Automatic Regulating Time Series for Multivariable Processes with Specifications on Rise Times

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Abstract: - In this paper, an auto-tuning method is proposed for multivariable processes. The considered system is first estimated by use of an on-line identification method, and then low-interacting precompensating matrix is applied , and finally nonlinear automatic regulating time series is applied to get the wanted response times. The performance of the compensated systems can be evaluated by wanted rising time and low-interacting characteristics. Three numerical examples give the proposed method provides better performance and less interaction effects than those of other four mentioned on-line computing and PI controlling methods.

Key-Words: - Auto-tuning, Low-interaction, Multivariable processes.

1 Introduction

The proportional-integration-derivative (PID) controllers have been used widely in industry because of robustness and simplicity, though there has been a lot of interest in research into and implementation of advanced controllers. On-line tuning controls are generally applicable for slow industry processes and can be retuned for system parameter variations. Åström and Hägglund [1-3] identified the ultimate process information from a relay feedback. Based on the identified information, parameters of PID controllers can be tuned by many developed techniques [4-9].

In this paper, a nonlinear discrete-time control sequence described by periodic numerical series $Q(jT_S)$ is proposed for industry processes[10]. They are sampled-data feedback control systems. T_S represents the sampling interval. The ratios of $Q((k+1)T_S)$ to $Q(kT_S)$ of the series are formulated as a function of the reference input command and the output of the plant. The value of $Q(kT_S)$ is the control input of the plant at time intervals between $(k-1)T_S$ and kT_S. Thus, the considered system is closed as a feedback control system by use of $Q(jT_S)$. It will be seen that the output of the plant tracks the reference input command exactly after ratios $Q((k+1)T_S)$ / $Q(kT_S)$ of the series being converged to unities. It implies that $Q(kT_S)$ will be converged to a steady-state value for a constant reference input applied. The stability of the closed-loop system is guaranteed by selecting the proper function of ratios $Q((k+1)T_S)/Q(kT_S)$. This function can be called as "Tuning Function". It will be proven that the considered system using $Q(kT_S)$ becomes a negative feedback control system for a stable plant[10]. The proposed method can be used as an on-line manner to the considered system to get zero tracking error and the wanted rising time. It provides a simple and effective way to the considered system.

Since decoupling interactions of MIMO processes are generally expected [8,11-16] for faster productions. Integrations of error signals in on-diagonal loops will always decouple interactions of MIMO processes for long-time characteristics. However, it is not true for short-time characteristics. Short-time characteristics depended on the system characteristics in the middle frequency band; i.e., the system characteristics around the gain crossover frequency. Decoupling the system characteristics in middle frequency band is the major problem of MIMO process designs[15]. There are several researches for auto-tuning of MIMO process by Wang et al. [8,12,14], Semino [15] and Ju[16]. Those researches gave good results while leaving decoupling behaviors can further be improved. Bad decoupling characteristics are usually resulted from using wrong critical frequencies [8,11-16] to find multivariable PID controllers. Critical frequencies found by relay identification process [1-3] are phase crossover frequencies. However gain crossover frequencies are usually needed. The major difference between the proposed method and other methods by previous researchers is the gain crossover frequency is used.

The proposed method is a technique for on-line identification and tuning in the framework of a MIMO auto-tuning procedure. A stable pre-compensator is first used to reduce coupling effects of the considered plant [18-21], and then nonlinear automatic regulating time series[10] is applied to each loop for getting the wanted response time. The proposed method are applied to three numerical examples. It will be seen that the proposed method provides better performance and less interaction effects than those of other mentioned on-line computing methods using PI and PID controllers.

2. The Proposed Method

2.1. Low interacting pecompensator design

Consider a $m \times m$ multivariable process $G(s)$ of a feedback control system shown in Fig.1, and given a stable pre-compensator $P(s)$ defined as

$$P(s) = G(s)^{-1} det(G(s))/det(G(0)) \tag{1}$$

for stable and nonsingular $G(s)$. Then the multivariable system $G(s)$ is decoupled ideally by use of $P(s)$; i.e.,

$$\begin{aligned} G(s)P(s) &= G(s)G(s)^{-1} det(G(s))/det(G(0)) \\ &= I_m det(G(s))/det(G(0)) \end{aligned} \tag{2}$$

where I_m is the identity matrix and $det(G(s))$ is the determinate of $G(s)$. Since $G(s)P(s)$ is a diagonal matrix, the multivariable design problem becomes m single-input single-output (SISO) design problems ideally. Eq.(1) gives $P(s)$ is stable for a stable and nonsingular plant $G(s)$. It implies that $P(s)$ and $G(s)P(s)$ are all stable for stable $G(s)$ and $P(s)$ can be realized by conventional RC networks.

Note that the estimated plant $\hat{G}(s)$ will replace $G(s)$. It will be evaluated from on-line identification processes. A diagonal dominant rather than full decoupled precompensated sysem $G(s)\hat{P}(s)$ will be made**[19]**. $\hat{P}(s)$ is found by Eq.(1) in which $G(s)$ is repalced by $\hat{G}(s)$; i.e.,

$$\hat{P}(s) = \hat{G}(s)^{-1} det(\hat{G}(s))/det(\hat{G}(0)) \tag{3}$$

The mathematical models used in $\hat{G}(s)$ are first order dynamic plus time delay (FOPDT) [21-23]; i.e., the $(i, j)th$ element of $\hat{G}(s)$ is in the form of

$$\hat{g}_{ij}(s) = \frac{K_{ij}}{T_{ij}s + 1} e^{-L_{ij}s} \tag{4}$$

Parameters(K_{ij}, T_{ij}, L_{ij}) are evaluated in the on-line identification processes. The critical frequency(ω_c)[1-3] will be used to find them. The $\hat{P}(s)$ decouples $\hat{G}(s)$ (not $G(s)$) in the critical frequency point only; i.e., not full decoupled for full frequencies. Eq.(4) gives the complexity of $\hat{P}(s)$ described by Eq.(3) is limited.

