The term structure model of corporate bond yields

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Abstract: - We build the term structure of corporate bond yields with N-factor affine model, and we estimate the parameters by using Kalman filtering. We choose weekly average corporate bond yields data in Shanghai Stock Exchange and Shenzhen Stock Exchange. We find the one-factor model and two-factor model could do one-step forward forecasting well, but the three-factor model could fit the observable data well.

Key-Words: - corporate bond; yields; term structure; Kalman filtering

1 Introduction

Many scholars research on term structure affine models of bonds. The literatures are as below. Some scholars find the three factor model fits observable data well. Dai, Singleton(2000) [1] analyzes the structural differences and goodness-of-fits of affine term structure models. Some models are good at modeling the conditional correlation, some are good at modeling volatilities of the risk factors. He extends N-factor affine model into N+1-factor affine model. Vasicek (1977) Cox, Ingersoll, and Ross (1985) [2,3]assume instantaneous short rate r(t) is the equation of N-factor state variable Y(t), and r(t)=$\alpha_0 + \alpha_y'$Y(t), and Y(t) follows Gaussian and square root diffusions. Some scholars extend Markov one factor short rate model, and add in a stochastic long-run mean $\theta(t)$ and a volatility v(t) of r(t), dr(t)=($\theta - r(t)$)dt+$\sqrt{v}dB(t)$. These models come from bond pricing and interest-rate derivatives. Duffee(2002)[4] considers affine model can't forecast treasury yields. He thinks assuming yields follow stochastic random walk and forecasting results are good. He considers the models failure for the reason that variation of risk compensation is related with interest-rate volatility. He raises essential affine model, and the model keeps the advantage of standard model, but it makes interest-rate variation independent from interest-rate volatility, and this is important for forecasting future yield. Jong(2000) [22] analyzes term structure affine model combining with time series and cross-section information, and he uses discretization continuous time to do Kalman filtering. He finds the three factors model could fit cross-section and dynamic term structure model. Duffie, Kan(1996) [5] finds yields with fixed maturity follow stochastic volatility multi-parameters Markov diffusion process by using continuous no arbitrage multi-factor model of interest-rate term structure. He uses jump-diffusion to solve interest-rate term structure model. Longstaff and Schwartz(1995) [6] evaluate corporate bonds value which have default risk and interest-rate risk by using simple methods. He finds the relation between default risk and interest-rate has important effect on credit spread. Also, he finds credit spread correlates with interest-rate negatively, and the risky bond duration depends on interest rate. He uses V to represent corporate total asset value, and it follows dynamic variation below: dv=$\mu V dt + \sigma v dz_1$, and σ is constant, and z_1 is

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$$\begin{bmatrix} \beta_{11} & 0 & 0 & 0 & 0 \\ 0 & \beta_{22} & 0 & 0 & 0 \\ 0 & 0 & \beta_{33} & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & \beta_{nn} \end{bmatrix} \begin{bmatrix} f_{1t} \\ f_{2t} \\ f_{3t} \\ \vdots \\ f_{nt} \end{bmatrix} dt$$

=

$$\begin{bmatrix} k_1-\beta_{11} & 0 & 0 & 0 & 0 \\ 0 & k_2-\beta_{22} & 0 & 0 & 0 \\ 0 & 0 & k_3-\beta_{33} & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & k_n-\beta_{nn} \end{bmatrix} \times$$

$$\left[\begin{bmatrix} k_1-\beta_{11} & 0 & 0 & 0 & 0 \\ 0 & k_2-\beta_{22} & 0 & 0 & 0 \\ 0 & 0 & k_3-\beta_{33} & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & k_n-\beta_{nn} \end{bmatrix}^{-1} \times \begin{bmatrix} \beta_{10} \\ \beta_{20} \\ \beta_{30} \\ \vdots \\ \beta_{n0} \end{bmatrix} - \begin{bmatrix} f_{1t} \\ f_{2t} \\ f_{3t} \\ \vdots \\ f_{nt} \end{bmatrix} \right] dt + \begin{bmatrix} \sigma_1 & 0 & 0 & 0 & 0 \\ 0 & \sigma_2 & 0 & 0 & 0 \\ 0 & 0 & \sigma_3 & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & \sigma_n \end{bmatrix} \begin{bmatrix} dw_{1t} \\ dw_{2t} \\ dw_{3t} \\ \vdots \\ dw_{nt} \end{bmatrix} \tag{4}$$

