A Bus Transport Network Model Based On Directed Random Walk

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Abstract: - On the basis of the analysis of the statistical data of bus transport networks of three major cities in China, we propose a novel evolution model of the bus transport network, which applies the strategy of random walk based on direction constrain matrix on two-dimensional grid and the mechanism of combining the geographical-close stations. We investigate some static properties, which include the topological structure and the spatial characteristics covering the neighboured road sections directions and the road section length distribution, and study the dynamic characteristic of the network traffic flow distribution analysed by the method of minimum spanning tree. The simulation results show that the statistical features of the model agree well with that of the actual networks.

Key-Words: - bus transport network model, directed random walk, combining the geographical-close stations, direction constrain matrix

1 Introduction

As an emerging discipline, Network science [1] has been got a rapid development. It usually takes the biological network [2], the social network [3-5], and the communication network [6,7] etc. as study targets, and utilizes statistical methods to analyze the complex systems that objectively exist in the world, and it also strives to explain the group behaviour on a system perspective. Recent years, a growing number of scholars are attracted to study the public transport network from the point of view of the network topology, including the subway network [8-10], the bus transport network [11-14], the air traffic network [15,16], the road network [17-19],the railway network [20, 22] and so on. These networks, as social fundamental service infrastructures, covering almost all the areas inside in cities and between regions, undertake abundant transportation missions, and occupy an irreplaceable position in the current society. Studying transport network in depth and exploring its spatial and temporal complexity and evolution mechanism is helpful to mitigate traffic congestion, optimize network structure and enhance network operational efficiency.

This paper focuses on the urban bus transport network (BTN), because the BTN has a large scale and covers a wide range of area, which occupies a dominant position in urban transportation. A BTN consists of bus routes and bus stations, and a bus route includes a certain number of bus stations. We adopt two methods to describe BTN, which are the L-Space method and the P-Space method [23, 24]. In the L-Space, a node represents a bus station; if both stations are neighboured in a bus route, there will be a link between both corresponding nodes in the network. In the P-Space, a node represents a station; if any two stations consisted in the same route, we link an edge between the both corresponding nodes. The network constructed by the L-Space reflects the geographical linking relationship among stations, while that constructed by the P-Space describes transfer relationship among stations when travelling in the network.

Based on public transport data of three major cities in China and constructing networks respectively in the L-Space and the P-Space, the statistical method in complex network is used to compute a serial of statistical parameters, including the spatial characteristics of the neighboured road sections directions, the road section length distribution, the dynamic characteristic of the traffic flow distribution which is used the method of minimum spanning tree to analyses, the topological properties that containing degree distribution, the direct routes distribution, clustering coefficient, average shortest path length, network diameter and so on. We find that the road section length is random and the neighboured road sections directions appear to always try to keep consistent, while a few trunk road sections are responsible for a large amount of even almost all traffic flow in the BTNs. In addition, statistical results indicate that the urban BTN is a typical small world network, and its evolution with the law of exponential distribution. According to the statistics, we put forward a novel method to reproduce the BTN's evolution process and construct a BTN model which considers the strategy of random walk based on direction constrain matrix on two-dimensional grid and the mechanism of combining the geographical-close stations; final results indicate our method works well.

This paper is organized as follows. In Sec.2, we analyse the spatial characteristics, the traffic flow distribution and the topology of public transport networks of three BTNs. In Sec.3, we propose a method to construct network model on two-dimensional grid, and compare the parameters with that of real networks. In Sec.4, a conclusion is given.

2 Statistical Properties of three BTNs

The data of BTNs used in this article are collected from the Internet [25, 26], including three cities which are Hangzhou, Shanghai and Beijing. The data only contains public bus routes, while excluding subway, light railways and other traffic patterns. Additionally, some stations with the same name and geographically adjacent to each other usually appear in BTNs, are not the same station and not included in the same route. Because it is very close to each other on geographic position, no routes connect among them. Travelling among these stations just need a short walk instead of transferring by bus. We combine these stations into one, and after combination, the geographical information of the single station is equivalent to the average value of latitude and longitude of all stations before merging.

After processing the data, the scale of three cities are listed as follows: Hangzhou containing 276 routes with 1828 stations, Shanghai 457 routes with 4498 stations, and Beijing 669 routes with 5299 stations.

We investigate the spatial characteristics, the traffic flow distribution and the topological characteristics in the BTNs as flows.

2.1 The analysis of road section length

Connecting any neighbored stations within a route by a straight line, the line segment between the both stations is called a road section without considering the routes' actual travelling pathway on the road which approximates to a linked edge in the L-Space. According to the stations' latitude and longitude information, we calculate the linear length between any two neighboured stations existing within the same line, and take it as the length of the road section constructed by corresponding stations. Here, $P(m)$ represents the probability that a road section length equals to m, where the road section can be selected randomly from an arbitrary route. As is shown in fig.1, $P(m)$ conforms to normal distribution, i.e.

$$P(m) \propto \frac{1}{\theta} \cdot e^{-(m-\nu)^2/2\theta^2} \quad (1)$$

where the fitting parameters ν and θ is shown in table 1.