Consider a multivariable process[16] described by

$$G(s) = \begin{bmatrix} \dfrac{0.66}{6.7s+1}e^{-2.6s} & \dfrac{-0.61}{8.64s+1}e^{-3.5s} \\ \dfrac{1.11}{3.25s+1}e^{-6.5s} & \dfrac{-2.36}{5s+1}e^{-3s} \end{bmatrix} \tag{5}$$

The elements of $G(s)$ are all described by FOPDT. The precompensating matrix found by Eq.(1) is

$$P(s) = \frac{-1}{0.8805} \begin{bmatrix} \dfrac{-2.36}{5s+1}e^{-3s} & \dfrac{+0.61}{8.64s+1}e^{-3.5s} \\ \dfrac{-1.11}{3.25s+1}e^{-6.5s} & \dfrac{0.66}{6.7s+1}e^{-2.6s} \end{bmatrix} \tag{6}$$

Eq.(2) gives

$$\begin{aligned} G(s)P(s) &= I_2 \det(G(s))/\det(G(0)) \\ &= I_2 \left[\frac{+1.7690}{(5s+1)(6.7s+1)} e^{-5.6s} - \frac{0.7690}{(10s+1)^2} e^{-10s} \right] \end{aligned} \tag{7}$$

Note that elements of $P(s)$ have common time delays. There are two way to speed up the compensated system. One is to elimnated 2.6s for four elements, the other is to elimate 3s for the first column and 2.6s for the second column. The first case gives

$$\begin{aligned} G(s)P(s)^{(1)} &= I_2 \det(G(s))/\det(G(0)) e^{2.6s} \\ &= I_2 \left[\frac{+1.7690}{(5s+1)(6.7s+1)} e^{-3s} - \frac{0.7690}{(10s+1)^2} e^{-7.4s} \right] \end{aligned} \tag{8}$$

The second case gives

$$G(s)P(s)^{(2)} = diag\{e^{3s}, e^{2.6s}\} \det(G(s))/\det(G(0)) \tag{9}$$

Eqs.(8) and (9) give the decoupling characteristics of precompensated system is kept after $P(s)$ is replaced by $P(s)^{(1)}$ or $P(s)^{(2)}$, and can be deduced to a general form

$$G(s)P(s)^{(2)} = diag\{e^{T_{c1}S} \quad e^{T_{c2}S} \quad \dots \quad e^{T_{cm}S}\} \times \det(G(s))/\det(G(0)) \quad (10)$$

where T_{cj} is the common time delay of jth column of $P(s)$. Note that first order dynamics in $P(s)$ can not be further manipulated for they are used for getting diagonal dominant properties of the precompensated system. Eq.(8) gives the design problem decomposed into two identical SISO problems. Eq.(9) gives the design problem decomposed into two different SISO problems.

2.2. Plant Estimation using FOPDT models

On-Off relays will be used in loops for on-line identifying multivariable processes[1-3]. The introducing relays will result in sustaining oscillation for stable $G(s)$ [9]. Then, the Laplace-transformation of $G(s)$ can be derived from computing integrations of input and output signals of relays [8,11-14]. Consider a $m \times m$ multivariable process in the frequency domain as

$$\begin{bmatrix} y_1(j\omega) \\ \vdots \\ y_m(j\omega) \end{bmatrix} = \begin{bmatrix} g_{11}(j\omega) & \cdots & g_{1m}(j\omega) \\ \vdots & \ddots & \vdots \\ g_{m1}(j\omega) & \cdots & g_{mm}(j\omega) \end{bmatrix} \begin{bmatrix} u_1(j\omega) \\ \vdots \\ u_m(j\omega) \end{bmatrix} \quad (11)$$

The $(i, j)th$ element of the process is described by

$$g_{ij}(j\omega) = y_i(j\omega)/u_j(j\omega)\Big|_{u_k=0,k\neq j} \quad (12)$$

The above equation represents it needs m individual procedures for identifying whole elements of a $m \times m$ multivariable process. Wang[8] proposed a decentralized relay feedback identification method. Although all relays are used simultaneously, but it still needs m procedures with perturbing the amplitude of relays to prevent linear dependent. If the oscillations in m loops have common frequency ω_c, then the direct-current(DC) components and the first harmonics of these periodic waves are extracted as

$$\hat{g}_{ij}(0) = \frac{\int_0^T y_i(t)dt}{\int_0^T u_j(t)dt}\Bigg|_{u_k=0,k\neq j} \quad (13)$$

and

$$\hat{g}_{ij}(j\omega_c) = \frac{\int_0^T y_i(t)e^{-j\omega_c t}dt}{\int_0^T u_j(t)e^{-j\omega_c t}dt}\Bigg|_{u_k=0,k\neq j} \quad (14)$$

$\hat{G}(0)$ and $\hat{G}(j\omega_c)$ can be found by Eqs. (13) and (14). For industry process, first order models plus time delay(FODPT)[20-24] are usually used for modeling and control[8-13]. The $(i, j)th$ element of the estimated model $\hat{G}(s)$ can be defined as

$$\hat{g}_{ij}(s) \equiv \hat{g}_{ij}(0)e^{-L_{ij}s}/(T_{ij}s+1), \quad (15)$$

and values of L_{ij} and T_{ij} are evaluated by $\hat{g}_{ij}(j\omega)$.The precompensator can be found by Eq.(3) or Eq.(9), The estimated model $\hat{G}(s)$ will be used to found the precompensating matrix $P(s)$ or $P(s)^{(m)}$. The precompensated system $G(s)P(s)$ or $G(s)P(s)^{(m)}$ is diagonal dominant and m automatic regulating time series can be applied.