$\beta_{10}, \beta_{20}, \beta_{30}, \dots \beta_{n0}$ denote the fixed interest rate risk premium. $\beta_{11}, \beta_{22}, \beta_{33}, \dots \beta_{nn}$ denote the time varying interest rate risk premium.

$$K = \begin{bmatrix} k_1-\beta_{11} & 0 & 0 & 0 & 0 \\ 0 & k_2-\beta_{22} & 0 & 0 & 0 \\ 0 & 0 & k_3-\beta_{33} & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & k_n-\beta_{nn} \end{bmatrix},$$

$$\theta = \begin{bmatrix} \frac{\beta_{10}}{k_1-\beta_{11}} & 0 & 0 & 0 & 0 \\ 0 & \frac{\beta_{20}}{k_2-\beta_{22}} & 0 & 0 & 0 \\ 0 & 0 & \frac{\beta_{30}}{k_3-\beta_{33}} & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & \frac{\beta_{n0}}{k_n-\beta_{nn}} \end{bmatrix},$$

$$\partial = \begin{bmatrix} \sigma_1 & 0 & 0 & 0 & 0 \\ 0 & \sigma_2 & 0 & 0 & 0 \\ 0 & 0 & \sigma_3 & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & \sigma_n \end{bmatrix}, \quad F_t = \begin{bmatrix} f_{1t} \\ f_{2t} \\ f_{3t} \\ \vdots \\ f_{nt} \end{bmatrix}$$

In the real probability P, state variables mean reversion follow the equation below.

$$dF_t = K(\theta - F_t)dt + \sigma dW_t \tag{5}$$

$$\emptyset = \begin{bmatrix} \exp(\beta_{11}-k_1) & 0 & 0 & 0 & 0 \\ 0 & \exp(\beta_{22}-k_2) & 0 & 0 & 0 \\ 0 & 0 & \exp(\beta_{33}-k_3) & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & \exp(\beta_{nn}-k_6) \end{bmatrix},$$

$$Q = \begin{bmatrix} \sigma_1^2 \frac{1-e^{-2(k_1-\beta_{11})}}{2(k_1-\beta_{11})} & 0 & 0 & 0 & 0 \\ 0 & \sigma_2^2 \frac{1-e^{-2(k_2-\beta_{22})}}{2(k_2-\beta_{22})} & 0 & 0 & 0 \\ 0 & 0 & \sigma_3^2 \frac{1-e^{-2(k_3-\beta_{33})}}{2(k_3-\beta_{33})} & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & \sigma_n^2 \frac{1-e^{-2(k_n-\beta_{nn})}}{2(k_n-\beta_{nn})} \end{bmatrix}$$

In real probability P, the conditional expectation and variance of state variables are below:

$$E(F_{t+1}|F_t) = \theta + \emptyset[F_t - \theta] \tag{6}$$

$$var(F_{t+1}|F_t) = Q \tag{7}$$

When short term interest rate and state variable are certain, bond price and long term interest rate will be determined by short term interest rate in risk neutral probability. According to literatures, the bond with maturity at time T and par value 1$, its pricing model is as below.

$$P_t^{(T-t)} = E_t^Q\left[\exp\left(-\int_t^T r_s ds\right)\right] =$$

$$E_t^Q\left[\exp\left(-\int_t^T (a_0 + \delta_1 f_{1t} + \delta_2 f_{2t} + \delta_3 f_{3t} + \dots + \delta_n f_{nt}) ds\right)\right] \tag{8}$$