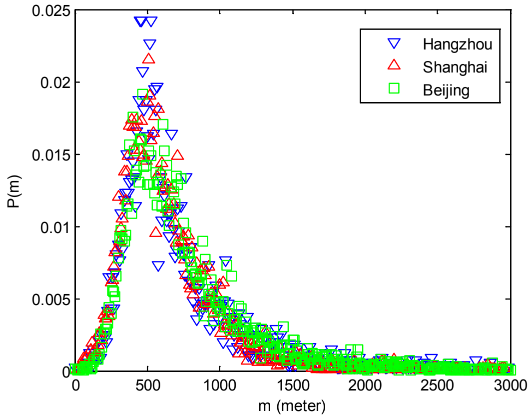

Fig.1 The road section length distribution in three BTNs

Table 1 Fitting parameters of three BTNS

	Hangzhou	Shanghai	Beijing
ν	649.6880	624.8500	688.5540
θ	310.8530	294.9480	327.4970
μ	21.2536	22.8840	27.9851
σ	6.7448	7.7202	12.7798
α	0.2596	0.3571	0.2150
β	0.3506	0.3558	0.3736
γ	0.0212	0.0225	0.0110
φ	0.7043	0.7156	0.6494

2.2 The analysis of the neighbored road sections directions

Next, we discuss the road section directions and the mutual relationship of neighboured road sections directions in bus routes. Firstly, with the latitude and longitude information coordinating, bus stations on the earth sphere surface are approximate to put down on the two-dimensional plane. Moreover, the two-dimensional plane is divided averagely into 8 directions which are A, B, C, D, E, F and G as shown in fig.2. The Table 2a, 2b, 2c show the direction statistical results based on the directions of neighboured road sections within a same route, and it can be explicated that if the direction of current of current road section s_1s_2 (s_1 points to s_2) is *x*, then the road section s_2s_3 (s_2 points to s_3) sharing the same station s_2 with road section s_1s_2. The probability of whose direction is *y* is the value of the position *(x,y)* in the table. Taking the *E* row in table 2b for example, assuming that a route crosses s_1 , s_2 , s_3 sequentially, the direction of current road section s_1s_2 (s_1 points to s_2) is E, and the probabilities of the direction of road section s_2s_3 (s_2 points to s_3) be A, B, C, D, E, F, G, H are as follows 0.0115, 0.0189, 0.0842, 0.1397, 0.4482, 0.1956, 0.0867, 0.0152. We find that a road section direction always tries to be consistent with its neighbours', which means there only exists a little difference between the neighboured road sections directions, i.e. making a further illustration that if the direction of current road section s_1s_2 (s_1 points to s_2) is E. Then the road section s_2s_3 (s_2 points to s_3) has much higher probability to direct with E, D and F, while with little chance to point to A, B and H. These results indicate why bus routes are highly directed in reality.

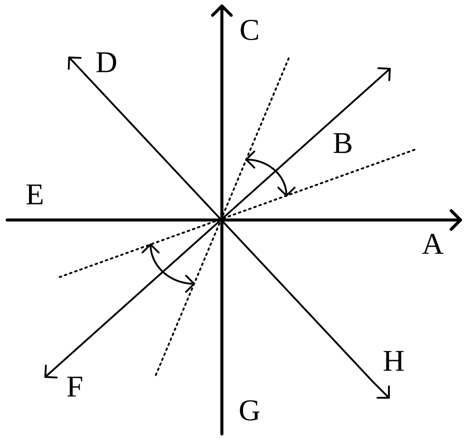

Fig.2 the dividing directions in two- dimensional plane

Table 2a Statistics of road section direction based on its neighbours' in Hangzhou BTN

	A	B	C	D	E	F	G	H
A	0.5572	0.1607	0.0622	0.0172	0.0068	0.0117	0.0683	0.1158
B	0.2191	0.4685	0.1962	0.0450	0.0155	0.0033	0.0114	0.0409
C	0.0688	0.1632	0.5195	0.1547	0.0802	0.0092	0.0007	0.0035
D	0.0277	0.0524	0.2261	0.4132	0.1922	0.0514	0.0082	0.0288
E	0.0088	0.0063	0.0629	0.1151	0.5654	0.1660	0.0591	0.0164
F	0.0067	0.0017	0.0084	0.0505	0.2212	0.4743	0.1960	0.0412
G	0.0736	0.0064	0.0014	0.0106	0.0722	0.1628	0.5230	0.1500
H	0.1882	0.0620	0.0165	0.0052	0.0300	0.0610	0.2275	0.4095