2.3. Automatic regulating time series[10]

The automatic time series $Q(n+1)$ is defined as in the form of

$$Q(n+1) = \left[(1-\beta)\left(\frac{R(n)}{Y_s(n)}\right)^{J_G} + \beta\right]Q(n); \quad (16)$$

and

$$u(n+1) = Q(n+1); \quad (17)$$

where β is the regulating parameter, $R(n)$ is the reference input, $Y_S(n)$ is the sampled output, and $u(n+1)$ is the input of the plant. Fig.2 shows $F(n) = Q(n+1)/Q(n)$ versus $R(n)/Y_S(n)$ represented by Eq.(16) for $\beta = 0.1$ and $J_G = 1,3,5$; respectively. Fig. 2 shows that the value of $F(n)$ is less than one for that of $Y_S(n)$ greater than that of $R(n)$, then the value of $Q(n+1)$ will be decreased; and the value of $F(n)$ is greater than one for that of $Y_S(n)$ less than that of $R(n)$, the value of $Q(n+1)$ will be increased. This implies that the controlled system connected using Eqs.(16) and (17) will be regulated to the equilibrium point ($Y_S(n)/R(n) = 1$) and gives a negative feedback control system for deviation from the equilibrium point. Taking the derivative of $F(n) = Q(n+1)/Q(n)$ with respect to $Y_S(n) = R(n)$, one has

$$\frac{\partial F(n)}{\partial(Y_S(n)/R(n))} = -(1-\beta)\left(\frac{Y_S(n)}{R(n)}\right)^{-(J_G+1)} \quad (18)$$

For negative value of Eq.(18), the value of $1-\beta$

must be greater than zero and J_G is odd. This implies the range of β is grater than zero and less than one. From Fig.2, one can select J_G to get desired regulating slope; i.e., regulating characteristic. Certainly, other tracking functions can be formulated and proposed also for the considered system, if its derivative with respect to $Y_S(n)/R(n)$ is negative. Note that the singularity of Eq.(16) must be avoided when $Y_S(n)=0$. It is easy to replace $Y_S(n)=0$ by a small value. A small value of $Q(n)$ is selected also to avoid null the time series. The maximal value of $Q(n)$ can be limited by an adaptive constraint $|R(n)|$ to minimize the control effort. The constraint of $Q(n)$ can not only for minimizing the control effort, but also for improving system performance.

Consider a stable SISO process has the transfer function[9]:

$$P_1(s)=\frac{e^{-s}}{(s+1)^2} \tag{19}$$

Fig.3 shows the time responses for $\beta=0.1$, $J_G=1$, sampling interval $T_S=25ms$, and values of reference inputs $R(n)$ between 0 and 20 seconds are equal to 1; between 20 and 40 seconds are equal to -0.7, between 40 and 60 seconds are equal to 0.5, and between 60 and 80 seconds are equal to -0.3. Fig. 3 gives the output Y(solid-line), the series $Q(n)$ (dotted-line) and ratios $F(n)$ (dash-dotted-line) . It shows that the ratios $F(n)$ are converged to be unities quickly, the proposed tuning rule can give good performance and no integration is needed to get zero tracking error.

2.4. On line Phase Lead Compensation

A conventional digital filter $C(z)$ can be applied for filtering $Q(n)$, if it is necessary. In general, phase-lead compensation is used for speeding up the time response. The first order phase-lead compensator can be expressed as

$$C(z)=\left.\frac{T_n s+1}{T_n s/\rho+1}\right|_{s=\frac{2}{T_S}\frac{z-1}{z+1}} \tag{20}$$

for $\rho>1$. The parameter T_n can be found by another numerical time series. It is

$$W(n+1)=\left[(1-\eta)\left(\frac{T_c}{T_{cs}}\right)^{J_W}+\eta\right]W(n) \tag{21}$$

$$T_n=W(n+1)$$

where T_c is the time constant of the closed loop system and T_{cs} is the wanted time constant. The wanted T_c and T_n will be found after $W(n+1)/W(n)$ converged to be unity.

Consider the illustrating example(Eq.(19)) described in the above paragraph, the time constant $T_c=3.151\sec$. of the system closed using Eqs.(16) and (17) is too slow. Therefore, phase-lead compensation is usually needed to speed up the system response. The time constant $T_{cs}=1.85\sec$ is selected. Parameters of Eqs.(16), (17), (20) and (21) are $\beta=0.1$, $J_G=1$, $T_S=25ms$, $\rho=50$, $\eta=0.8$ and $J_W=1$. Fig.4 shows on-line adjusting processes for finding $C(z)$. The initial guess of T_n is equal to 1.00 and converged to 0.5792 after two adjusting processes. The found lead compensator is

$$C(z)=\left.\frac{0.5792s+1}{0.5792s/\rho+1}\right|_{s=\frac{2}{T_S}\frac{z-1}{z+1}} \tag{22}$$

Time constant of each evaluation step is 1.4032, 1.8494, and 1.8500, respectively. Fig.4 shows the proposed method can provide an on-line tuning procedure to meet the wanted design specification. It gives good performance and zero steady-state error. Fig.5 shows converging properties of Eq.(21) with different initial guesses for finding $C(z)$. It shows T_n s are converged to 0.5792 quickly.

The adequate converging property can apply Word of Mouth technology[28] with users' database; i.e., β, T_n, T_c.

