface roughness is high2
24	IF voltage IS high AND feed rate IS medium AND pulse-off time IS high THEN recast layer thickness is high1 AND surface roughness is high2
25	IF voltage IS high AND feed rate IS high AND pulse-off time IS low THEN recast layer thickness is high1 AND surface roughness is medium
26	IF voltage IS high AND feed rate IS high AND pulse-off time IS medium THEN recast layer thickness is high2 AND surface roughness is high2
27	IF voltage IS high AND feed rate IS high AND pulse-off time IS high THEN recast layer thickness is low3 AND surface roughness is high2

## 5.7 Illustration of the use of the fuzzy-nets prediction model

The example to illustrate the inference model considers the input data set in Table 9.

Table 9: Data set for fuzzy inference model

V [V]	f [mm/min]	T <sub>off</sub> [μs]
130	11	6

According to Fig. 7.a, voltage has a non-null membership degree in two regions: low (L) and medium (MD) as shown in Fig. 8.

Therefore the membership degree of the voltage in both regions is calculated as follows:

$$\begin{cases} \mu_L(Voltage) = 1 - \frac{130 - 110}{150 - 110} = \frac{20}{40} = 0.5 \\ \mu_{MD}(Voltage) = 1 - \frac{150 - 130}{150 - 110} = \frac{20}{40} = 0.5 \end{cases}$$

Feed rate has non null membership function degree in one region MD as shown in Fig. 9. The membership degree is one ( $\mu_{MD}(FeedRate) = 1$ ) as the value of feed rate falls on the center of the region MD.

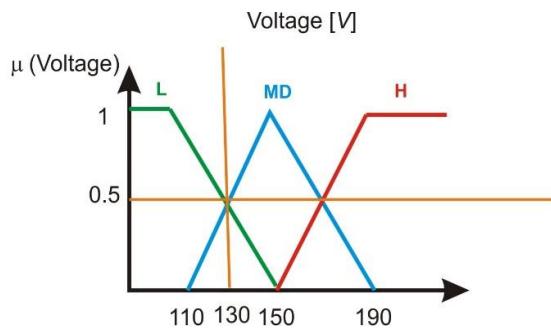


Fig. 8: Voltage membership degree determination

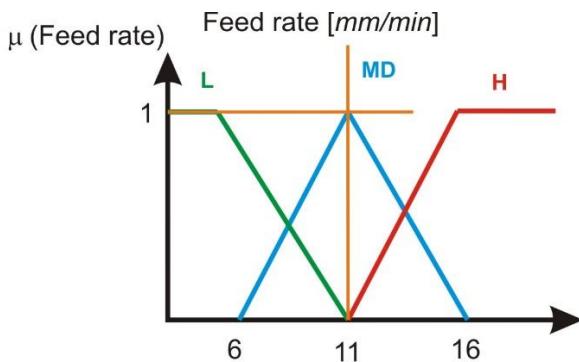


Fig. 9: Feed rate membership degree determination

Pulse-off time has non null membership function degree in one region MD as shown in Fig. 10. Also in this case the membership degree is one ( $\mu_{MD}(Pulse-off\ Time)=1$ ) due to the value of pulse-off time which is equal to the center value of the membership function MD.

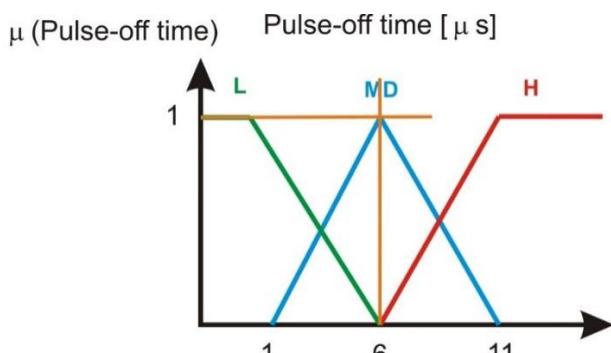


Fig. 10: Pulse off time membership degree determination

The two fired fuzzy rules are the rules number 6 and 14 listed in Table 8: (i) IF voltage is low AND feed rate is medium AND pulse-off time is medium THEN recast layer thickness is low1 AND surface roughness is low1, and (ii) IF voltage is medium AND feed rate is medium AND pulse-off time is medium THEN recast layer thickness is low1 AND surface roughness is medium.

The membership degrees of surface roughness and recast layer thickness with respect to these two fuzzy rules if 0.5.