Table 2b Statistics of road section direction based on its neighbours' in Shanghai BTN

	A	B	C	D	E	F	G	H
A	0.4470	0.1845	0.0830	0.0155	0.0098	0.0168	0.0892	0.1542
B	0.2063	0.4059	0.1970	0.0645	0.0190	0.0133	0.0265	0.0676
C	0.0745	0.1548	0.4669	0.1693	0.0738	0.0211	0.0243	0.0153
D	0.0196	0.0883	0.2385	0.3353	0.1988	0.0825	0.0227	0.0143
E	0.0115	0.0189	0.0842	0.1397	0.4482	0.1956	0.0867	0.0152
F	0.0199	0.0119	0.0279	0.0743	0.2009	0.4071	0.1863	0.0717
G	0.0740	0.0238	0.0245	0.0177	0.0718	0.1617	0.4664	0.1599
H	0.1850	0.0895	0.0280	0.0119	0.0221	0.0798	0.2487	0.3350

Table 2c Statistics of road section direction based on its neighbours' in Beijing BTN

	A	B	C	D	E	F	G	H
A	0.6669	0.0992	0.0572	0.0072	0.0043	0.0074	0.0592	0.0986
B	0.2486	0.3490	0.2774	0.0411	0.0205	0.0082	0.0141	0.0411
C	0.0688	0.0872	0.6714	0.0994	0.0566	0.0048	0.0049	0.0068
D	0.0160	0.0386	0.2678	0.3717	0.2472	0.0366	0.0156	0.0066
E	0.0036	0.0041	0.0618	0.0993	0.6662	0.0886	0.0705	0.0060
F	0.0115	0.0055	0.0087	0.0459	0.2807	0.3482	0.2554	0.0441
G	0.0596	0.0029	0.0049	0.0056	0.0556	0.0969	0.6723	0.1022
H	0.2530	0.0400	0.0151	0.0098	0.0167	0.0375	0.2603	0.3676

1) Using edge betweenness to represent traffic flow. For BTNs, the shortest path in the P-Space means least transfers when travelling. In the P-Space, we use BSF (Breadth-First Search) algorithm to get shortest paths between any node pairs. Then, according to the detail connections relationship among stations in routes, edge betweenness in the L-Space is got by a further computation, which stands for the traffic flow on the edge.
2) Using the number of routes between neighbored stations that in a route to represent traffic flow. In BTNs, stations are not equal at all, for example, the stations in the urban center are usually more important to connect more routes. It’s reasonable we believe that in the L-Space, the more routes between neighbored stations in any route, then the lager of the traffic flow on the corresponding road sections. So, the number of routes between neighbored stations is used to stands for traffic flow on the edge in the L-Space.

The parameters of MST’s traffic flow-sharing-rate ρ and relative-importance τ listed in table 3 indicate that, regardless of edge betweenness or the number of routes between neighboured stations that in a route representing traffic flow, the main road sections in the BTNs corresponded by respective MST share most of the traffic flow of the network, and play a leading role in the transportation of entire network.

2.4 The analysis of the number of stations included in a route

In BTNs, each route connects a certain number of stations. $P(z)$ stands for the probability that a route containing z stations, according to fig.3, $P(z)$ conforms to normal distribution, i.e.

$$P(z) \propto \frac{1}{\sigma} \cdot e^{-(z-\mu)^2/2\sigma^2} \tag{4}$$

which means that the number of stations included in a route is limited, which usually changes around the average value μ at a certain range of $\sigma/2$, where the fitting result of the parameters μ and σ is shown in table 1.

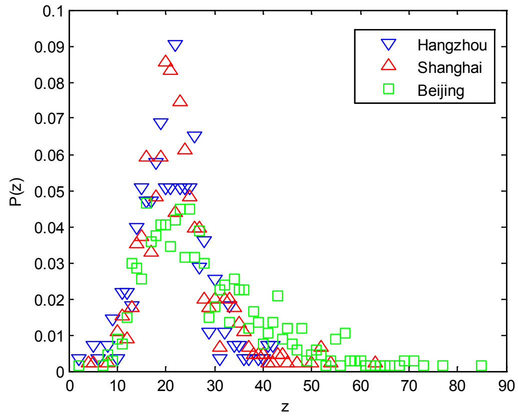

Fig3. The distribution of the number of stations that included in a route in three BTNs

2.5 The analysis of the number of routes that a station belongs to

In order to transfer conveniently, a part of stations usually possess several routes. But taking the roads condition and the buses operation cost into consideration, the stations owning many routes only occupy a relatively smaller proportion. On the basis of statistics, the distributions of the number of bus routes that a station belongs to in three BTNs fit exponential distributions and their cumulative distributions are exponent distributions that owning the same exponent, which can be described that

$$P(s) \propto e^{-\alpha s} \tag{5}$$

where $P(s)$ represents the probability of the number of routes at least s that a station belongs to, and α is the exponent of the cumulative distribution. In order to decrease large fluctuation, cumulative distribution is applied to simulate, and the result is shown in fig.4, while the fitting result of α listed in table 1 and the average number of routes $<s>$ that a station belongs to shown in table 3.