The proposed method will be applied to three multivariable processes. On line identification processes are first used to find estimated plant models of considered systems, and then precompensating matrices are used to get diagonal dominant systems, finally automatic regulating time series are applied to get wanted specifications on time constant(i.e., bandwidth).

3. Numerical Examples

Example 1. Consider the well-known Wood and Berry binary distillation column plant[25,26]

$$G(s)=\begin{bmatrix}\frac{12.8}{16.7s+1}e^{-s} & \frac{-18.9}{21s+1}e^{-3s}\\ \frac{6.6}{10.9s+1}e^{-7s} & \frac{-19.4}{14.4s+1}e^{-3s}\end{bmatrix} \quad (23)$$

On-off relays with magitude H are used in the plant idenfication process. The sustain oscillation frequency $\omega_{c1}=0.439 rad/\sec$ for loop 1 closed with a relay(H=0.1) and time lag 3.0seconds. The time delay is used to slow down the oscillating frequency [1-3] for reducing the computing efforts of Eqs. (13) and (14). Using Eqs. (13) and (14), the found $\hat{G}_{j1}(0)$, and $\hat{G}_{j1}(j\omega_{c1})$ are

$$\hat{G}_{j1}(0)=\begin{bmatrix}12.8\\6.6\end{bmatrix} \quad (24a)$$

and

$$e^{-1.2987j}\hat{G}(0.4329j)=\begin{bmatrix}e^{-0.02837j}/0.570\\ e^{3.730j}/0.7105\end{bmatrix} \quad (24b)$$

The sustain oscillation frequency $\omega_{c2}=0.3066 rad/\sec$ for loop 2 closed with relay and time lag 3.0seconds. Using Eqs. (13) and (14), the found $\hat{G}_{j2}(0)$, and $\hat{G}_{j2}(j\omega_{c2})$ are

$$\hat{G}_{j2}(0)=\begin{bmatrix}-18.9\\-19.4\end{bmatrix} \quad (25a)$$

and

$$e^{-0.9199j}\hat{G}(0.3066j)=\begin{bmatrix}e^{-0.1201j}/0.3450\\ e^{-0.05147j}/0.2335\end{bmatrix} \quad (25b)$$

Using dynamic model given by Eq.(15), one has

$$\hat{G}(s)=\begin{bmatrix}\frac{12.8}{16.695s+1}e^{-1.012s} & \frac{-18.9}{21.014s+1}e^{-3.017s}\\ \frac{6.6}{10.895+1}e^{-7.009s} & \frac{-19.4}{14.410s+1}e^{-3.017s}\end{bmatrix} \quad (26)$$

and

$$\hat{P}(s)=\frac{-1}{123.58}\begin{bmatrix}\frac{-19.4}{14.410s+1}e^{-3.017s} & \frac{+18.9}{21.014s+1}e^{-3.017s}\\ \frac{-6.6}{10.895s+1}e^{-7.009s} & \frac{12.8}{16.695s+1}e^{-1.012s}\end{bmatrix} \quad (27)$$

Eq. (26) gives the estimated model $\hat{G}(s)$ agrees with the considered plant $G(s)$ described by Eq. (23). Eliminating the common time delay for faster responses, Eq.(10) becomes

$$\hat{P}(s)^{(2)}=\frac{-1}{123.58}\begin{bmatrix}\frac{-19.4}{14.410s+1} & \frac{+18.9}{21.014s+1}e^{-2.005s}\\ \frac{-6.6}{10.895s+1}e^{-3.992s} & \frac{12.8}{16.695s+1}\end{bmatrix} \quad (28)$$

Frequency responses of the precompensated system $G(s)\hat{P}(s)^{(2)}$ are shown in Fig.6. They show good diagonal dominant properties. Therefore, two automaic regulating time series described by the illustrating example can be applied.
Applied the automatic regulating time series to the precompensated system. Parameters of Eqs.(16), (17), (20) and (21) are $\beta=0.9$, $J_G=1$, $T_S=200ms$, $\rho=200$, $\eta=0.8$ and $J_W=1$, $T_{cs1}=5.0\sec$, $T_{cs2}=8.5\sec$, and the found lead compensators are

$$C_1(z)=\left.\frac{21.102s+1}{21.102/\rho+1}\right|_{s=\frac{2}{T_S}\frac{z-1}{z+1}} \quad (29a)$$

and

$$C_2(z)=\left.\frac{18.303s+1}{18.303/\rho+1}\right|_{s=\frac{2}{T_S}\frac{z-1}{z+1}} \quad (29b)$$

Time response of the controlled system are shown in Fig.7. For comparison purposes, there are four other well-known one-line identification and design results given below:

Z-N Method[4]:

$K_P \equiv 9.035/2.2=4.1089; K_i=1.2/21.4=0.056$ with $\hat{P}(s)$

S-Y Method[5]:

$K_P \equiv 9.035/3=3.0132; K_i=0.5/21.4=0.0234$ with $\hat{P}(s)$

Wang's Method[8]:

$$K(s)=\begin{bmatrix}0.184(1+1/3.92s) & -0.0102(1+1/0.445s-0.804s)\\ -0.0674(1-1/4.23s+0.796s) & -0.0660(1+1/4.25s)\end{bmatrix} \quad (30)$$

Tsay's Method(PI/Lead)[19]:

$$K(s)=\begin{bmatrix}(\mathbf{4.6978}+0.3124/s)\times\frac{3.3098s+1}{0.0331s+1} & 0\\ 0 & (\mathbf{2.8955}+0.1541/s)\times\frac{4.2576s+1}{0.04257s+1}\end{bmatrix} \text{ with } \hat{P}(s)^{(2)}.$$

The gain margin of $G(s)\hat{P}(s)^{(2)}$ is 9.035, and oscillation period is 21.4seconds with relays in loops. Base on these two values, the Z-N and S-Y design methods [4, 5] give above values of K_P and

K_i . Time responses of the system controlled by four other methods are also shown in Fig. 7. Fig.7 shows the system performance controlled by the proposed method gives both better performance and good decoupling results than those of three mentioned methods and compatible to that of Tsay's method[19]. However the proposed method is simple and easy to implement.