After derivation, the bond with term τ, at time t, the spot interest rate is below:

$$y_t^\tau = a_0 - \frac{\sigma_1^2}{k_1^2} - \frac{\sigma_2^2}{k_2^2} - \frac{\sigma_3^2}{k_3^2} - \cdots - \frac{\sigma_n^2}{k_n^2} + \frac{2\sigma_1^2(1-e^{-k_1\tau})}{k_1^3\tau} - \frac{\sigma_1^2(1-e^{-2k_1\tau})}{2k_1^3\tau} + \frac{2\sigma_2^2(1-e^{-k_2\tau})}{k_2^3\tau} - \frac{\sigma_2^2(1-e^{-2k_2\tau})}{2k_2^3\tau} + \frac{2\sigma_3^2(1-e^{-k_3\tau})}{k_3^3\tau} - \frac{\sigma_3^2(1-e^{-2k_3\tau})}{2k_3^3\tau} + \cdots + \frac{2\sigma_n^2(1-e^{-k_n\tau})}{k_n^3\tau} - \frac{\sigma_n^2(1-e^{-2k_n\tau})}{2k_n^3\tau} + \frac{1-e^{-k_1\tau}}{k_1\tau} f_{1t} + \frac{1-e^{-k_2\tau}}{k_2\tau} f_{2t} + \frac{1-e^{-k_3\tau}}{k_3\tau} f_{3t} + \cdots + \frac{1-e^{-k_n\tau}}{k_n\tau} f_{nt} \quad (9)$$

4. Kalman filtering

Kalman filtering is made up of recursive mathematical formulas, and the signal equation indicates the relation between bond yields which could be observed and state variables which can't be observed. The state equation indicates the changing process of state variables. We give initial value for state variable, and we can estimate the parameters combining with maximum likelihood estimation model. According to equation (9), we mark

$$C_0^\tau = a_0 - \frac{\sigma_1^2}{k_1^2} - \frac{\sigma_2^2}{k_2^2} - \frac{\sigma_3^2}{k_3^2} - \cdots - \frac{\sigma_n^2}{k_n^2} + \frac{2\sigma_1^2(1-e^{-k_1\tau})}{k_1^3\tau} - \frac{\sigma_1^2(1-e^{-2k_1\tau})}{2k_1^3\tau} + \frac{2\sigma_2^2(1-e^{-k_2\tau})}{k_2^3\tau} - \frac{\sigma_2^2(1-e^{-2k_2\tau})}{2k_2^3\tau} + \frac{2\sigma_3^2(1-e^{-k_3\tau})}{k_3^3\tau} - \frac{\sigma_3^2(1-e^{-2k_3\tau})}{2k_3^3\tau} + \cdots + \frac{2\sigma_n^2(1-e^{-k_n\tau})}{k_n^3\tau} - \frac{\sigma_n^2(1-e^{-2k_n\tau})}{2k_n^3\tau}$$

$$C_1^\tau = \frac{1-e^{-k_1\tau}}{k_1\tau}, \quad C_2^\tau = \frac{1-e^{-k_2\tau}}{k_2\tau}, \quad C_3^\tau = \frac{1-e^{-k_3\tau}}{k_3\tau}, \ \ldots \ C_n^\tau = \frac{1-e^{-k_n\tau}}{k_n\tau}。$$

Equation (9) could be written as below:

$$y_t^\tau = C_0^\tau + C_1^\tau f_{1t} + C_2^\tau f_{2t} + C_3^\tau f_{3t} + \cdots + C_n^\tau f_{nt} \quad (10)$$

We choose corporate bond yields data from Shanghai Exchange and Shenzhen Exchange and the bonds with maturity of 3 years, 5 years, 7 years and 10 years.