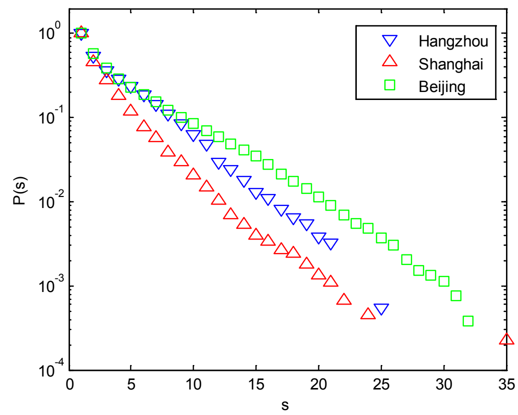

Fig4. The cumulative distribution of the number of bus routes that a station belongs to in three BTNs.

2.6 The analysis of degree distribution

The degree of a node means the number of nodes that connecting to it, which reflects the density of connecting between the node and others. The degree distribution function $f(k)$ is used to describe the probability that any node whose degree is k, and it's a significant standard of judging the network topological structure. Regardless of the statistics in the L-Space or in the P-Space, Both the degree distribution conform exponential distributions. The expressions

$$P_L(k) \propto e^{-\beta k}, \quad (6)$$

$$P_P(k) \propto e^{-\gamma k} \quad (7)$$

are cumulative degree distribution functions respectively in the L-Space and in the P-Space, whose simulations are shown in fig.5 and fig.6, and the fitting parameters β and γ are listed in table 1. The average degree refers to the average value of the degrees of all the nodes in the whole network, which reflects the density of linking among all nodes. $<k_L>$ and $<k_P>$ are average degree in the L-Space and in the P-Space which are listed in table 3.

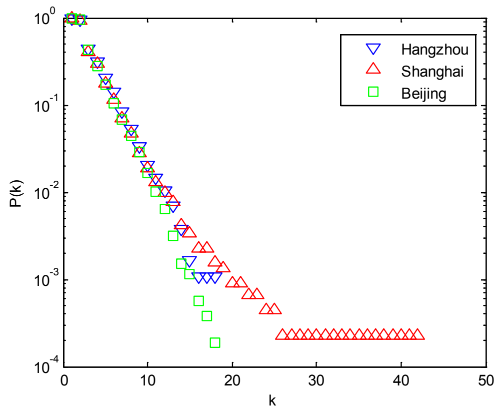

Fig5.Cumulative degree distribution in L-Space of three BTNs

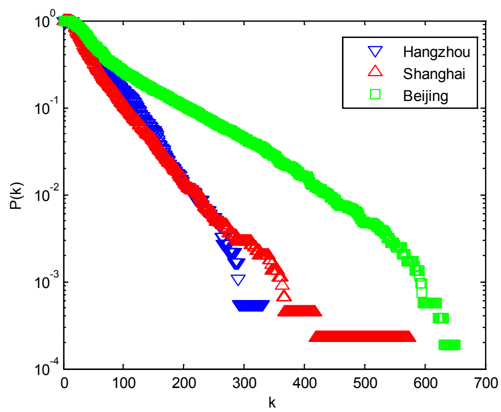

Fig6.Cumulative degree distribution in P-Space of three BTNs

2.7 The analysis of the number of direct routes

Whether existing direct routes or not any both stations and the number of direct routes influences obviously on convenient extent when travelling between them. In the P-Space, a link proves that there is at least one direct route between both corresponding stations. We compute the distribution of the number of direct routes among node pairs with the condition that at least one direct route between any pair nodes, and the function $f(h)$ represents the probability that h routes between a pair nodes. Statistics indicates that $f(h)$ is approximately exponent distribution. As is shown in fig.7, $P(h)$ is the cumulative distribution of $f(h)$, which can be described as

$$P(h) \propto e^{-\varphi h} \quad (8)$$

where the fitting parameter of exponent φ is in fig.1. Moreover, the parameter $<h>$ shown in table 3 is the average of direct routes among node pairs with the condition that at least one direct route between anyone pair nodes.