Example 2. Consider the multivariable process given by Palmor et al. [27] as

$$G(s)=\begin{bmatrix} \frac{0.5}{(0.1s+1)^2(0.2s+1)^2} & \frac{-1}{(0.1s+1)(0.2s+1)^2} \\ \frac{1}{(0.1s+1)(0.2s+1)^2} & \frac{2.4}{(0.1s+1)(0.2s+1)^2(0.5s+1)} \end{bmatrix} \quad (31)$$

The estimating process with relays (H=0.5) is evaluated. The sustaining oscillation frequency $\omega_{c1}= 0.8878 rad/\sec$ for loop 1 closed with relay and time lag 3.0seconds. Using Eqs. (13) and (14), the found $\hat{G}_{j1}(0)$, and $\hat{G}_{j1}(j\omega_{c1})$ are

$$\hat{G}_{j1}(0)=\begin{bmatrix} 0.5 \\ 1.0 \end{bmatrix} \quad (32a)$$

and

$$e^{-2.6634j}\hat{G}(0.8878j)=\begin{bmatrix} e^{-0.04617j}/2.079 \\ e^{0.03939j}/1.0360 \end{bmatrix} \quad (32b)$$

The sustain oscillation frequency $\omega_{c2}= 0.8124 rad/\sec$ for loop 2 closed with relay and time lag 3.0seconds. The found $\hat{G}_{j2}(0)$, and $\hat{G}_{j2}(j\omega_{c2})$ are

$$\hat{G}_{j2}(0)=\begin{bmatrix} -1.0 \\ +2.4 \end{bmatrix} \quad (33a)$$

and

$$e^{-2.4372j}\hat{G}(0.8124j)=\begin{bmatrix} e^{-2.838j}/1.030 \\ e^{-0.07952j}/0.463 \end{bmatrix} \quad (33b)$$

The estimated model $\hat{G}(s)$ of the plant is

$$\hat{G}(s)=\begin{bmatrix} \frac{0.5}{0.320s+1}e^{-0.279s} & \frac{-1.0}{0.304s+1}e^{-0.196s} \\ \frac{1.0}{0.305s+1}e^{-0.196s} & \frac{2.4}{0.597s+1}e^{-0.409s} \end{bmatrix} \quad (34)$$

The precompensator $\hat{P}(s)$ is

$$\hat{P}(s)=\frac{1}{2.2}\begin{bmatrix} \frac{2.4}{0.597s+1}e^{-0.409s} & \frac{+1.0}{0.304s+1}e^{-0.196s} \\ \frac{-1.0}{0.305s+1}e^{-0.196s} & \frac{0.5}{0.320s+1}e^{-0.279s} \end{bmatrix} \quad (35)$$

Eliminating common time delay 0.196second of Eq.(35), giving

$$\hat{P}(s)^{(2)}=\frac{1}{2.2}\begin{bmatrix} \frac{2.4}{0.597s+1}e^{-0.213s} & \frac{+1.0}{0.304s+1} \\ \frac{-1.0}{0.305s+1} & \frac{0.5}{0.320s+1}e^{-0.083s} \end{bmatrix} \quad (36)$$

The frequency responses of the precompensated system $G(s)\hat{P}(s)^{(2)}$ are shown in Fig.8. They show good diagonal dominant properties.

Applied the automatic regulating time series to the precompensated system. Parameters of Eqs.(16), (17), (20) and (21) are $\beta = 0.9$, $J_G = 1$, $T_S = 20ms$, $\rho = 100$, $\eta = 0.8$ and $J_W = 1$, $T_{cs1} = 0.55\sec$, $T_{cs2} = 0.60\sec$, and the found lead compensators are

$$C_1(z)=\left.\frac{0.6786s+1}{0.6786/\rho+1}\right|_{s=\frac{2}{T_S}\frac{z-1}{z+1}} \quad (37a)$$

and

$$C_2(z)=\left.\frac{0.6712s+1}{0.6712/\rho+1}\right|_{s=\frac{2}{T_S}\frac{z-1}{z+1}} \quad (37b)$$

Time response of the controlled system are shown in Fig.9. For comparison purposes, there are four other well-known one-line identification and design results given below:

<u>*Z-N Method[4]*</u>:

$K_P \equiv 4.235/2.2 = 1.925; K_i = 1.2/1.629 = .736$ with $\hat{P}(s)$;

<u>*S-Y Method[5]*</u>:

$K_P \equiv 4.235/3 = 1.4117; K_i = .5/1.629 = .3069$ with $\hat{P}(s)$;

<u>*Wang's Method[8]*</u>:

$$K(s)=\begin{bmatrix} 2.83(1+1/0.285s) & 1.51(1+1/0.865s+0.0911s) \\ -3.25(1+1/0.785s+0.182s) & 0.667(1+1/0.776s) \end{bmatrix} \quad (38)$$

<u>*Tsay's Method(PI/Lead)[20]*</u>:

$$K(s)=\begin{bmatrix} (1.2755+1.835/s)\times\frac{0.2234s+1}{0.00223s+1} & 0 \\ 0 & (1.2755+1.835/s)\times\frac{0.2234s+1}{0.00223s+1} \end{bmatrix} \text{ with } \hat{P}(s)^{(2)}.$$

The gain margin of $G(s)\hat{P}(s)^{(2)}$ is 4.235, and oscillation period is 1.629 seconds with relays in loops. Base upon these two values, the Z-N and S-Y design methods give above values of K_P and K_i. Time responses of the system controlled by four other methods are also shown in Fig. 9. Fig.9 shows the proposed method gives both better performance and good decoupling results than those of three mentioned methods and compatible to that of the Tsay's method[19]. However, the proposed method is simple and easy to implement.