In both of the fired fuzzy rules, the fuzzy region of recast layer thickness is low1. Therefore the center value of region low1 for recast layer thickness is 12.95 [ $\mu\text{m}$ ]. The crisp values of recast layer thickness is calculated as follows:

$$y_{RLT} = \frac{0.5 \times 12.95 + 0.5 \times 12.95}{0.5 + 0.5} = \frac{12.95}{1} = 12.95$$

The fuzzy regions of surface roughness in the first and second fired fuzzy rule are respectively low1 and medium, and the corresponding center values of these regions are respectively 0.63 [ $\mu\text{m}$ ] and 0.79 [ $\mu\text{m}$ ]. The crisp values of recast layer thickness is calculated as follows:

$$y_{Ra} = \frac{0.5 \times 0.63 + 0.5 \times 0.79}{0.5 + 0.5} = \frac{0.71}{1} = 0.71.$$

## 6 Conclusion

We presented for the first time a general Multiple-Input Multiple-Output (MIMO) fuzzy net architecture and approach; we demonstrated the application of the new MIMO framework for part quality prediction in WEDM and input process parameters planning. We have identified the process parameters influencing the part surface roughness (pulse off time, feed rate and voltage,) and those influencing the recast layer thickness (voltage and feed rate).

The proposed fuzzy logic based approach for part quality prediction and process planning in WEDM was demonstrated through its application to a WEDM machine (CUT 200 sp) from GF Machining Solutions where the considered part quality characteristics are recast layer thickness and surface roughness. This resulted in a paradigm change in WEDM process planning where not only the desired surface roughness can be targeted but also the average recast layer thickness. This is very important for practical applications since it is well known that the WEDM is often not recommended in safety critical applications due to the uncontrollable generation of recast layer thickness which could be detrimental to the respective component fatigue strength.

In the future work we will investigate the combination of the MIMO fuzzy-nets prediction approach with an optimization method to determine the set of initial conditions which provides the best

performance with respect to recast layer thickness and surface roughness simultaneously.

## 7 Acknowledgements

The work presented in this paper has been partly financed by the Swiss commission for technology and innovation (CTI).

### References:

- [1] G. Chryssolouris, Manufacturing Systems, Theory and Practice, Springer-Verlag, New-York, 1992.
- [2] Aspinwall, D. K., Soo, S. L., Berrisford, A. E., Walder, G., Workpiece surface roughness and integrity after WEDM of Ti-6Al-4V and Inconel 718 using minimum damage generator technology, CIRP Annals - Manufacturing Technology, Vol. 57 (2008), pp. 187-190.
- [3] R. J. Hammell II, T. Sudkamp. Learning fuzzy rules from data, Proc. NATO RTO Meeting 3: Appl. Inf. Technol. (Comput. Sci.) Mission Syst., 1998, pp. 8-1..8-10.
- [4] L.A. Zadeh, Fuzzy sets, Information and Control 8 (1965) 338-353.
- [5] Zimmermann, H.-J. (1996). Fuzzy set theory and its applications. Boston. Kluwer Academic Publishers.
- [6] Li-Xin Wang and Jerry M. Mendel, 1992, Generating fuzzy rules by learning from examples, IEEE Transactions on Systems, Man and Cybernetics 22 (6), pp. 1414-1427.
- [7] J. C. Chen, J. T. Black. A fuzzy-nets in-process (FNIP) system for tool-breakage monitoring in end-milling operations, International Journal of Machine Tools and Manufacture 37 (1997), pp. 783-800.
- [8] J. C. Chen, M. S. Lou. Fuzzy-nets based approach using an accelerometer for in-process surface roughness prediction system in milling operations, Journal of Computer Integrated Manufacturing Systems 13 (2000), pp. 358-368.
- [9] J. C. Chen, M. Savage. Fuzzy-net-based multi-level in-process surface roughness recognition system in milling operations. Journal of Advanced Manufacturing Technology 17 (2001), pp. 670-676.
- [10] Kirby, E. D. and Chen, J. C., (2007) Development of a fuzzy-nets-based surface roughness prediction system in turning operations. International Journal of Computer & Industrial Engineering, 53, pp. 30-42.
- [11] Kirby, E. D. & Chen, J. C. & J. Zhang, (2006) Development of a fuzzy-nets-based in-process surface roughness adaptive control system in turning operations. Expert Systems with Applications, 30, pp. 592-604.
- [12] J.L. Yang (2002) Development of a fuzzy-nets-based adapted surface roughness control (FNASRC) system in end-milling operations. Dissertation, Iowa State University.
- [13] Yang J. L. and Chen, J.C. (2003) Statistical assisted fuzzy-nets based in-process surface roughness prediction (S-FNIPSRP) system in end milling operations. Journal of Chinese Institution of Industrial Engineering, 20(5), 494-510.
- [14] L.D. Yang, J.C. Chen, H.M. and C.T. Lin, Fuzzy-nets based in process surface roughness adaptive control system in end-milling operations, Int. J. Adv. Manuf. Technol. (2006), 28: 236-248.
- [15] Zhu, J. and Chen, J.C. (2006) In-process mixed materials caused flash prediction (FNN-IPMFP) system in injection moulding operations. International Journal of Advanced Manufacturing Technology, 29, 308-316.
- [16] J.C. Chen, An effective fuzzy-nets training scheme for monitoring tool breakage, Journal of Intelligent Manufacturing (2000) 11, 85-101.
- [17] J. Jantzen. Design Of Fuzzy Controllers, Tech. Rep. no 98-E864, Technical University of Denmark, 1998.
- [18] H. Li, C.L. Philip Chen, H.-. Huang. Fuzzy Neural Intelligent Systems: Mathematical Foundation and the Applications in Engineering, CRC Press, 2000.
- [19] Mamdani, E.H. and Assilian, S., (1975). An experiment in linguistic synthesis with a fuzzy logic controller. International Journal of Man-Machine Studies, 7(1), 1-13.
- [20] K.M. Passino and S. Yurkovich, Fuzzy control, Addison Wesley Longman Inc. (1998) ISBN 0-201-18074-X.
- [21] Kovačić, Z. and Bogdan, S., (2006). Fuzzy Controller Design: Theory and Applications. CRC Press, Boca Raton, FL.
- [22] C.C. Lee, (1990), Fuzzy Logic in control systems: Fuzzy Logic controller part I", IEEE Transactions on Systems, Man and Cybernetics 20 (2), pp. 404 - 418.
- [23] T. Takagi, M. Sugeno. Fuzzy identification of systems and its applications to modeling and control, IEEE Transactions on Systems, Man and Cybernetics 15 (1985), pp. 116-132.
- [24] M.-P. Morand, Experimental investigation of surface integrity and defects for a quality assurance system of WEDMed aerospace parts, EPFL, 2014.