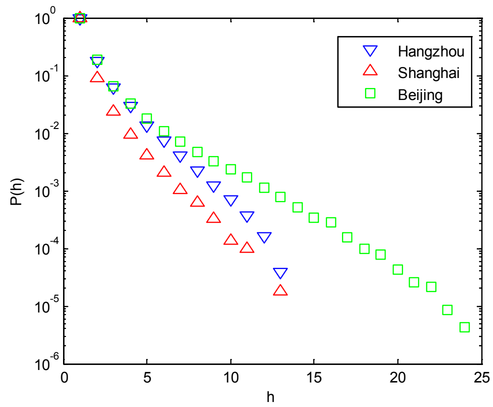

Fig7. Cumulative distribution of the number of direct routes in three BTNs

Table 3 Statistical parameters of three BTNs

	Hangzhou	Shanghai	Beijing
ρ_b	0.7938	0.7735	0.8286
τ_b	2.4671	1.9347	2.7158
ρ_r	0.7404	0.7309	0.7792
τ_r	1.8345	1.5404	1.9910
$<s>$	3.2090	2.3250	3.5331
$<k_L>$	3.2867	3.1342	3.1281
$<k_P>$	55.1488	50.3628	87.0538
$<h>$	1.3019	1.1301	1.3313
C	0.7215	0.7469	0.7179
L	2.6699	2.9679	2.8730
d	5	5	6

2.8 Clustering coefficient, average shortest path length and network diameter

A node's clustering coefficient [27] reflects the density of the connecting among all the node's neighbours. Without considering multiple edges, supposing that a node i has k_i neighbours, and the clustering coefficient C_i of the node i is defined as follows: $\mathrm{C_i} = E_i / (k_i(k_i - 1)/2)$, where E_i is the number of edges that exist in reality among the k_i neighbours, while the expression $k_i(k_i - 1)/2$ stands for the maximum number of edge that may exist among the k_i neighbours. The clustering coefficient C of network refers to the average value of all nodes' clustering coefficient in the whole network, i.e.

$$C = \frac{1}{n} \cdot \sum_{1}^{n} E(i) \tag{9}$$

where n is the number of nodes in the network. In the P-Space, the network's clustering coefficient reflects the transfer convenient extent of travelling.

In un-weighted networks, the shortest path length d_{ij} between any both nodes i and j refers to the minimum links from i to j. The average path length [24] L is the average value of the shortest path length between any both nodes, i.e.

$$L = \frac{1}{n(n-1)/2} \sum_{i<j} d_{ij} . \tag{10}$$

In BTNs, the expression that $L-1$ is the average transfer times from any station to another.

The network diameter d refers to the maximum value among all the shortest path length between any both nodes, i.e. $d = \max(d_{ij})$. In BTNs, the minimum transfer times when travelling between any both stations will not exceed $d-1$.

In the P-Space, the clustering coefficient C, the average shortest path length L and the network diameter d are calculated and listed in table 3, which indicate that in the P-Space, the BTNs is a typical small world [27] with high clustering coefficient and small average shortest path length. This characteristic of BTNs provides a promise that travelling between any both stations can be accomplished within a tolerable and limited transfer times.

Synthesizing all above-mentioned simulations and statistics, we make a conclusion as follows. First, as a spatial network, the road section length is random in BTNs whose distribution is a normal distribution, and the neighboured road sections directions are shown as consistent as possible. All those explain that why in real BTNs the lengths between neighboured stations are usually not equal but without a large difference and the routes have strong directivity instead of running non-directed. Second, the main road sections in BTNs undertake a large amount of transportation missions, and usually these road sections are easily to appear congestions. Moreover, according to the topological statistics, the characteristic of exponent distributions appear existing which is due to that from the system point and selecting original stations with a random mechanism from existing BTNs when joining a new bus route.

3 BTN model based on directed random walk strategy

The BTNs are typically growing spatial networks with the expansion of cities and the bus routes increasing, whose sizes are becoming larger and larger. The incensement of BTNs doesn't drive on joining in a node each time, but base on joining a route which usually contains some old stations that existed in the original networks and some new stations that will be established with joining the current route. Barabási [28] points out that a network with exponent degree distribution can be generated by the mechanism of increasing and random connecting. Due to the characteristic of exponent distribution of real BTNs, the mechanism of random connecting is used to construct the BTN model. Meanwhile, considering the statistics of the directions of neighboured road sections and the road section length distribution in real BTNs, we propose a method that using the strategy of random walk based on direction constrain matrix on two-dimensional grid and the mechanism of combining the geographical-close stations to construct the bus transport network model.

3.1 Direction constrain matrix

As known from the discussion in the section 2.2, in the real BTNs, the neighboured road sections directions always try to keep consistent, and the routes are highly directed. To reproduce this characteristic, we need a direction constrain matrix to make sure that selecting a direction randomly with unequal probabilities based on current road section direction and then getting the neighboured road section direction. It's easy to find that, in section 2.2, we compute the statistics of the road section direction based on its neighbours' from the real BTNs, but constructing a BTN model is an opposite process, so we need to take the statistics of road section direction based on its neighbours' as direction constrain matrices to construct a BTN model. The tables 2a, 2b and 2c all can be taken as direction constrain matrices.