Example 3: Consider the solid-fuel boiler plant given by Johansson and Koivo [8]

$$G(s)=\begin{bmatrix} \frac{-1}{10.0s+1}e^{-2s} & \frac{-1}{10s+1} \\ 0 & \frac{1}{60s+1}e^{-10s} \end{bmatrix} \quad (39)$$

Estimating processes with relays (H=0.5). The sustain oscillation frequency $\omega_{c1}=0.3770 rad/\sec$ for loop 1 closed with relay and time lag 3.0seconds. Using Equations (13) and (14), the found $\hat{G}_{j1}(0)$, and $\hat{G}_{j1}(j\omega_{c1})$ are

$$\hat{G}_{j1}(0)=\begin{bmatrix} -1.0 \\ 0.0 \end{bmatrix} \quad (40a)$$

and

$$e^{-1.131j}\hat{G}(0.3770j)=\begin{bmatrix} e^{-0.05819j}/3.887 \\ 0 \end{bmatrix} \quad (40b)$$

The sustain oscillation frequency $\omega_{c2}=0.1326 rad/\sec$ for loop 2 closed with relay and time lag 3.0seconds. The found $\hat{G}_{j2}(0)$, and $\hat{G}_{j2}(j\omega_{c2})$ are

$$\hat{G}_{j2}(0)=\begin{bmatrix} -1.0 \\ +1.0 \end{bmatrix} \quad (41a)$$

and

$$e^{-0.3978j}\hat{G}(0.1326j)=\begin{bmatrix} e^{-1.324j}/1.660 \\ e^{-0.02913j}/8.016 \end{bmatrix} \quad (41b)$$

The found $\hat{G}(s)$ and $\hat{P}(s)$ are

$$\hat{G}(s)=\begin{bmatrix} \frac{-1.0}{9.963s+1}e^{-2.011s} & \frac{-1.0}{9.992s+1} \\ 0 & \frac{1}{59.984s+1}e^{-10.009s} \end{bmatrix} \quad (42)$$

and

$$\hat{P}(s)=\begin{bmatrix} \frac{1}{59.984s+1}e^{-10.009s} & \frac{+1.0}{9.992s+1} \\ 0 & \frac{-1.0}{9.963s+1}e^{-2.011s} \end{bmatrix} \quad (43)$$

$$\hat{P}(s)^{(2)}=\begin{bmatrix} \frac{1}{59.984s+1} & \frac{+1.0}{9.992s+1} \\ 0 & \frac{-1.0}{9.963s+1}e^{-2.011s} \end{bmatrix} \quad (44)$$

The frequency responses of the precompensated system $G(s)\hat{P}(s)^{(2)}$ are shown in Fig.10. They show good diagonal dominant properties.

Applied the automatic regulating time series to the precompensated system. Parameters of Eqs.(16), (17), (20) and (21) are $\beta=0.9$, $J_G=1$, $T_S=100ms$, $\rho=100$, $\eta=0.8$ and $J_W=1$, $T_{cs1}=12.3\sec$, $T_{cs2}=22.55\sec$, and the found lead compensators are

$$C_1(z)=\left.\frac{60.563s+1}{60.563/\rho+1}\right|_{s=\frac{2}{T_S}\frac{z-1}{z+1}} \quad (45a)$$

and

$$C_2(z)=\left.\frac{61.039s+1}{61.039/\rho+1}\right|_{s=\frac{2}{T_S}\frac{z-1}{z+1}} \quad (45b)$$

Time response of the controlled system are shown in Fig.11. For comparison purposes, there are four other well-known one-line identification and design results given below:

<u>Z-N Method[4]:</u>

$K_P \equiv 6.9562/2.2=3.162; K_i=1.2/72.95=.0164$ with $\hat{P}(s)$;

<u>S-Y Method[5]:</u>

$K_P \equiv 6.9562/3=2.3187; K_i=.5/72.95=.00685$ with $\hat{P}(s)$

<u>Wang's Method[8]:</u>

$$K(s)=\begin{bmatrix} 2.61(1+1/10s) & 3.08(1+1/58.8s+2.04s) \\ 0 & 3.14(1+1/60s) \end{bmatrix} \quad (46)$$

<u>Tsay's Method(PI/Lead)[19]:</u>

$$K(s)=\begin{bmatrix} (2.541+0.04744/s)\times\frac{11.37s+1}{0.1137s+1} & 0 \\ 0 & (2.541+0.04744/s)\times\frac{11.37s+1}{0.1137s+1} \end{bmatrix} \text{ with } \hat{P}(s).$$

The gain margin of $G(s)\hat{P}(s)^{(2)}$ is 6.956, and oscillation period is 72.95seconds with relays in loops. Base on these two values, the Z-N and S-Y design methods give above values of K_P and K_i. Time responses of the system controlled by four other methods are also shown in Fig. 11. Fig.11 shows the proposed method gives both better performance and good decoupling results than those of other four methods.

4 Conclusions

In this paper, an on-line identification and auto-tuning method has been proposed for analyses and designs of multivariable processes. The considered system was first estimated by use of an on-line identification method with FOPDT models, and then low-interacting precompensating matrix was applied to get diagonal dominant , and finally nonlinear automatic regulating time series was applied for each loop to get the wanted response times. No integration is needed to get zero tracking erros. The performance of the compensated systems were evaluated by wanted rising time and low-interacting characteristics. Three numerical examples gave the proposed method can provide better performance and less interaction effects than those of other mentioned on-line computing methods.