$$Y_t = \begin{bmatrix} y_t^3 \\ y_t^5 \\ y_t^7 \\ y_t^{10} \end{bmatrix}, \quad C_0 = \begin{bmatrix} C_t^3 \\ C_t^5 \\ C_t^7 \\ C_t^{10} \end{bmatrix}, \quad e_t = \begin{bmatrix} e_t^3 \\ e_t^5 \\ e_t^7 \\ e_t^{10} \end{bmatrix},$$

$$C_1 = \begin{bmatrix} C_1^3 & C_2^3 & C_3^3 & C_4^3 & C_5^3 & C_6^3 & \cdots & C_n^3 \\ C_1^5 & C_2^5 & C_3^5 & C_4^5 & C_5^5 & C_6^5 & \cdots & C_n^5 \\ C_1^7 & C_2^7 & C_3^7 & C_4^7 & C_5^7 & C_6^7 & \cdots & C_n^7 \\ C_1^{10} & C_2^{10} & C_3^{10} & C_4^{10} & C_5^{10} & C_6^{10} & \cdots & C_6^3 \end{bmatrix}$$

The signal equation is as below:

$$Y_t = C_0 + C_1 F_t + e_t \quad (11)$$

According to financial theory, interest rate is determined by state variables. The mean value of e_t is 0, and it follows the equation below:

$$\text{var}(e_t) = \text{M}$$

According to (5), we get the state equation below:

$$F_{t+1} = \theta + \emptyset(F_t - \theta) + \varepsilon_{t+1} \quad (12)$$

ε_{t+1} is the stochastic error of state variable, and its mean value is 0, and its variance is Q. F_t has initial value and initial variance as below:

$$\widetilde{F_0} = \theta, \ \ \text{vec}(\hat{P}_0) = [I - (\emptyset \times \emptyset)]^{-1} vec(Q)$$

The predicting equation of $\hat{F_t}$ is below:

$$F_{(t|t-1)} = \theta + \emptyset(F_{t-1} - \theta) \quad (13)$$

The conditional variance of predicting value is below:

$$P_{(t|t-1)} = \emptyset P_{t-h} \acute{\emptyset} + Q \quad (14)$$

$$U_t = Y_t - C_0 - C_1 F_{(t|t-1)}, \quad V_t = C_1 P_{(t|t-1)} \acute{C_1} + M$$

and U_t follows normal distribution, so the likelihood equation is below:

$$-2\ln L_t = ln|V_t| + \acute{U_t} V_t^{-1} U_t \quad (15)$$

The parameters meet the condition below:

$$\min_{(parameters)} \sum_t 2lnL_t$$

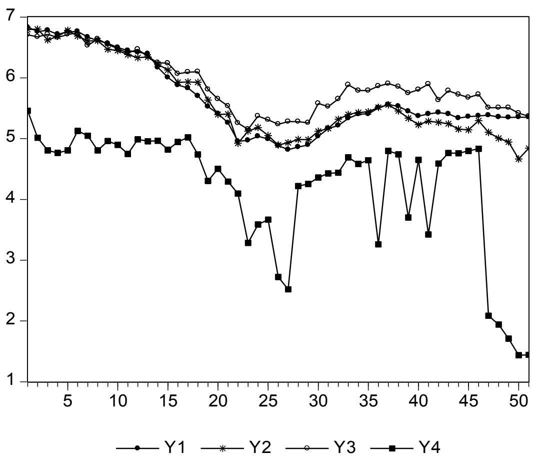
In Kalman filtering analysis,

$$K_t = P_{(t|t-1)} \acute{C_1} V_t^{-1}, \quad L_t = I - K_t C_1$$

Recursive Algorithm is below:

$$\hat{F_t} = F_{(t|t-1)} + K_t U_t, \quad \hat{P_t} = L_t P_{(t|t-1)}$$

5. Empirical results analysis



Graph1 observable bonds yields

Graph1 indicates corporate bond weekly average yields in Shanghai Exchange and Shenzhen Exchange, and Y1 shows corporate bond yields with 3 years maturity, and Y2 shows corporate bond yields with 5 years maturity, and Y3 shows corporate bond yields with 7 years maturity, and Y4 means corporate bonds yields with 10 years maturity. We can see bonds with short term have higher weekly average yields.