3.2 Modelling method

The detailed modelling method is described as follows, and its diagrammatic sketch is shown in fig.8.

1) On the two- dimensional grid, there is only one station in the original network.
2) Constructing a road section of the current joining route.
   a) Select a station S_1 randomly from the existing network and get a direction *D* by a random number that ranges from 0 to 1 generated by uniform distribution function;
   b) Get a random number *dis* generated by the normal distribution function $normal(\nu,\theta)$, and extend *dis* units along the direction *D*, then take the reaching position as the coordinate of the station S_2;
   c) Adjust the coordinate of S_2 to the nearest position with the integer coordinate. If there isn't a station at the position of S_2, and then establish a new one; if the coordinate of S_2 is as same as that of S_1, then return to the step b);
   d) Connect the station S_1 and S_2, and generate a new road section, then record the both directions D_1 and D_2 of the road section, where D_1 is form S_1 to S_2, and D_2 is form S_2 to S_1;
3) Use the normal distribution function $normal(\mu,\sigma)$ to generate a random number *A* as the total number of stations in the current joining route, use a uniform distribution function to generate a random number A_1 $(0\le A_1\le A\text{-}2)$ as the number of stations extend on the extreme point S_2 and then compute $A_2=A\text{-}2\text{-}A_1$, which is the number of stations that extend on the extreme point S_1.
4) Extend the A_1 stations on the extreme point S_2.
   a) Make sure that the current road section direction $curD=D_1$ and $S=S_2$, where *S* is the starting station of extending;
   b) Use uniform distribution function to generate a random number that ranges from 0 to 1 and then based on the direction *curD* and the restrictive direction matrix, compute the extending direction *D*, where *D* is from *S* to the current joining station.
   c) Get a random number *dis* generated by the normal distribution function $normal(\nu,\theta)$, and extend *dis* units along the direction *curD*, and then take the reaching position as the coordinate of the station S_{add};
   d) Adjust the coordinate of S_{add} to the nearest position with the integer coordinate. If there isn't a station at the position of S_{add}, and then establish a new one; if the coordinate of S_{add} is as same as that of S, then return to the step b);
   e) Connect the stations *S* and S_{add}, and update the information that $curD=D$ and $S=S_{add}$;
   f) Repeat the step b)—e), until A_1 stations are completed extending.
5) According to step 4), extend the A_2 stations on the extreme point S_1.
6) Repeat the step 2)—5), until all the routes joined into the network.
7) Combine the geographical-close stations that are not included in the same routes. The network constructed by the above-mentioned steps contains a large number of geographical-close stations. If stations that among whom any both are close to each other and the distance between them is less than a given value *comDis*, we combine them into one, and after combining, its coordinate equals the average of coordinate of all stations before combining.

In accordance with the above-mentioned method, the statistics of Shang BTN is restrictive conditions to construct BTN model, which are that the total routes is 457, the function $normal(\nu,\theta)$ that generating the length of road section is $normal(624.8500, 294.9480)$, the function $normal(\mu,\sigma)$ that generating the number of stations in a route is $normal(22.8840, 7.7202)$, as the direction constrain matrix is shown in table 3b. We also make the parameter *comDis* equal to 200. Based on the above parameters, the BTN model is constructed, which contains 4637 stations. The simulations and statistics are shown in fig.9 to fig.15 and table 4 to table 6.