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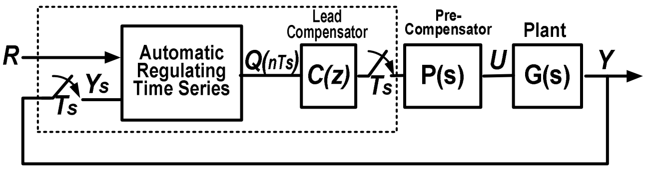

Fig.1. Multivariable Feedback Systems.

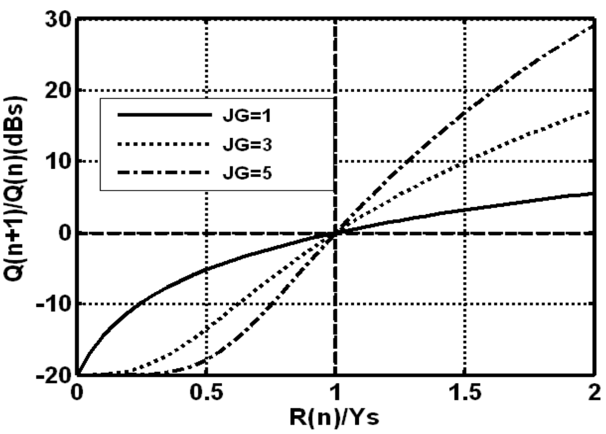

Fig.2. $Q(n+1)/Q(n)$ Versus R/Y_S for $\beta = 0.1$ and $J_G = 1,3,5$.

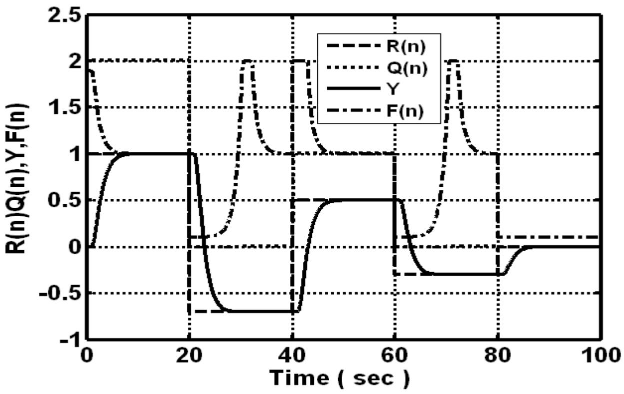

Fig.3. Time responses of the illustrating Example.

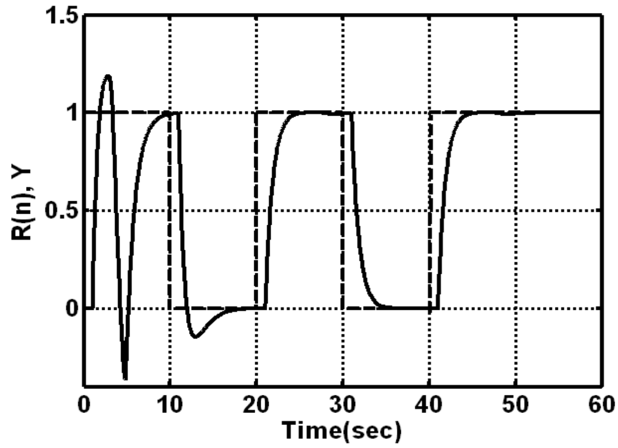

Fig.4. Time responses of the illustrating Example for finding C(z).

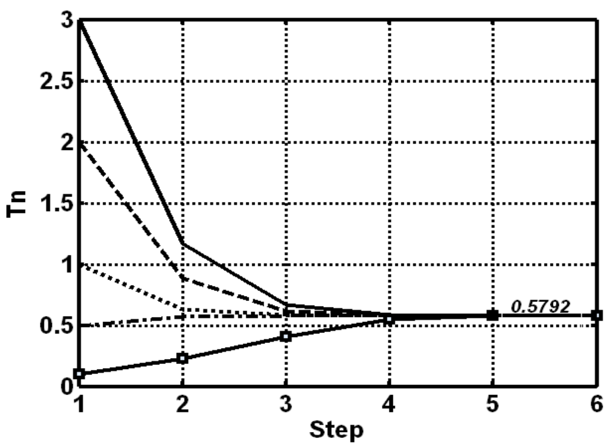

Fig.5. Converging properties for finding C(z) with different initial guesses.

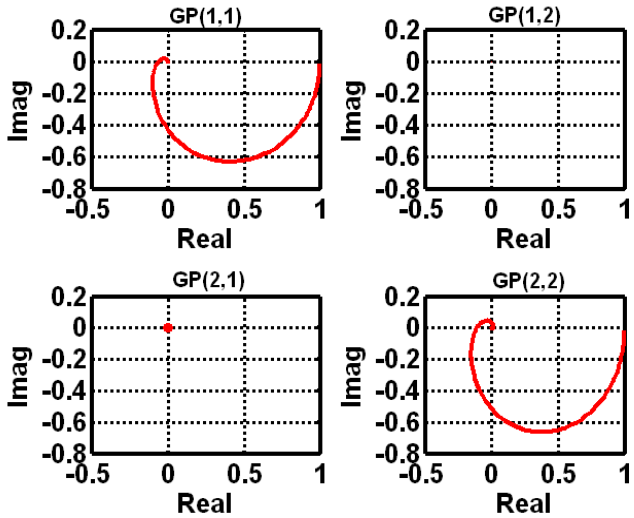

Fig.6. Frequency Responses of precompensated system $G(s)\hat{P}(s)^{(2)}$.