## 8 Appendix

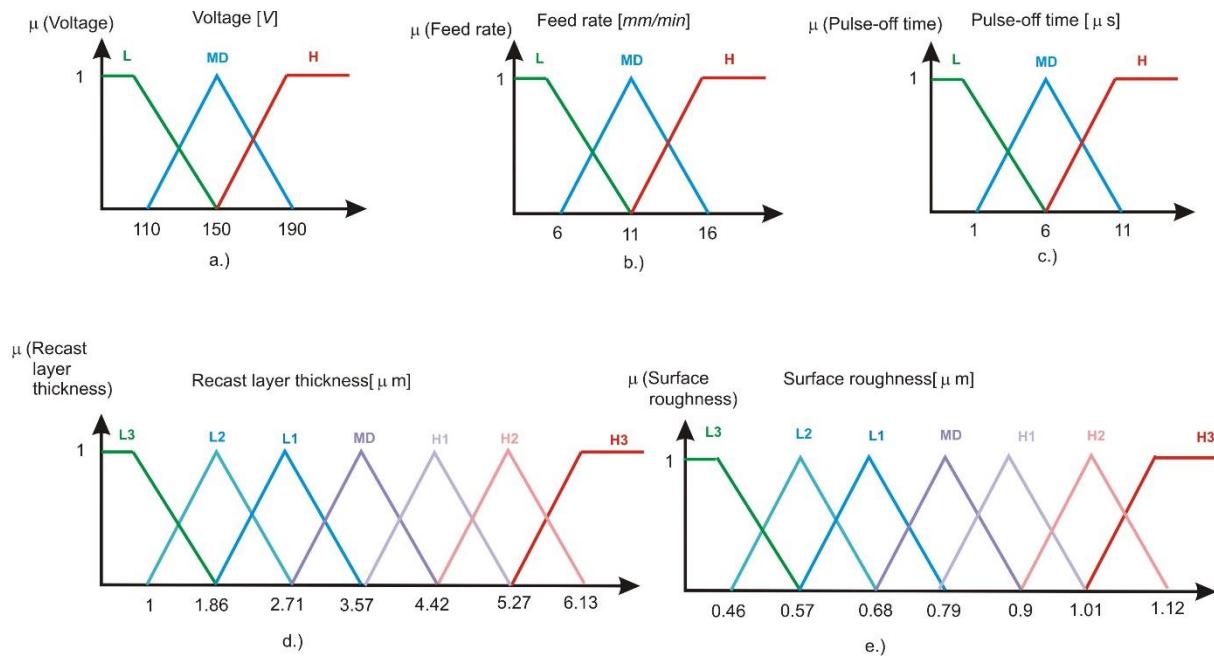


Fig. 7: Fuzzy regions and related membership functions of a.) Voltage [V] b.) Feed rate [mm/min] c.) Pulse-off time [ $\mu$ s] d.) Average recast layer thickness [ $\mu$ m] e.) Surface roughness [ $\mu$ m]