5.1 one-factor empirical analysis

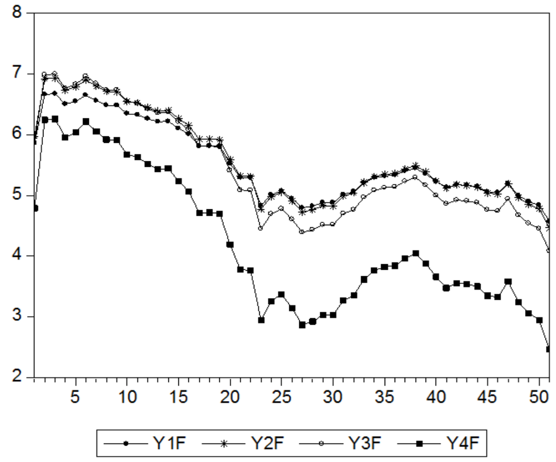
With given initial values of parameters, we get parameters in table2. From table2 we know a_0 is significant at 5% level. σ_1 is significant at 1% confidence level, and it means corporate bond yields fluctuate. k_1 is significant at 1% confidence level, and it means bond yields have mean reversion, but they reverse slowly. β_1 isn't significant at 1%.

Table2 one-factor affine model results

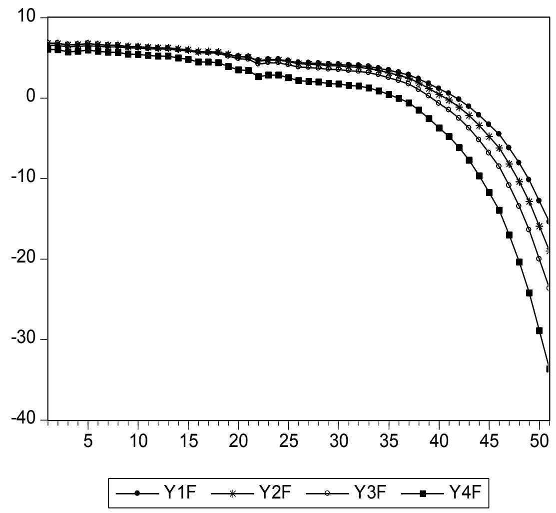
	parameters	**St.d**	**Z**	**Prob.**
$\mathbf{a_0}$	3.691**	1.581	2.33	0.0196
σ_1	0.161***	0.036	4.50	0.0000
k_1	-0.146***	0.018	--8.24	0.0000
β_1	-0.265	0.178	-1.48	0.1377

*** denotes statistical variables are significant at 1% confidence level and ** denotes statistical variables are significant at 5% confidence level.

From graph2 we can see, it's one-step forward forecasting of corporate bond weekly average yields in Shanghai Exchange and Shenzhen Exchange. The yields curves in graph2 are similar with the yields curves in graph1, so the model fits one-step forward forecasting well. Graph3 indicates the modeling of real yields in graph1, we can see it can't fit the real curve well. So one-factor Kalman filtering model can't fit real value well.



Graph2 one-step forward forecasting of yields



Graph3 modeling real curve of yields

5.2 Two-factor empirical analysis

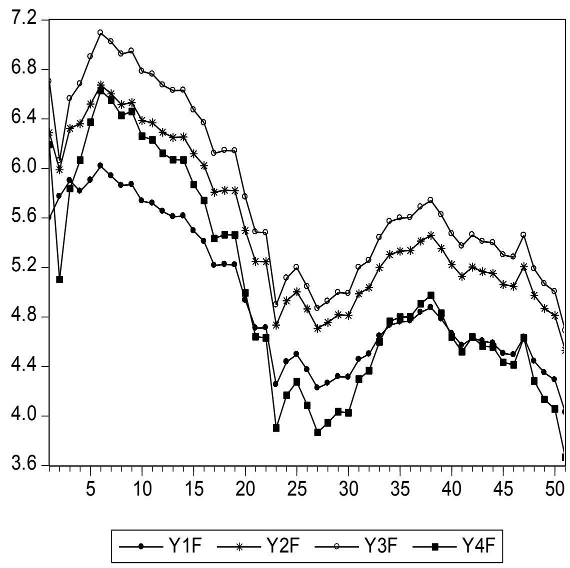
From table3 we can see a_0 isn't significant. σ_1 is significant at 1% confidence level, and σ_2 is not significant, and we infer may be they represent default risk and liquidity risk. k_1 is significant at 1% confidence level, and k_2 is not significant. β_1 is significant at 1% confidence level, also β_2 is significant at 1% confidence level, so there are risk premium in both state variable 1 and state variable 2.