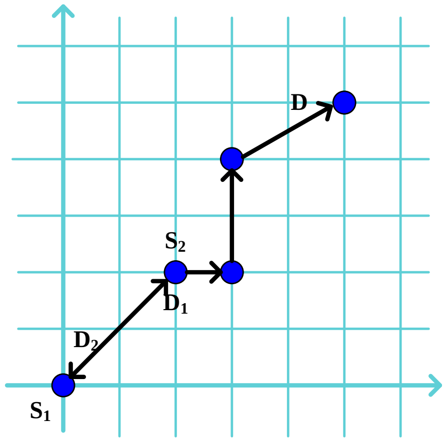

Fig.8 The diagrammatic sketch of the modelling method

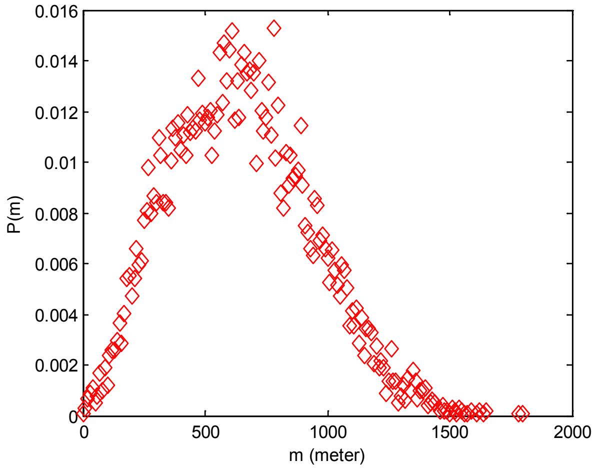

Fig.9 The road section length distribution in BTN model

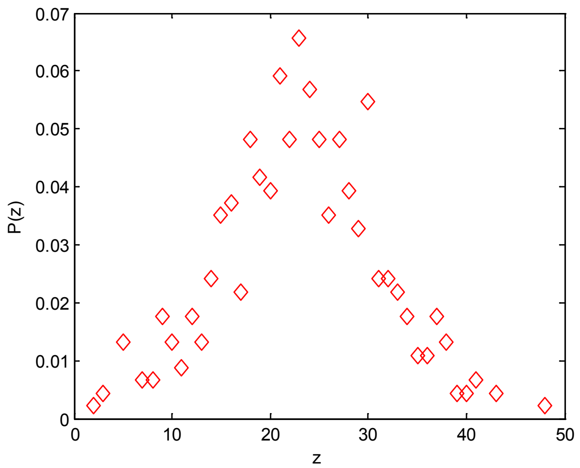

Fig.10 The distribution of the number of stations that included in a route in BTN model

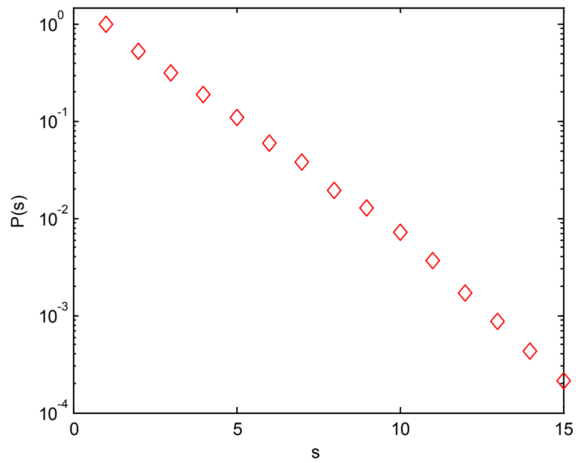

Fig.11 The cumulative distribution of the number of bus routes that a station belongs to in BTN model

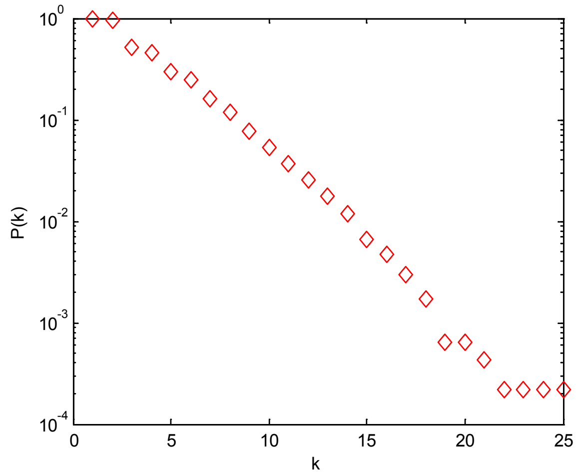

Fig.12 Cumulative degree distribution in L-Space of of BTN model

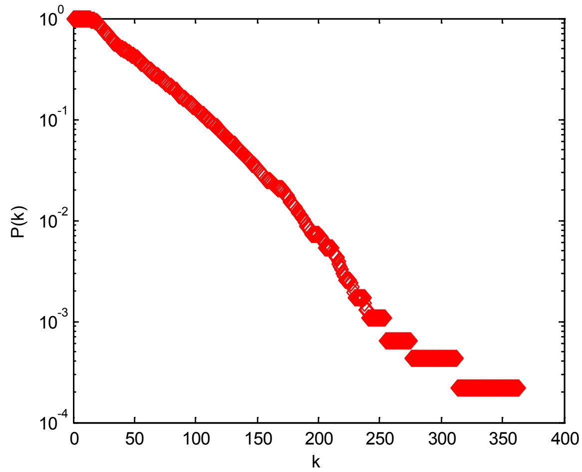

Fig.13 Cumulative degree distribution in P-Space of BTN model

Table 4 Fitting parameters of BTN model

μ	σ	α	β	γ	φ	ν	θ
23.1707	7.8294	0.4266	0.2860	0.0239	0.9495	646.3910	285.7860

Table 5 Statistics of road section direction based on its neighbors' in BTN model