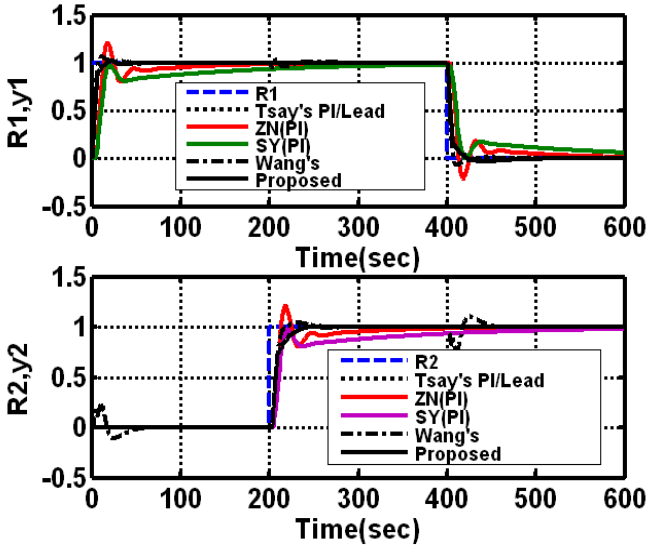

Fig.7. Time responses of Example 1 using diferent methods.

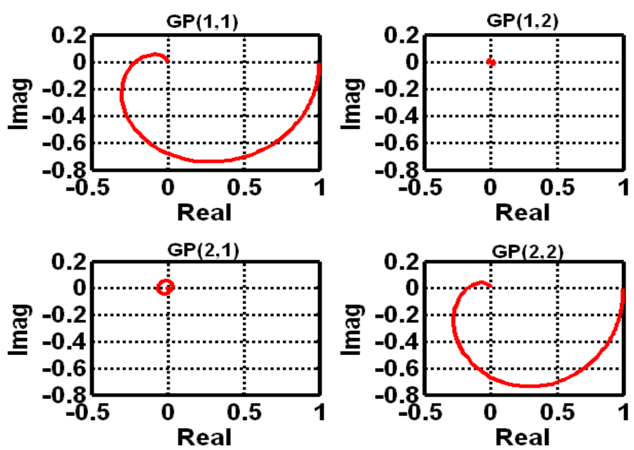

Fig.8. Frequency Responses of precompensated system $G(s)\hat{P}(s)^{(2)}$

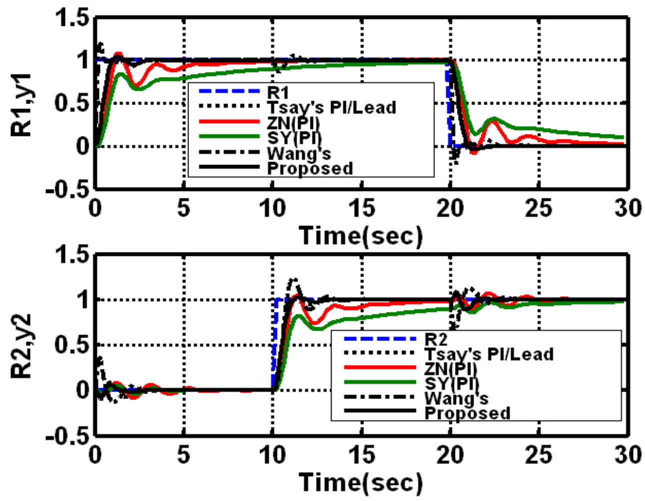

Fig.9. Time responses of Example 2 using diferent methods.

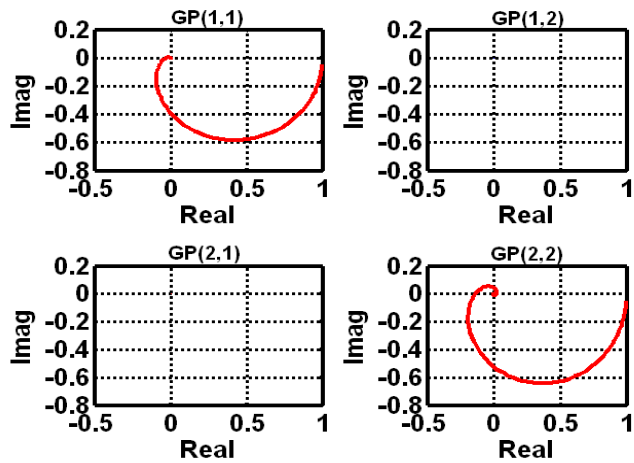

Fig.10. Frequency Responses of precompensated system $G(s)\hat{P}(s)^{(2)}$

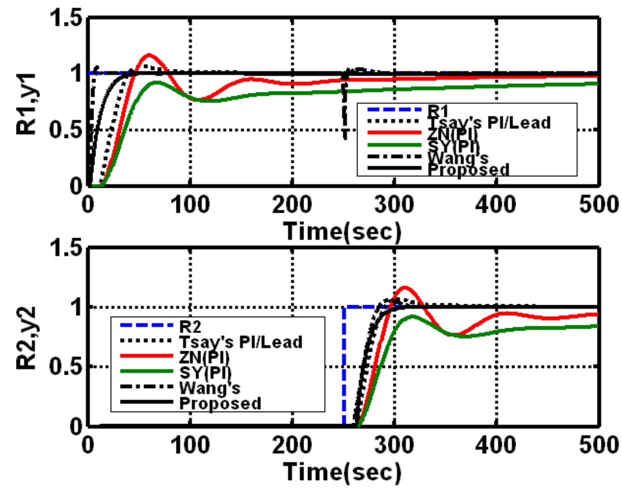

Fig.11 Time responses of Example 3 using diferent methods