Table3 two-factor affine model results

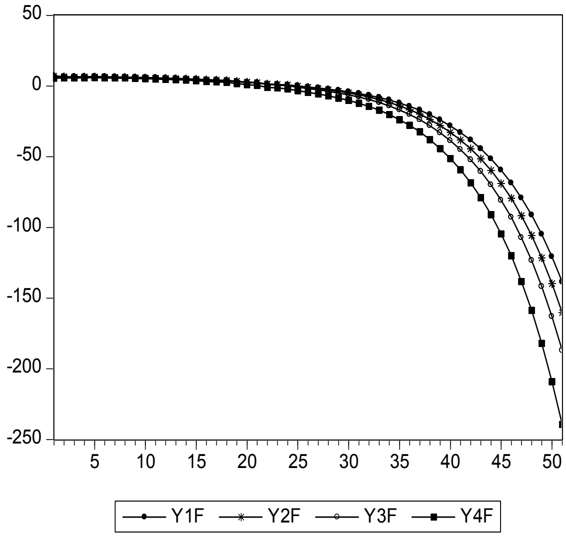
	parameters	St.d	Z	Prob.
a_0	1.087	8.948	0.122	0.9033
σ_1	0.182***	0.0238	7.664	0.0000
k_1	-0.215***	0.006	-38.93	0.0000
β_1	-0.215***	0.011	-19.86	0.0000
σ_2	2.330	6.890	0.338	0.7353
k_2	1.330	2.495	0.533	0.5941
β_2	-4.484***	1.679	-2.671	0.0076

*** denotes statistical variables are significant on the 1% confidence level.

From graph4 we can see it's the one step-forward forecasting of corporate bond weekly average yields in Shanghai Exchange and Shenzhen Exchange, graph4 is similar with graph1, and it means Kalman filtering two-factor model could forecast yields well. Graph5 is modeling the real yields, and we can see graph5 and graph1 is very different, so the two-factor Kalman filtering model can't fit real curve well.