	A	B	C	D	E	F	G	H
A	0.4210	0.1981	0.0952	0.0168	0.0080	0.0196	0.0884	0.1529
B	0.2055	0.3919	0.1900	0.0706	0.0200	0.0155	0.0290	0.0775
C	0.0791	0.1709	0.4194	0.1752	0.0795	0.0255	0.0288	0.0216
D	0.0219	0.0857	0.2387	0.3162	0.1948	0.0969	0.0306	0.0153
E	0.0161	0.0194	0.0952	0.1359	0.4242	0.2032	0.0887	0.0173
F	0.0196	0.0147	0.0274	0.0809	0.2022	0.3926	0.1940	0.0686
G	0.0850	0.0241	0.0216	0.0270	0.0857	0.1679	0.4200	0.1686
H	0.1725	0.1013	0.0384	0.0113	0.0215	0.0885	0.2492	0.3173

Table 6 Statistical parameters of BTN model

$<s>$	$<k_L>$	$<k_P>$	$<h>$	C	L	d	ρ_b	τ_b	ρ_r	τ_r
2.2836	3.9983	53.9099	1.0510	0.6984	3.6632	10	0.6451	1.8161	0.5406	1.1756

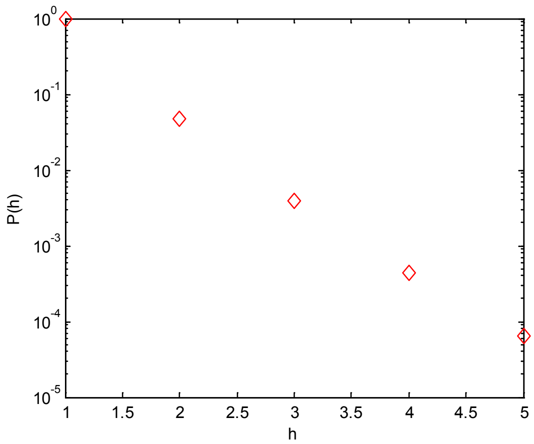

Fig.14 Cumulative distribution of the number of direct routes in BTN model

We make a comparison on the simulations and statistics between the BTN model and the Shanghai BTN:

1) Constructed in light of the same number of routes, the size of the BTN model is approximately equal to that of the real BTN;
2) Respectively compare fig.9 and fig1, fig.10 and fig.3, fig.11 and fig.4, fig.14 and fig.7, results indicate that each pair of simulations fits well, and their tendency keep consistent;
3) As shown in fig.12 and fig.13 , the results show that the degree distributions conform exponent distribution regardless of in the L-Space or in the P-Space, and the simulations respectively fit well with fig.5 and fig.6, which indicate that the BTN model's topological structure is same to the real's.
4) Compare table 4 and table 1, the fitting parameters approximately equal.
5) Compare table 5 and table 2b, the direction statistics of BTN model are almost the same to the real's, which indicate that the routes are as directed as the real's in BTN model instead of random.
6) Compare table 3 to table 6, statistics fit well. It's worthwhile to note that in the P-Space, the average shortest path length and the network diameter in the BTN model are larger than that in the Shanghai BTN. k in real BTNs, usually there are a few long-distance routes that almost all road sections in these routes with large length in order to travel conveniently in order to travel conveniently, and it causes that the long-distance routes cover a large area of the city. When travelling between some stations at a far distance, the transfer times decrease sharply by crossing the long-distance routes which cause that the average shortest path length and network diameter decrease on the whole in real BTNs. However, in BTN model, when constructing routes, the length of road section is generated randomly and the probability of getting large value is so small. It's impossible to promise that continually generating road sections with

large length to construct long-distance routes which cause that the number of long-distance routes in the BTN model is much less than that in the real BTN, so a little difference emerges.

According to above-mentioned result, the BTN model constructed by our method possesses the same properties with the real BTNs. So it's certain that the method applied to construct BTN model reflects the dominant mechanism in the process of evolution of real BTNs.

4 Conclusion

In this paper, we study three BTNs in China. The results shows that in different BTNs, there are the same properties including topological structure, spatial characteristics and the characteristic of network traffic flow distribution, i.e. degree distributions conform exponent distribution. The neighboured road sections directions always try to keep consistent and the routes are highly directed, and the road section length is random and its distribution is normal distribution, the small word characteristic, etc. All facts indicate that there are same and inherent mechanisms in the BTNs evolutionary process, even the BTNs corresponding different cities. According to the statistics, we propose a method using the strategy of random walk based on directions on two-dimensional grid, the mechanism of combining the geographical-close stations to construct the bus transport network model and the comparisons state clearly that our method reflects the main factor and indispensable mechanism in the BTNs evolutionary process.

Public transport network which is the traffic artery plays an irreplaceable role in the urban transportation. The study in this paper is useful to forecast the trend of the urban public transport network's development and network congestion, to a further significance, there is positive reference worth to optimize network structure and improve the efficiency of network operation.

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