Graph4 two-factor one-step forward forecasting



Graph5 modeling real curve of yields

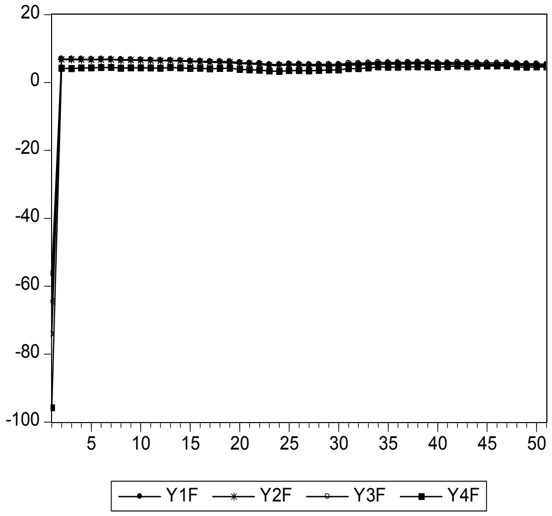
5.3 Three-factor empirical analysis

From table4 we can see a_0 is significant. σ_1 is significant at 1% confidence level, and it means the state variable 1 fluctuates with time, and σ_2 isn't significant, also σ_3 isn't significant. k_1 is significant at 1% confidence level, and it means state variable 1 follows mean reversion, and k_2 is significant at 5% confidence level, and it means state variable 2 follows mean reversion, and $k_2 > k_1$, means state variable 2 reverses more quickly than state variable 1, and k_3 is significant at 1% confidence level, and means state variable 3 follows mean reversion, but it reverses more slowly than variable2. β_{11} is significant at 10% confidence level, and means state variable 1 has time varying risk premium, and β_{22} is significant at 5% confidence level, and means state variable 2 has time varying risk premium, also β_{33} is significant at 1% confidence level, and it means state variable 3 has time varying risk premium, and state variable 2 has the largest time varying risk premium. β_{10} is significant at 10% confidence level, and it means state variable 1 has fixed risk premium, but both β_{20} and β_{30} aren't significant.

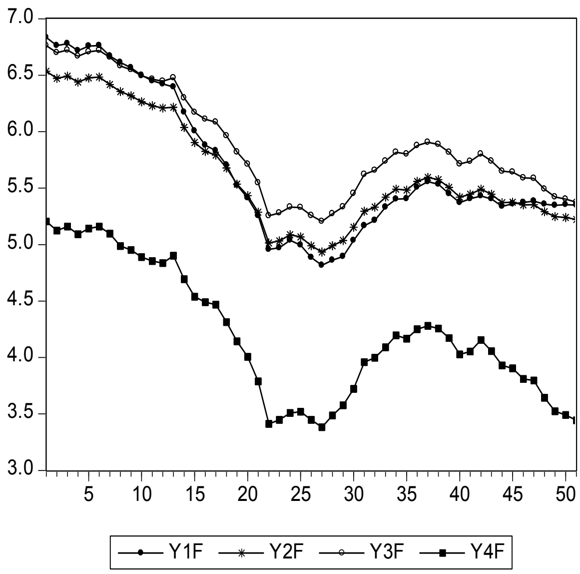
Table4 three-factor affine model results

	parameters	St.d	Z	Prob.
a_0	3.681	24.374	0.151	0.8800
σ_1	0.152***	0.058	2.631	0.0085
k_1	-0.229***	0.072	-3.182	0.0015
β_{11}	-0.635*	0.383	-1.658	0.0974
σ_2	-0.764	16.977	-0.045	0.9641
k_2	0.969**	0.388	2.497	0.0125
β_{22}	0.865**	0.434	1.992	0.0464
σ_3	0.103	9.854	0.010	0.9917
k_3	0.203***	0.054	3.735	0.0002
β_{33}	0.187***	0.056	3.367	0.0008
β_{10}	0.976*	0.547	1.787	0.074
β_{20}	1.162	4.954	0.234	0.815
β_{30}	-0.033	2.001	-0.017	0.987

*** denotes statistical variables are significant at the 1% confidence level. ** denotes statistical variables are significant at the 5% confidence level. * denotes statistical variables are significant at 10% confidence level.



Graph 6 Three-factor one-step forward forecasting



Graph 7 modeling real curve of yields

From graph 6 we can see it's one-step forward forecasting of average weekly corporate bond yields in Shanghai Exchange and Shenzhen Exchange, and it's very different with graph 1, so the forecasting isn't good. Graph 7 is the modeling of real curve, and it's similar with graph 1, so the three-factor model could fit real data well.

6. Conclusion

We analyze corporate bond yields term structure in Shanghai Exchange and Shenzhen Exchange by using Kalman filtering model. We build N-factor affine term structure model, and then we use Kalman filtering to estimate the parameters of one-factor model, two-factor model and three-factor model. The results indicate one-factor model and two-factor model could do one-step forward forecasting well, and they have fixed risk premium, but they can't fit the real data well. Three-factor model can't forecast well, but it could fit real data well, and we add the time varying risk premium factor into three-factor model, and find they are all significant, so the three state variables have time varying risk premium. But only state variable 1 has the significant fixed risk premium. And the results are similar with other scholars. I would do further research on corporate bond spread by using Kalman filtering.

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