$$K\left(z^{-1}\right)=1-z^{-1}. \qquad (22)$$

The feedback controller polynomials and the desired polynomial have the form

$$P\left(z^{-1}\right)=1+p_1z^{-1}; \quad Q\left(z^{-1}\right)=q_0+q_1z^{-1}+q_2z^{-2} \qquad (23)$$

$$D\left(z^{-1}\right)=1+d_1z^{-1}+d_2z^{-2}. \qquad (24)$$

By solving equation (21), a system of linear equations can be obtained using the method of indeterminate coefficients

$$\begin{bmatrix} \hat{b}_1 & 0 & 0 & 1 \\ \hat{b}_2 & \hat{b}_1 & 0 & \hat{a}_1-1 \\ 0 & \hat{b}_2 & \hat{b}_1 & \hat{a}_2-\hat{a}_1 \\ 0 & 0 & \hat{b}_2 & -\hat{a}_2 \end{bmatrix}\begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ p_1 \end{bmatrix}=\begin{bmatrix} d_1+1-\hat{a}_1 \\ d_2+\hat{a}_1-\hat{a}_2 \\ \hat{a}_2 \\ 0 \end{bmatrix} \qquad (25)$$

where $\hat{a}_1$, $\hat{a}_2$, $\hat{b}_1$, and $\hat{b}_2$ are current estimates of the process model parameters.

From the matrix equation (25) it is possible to compute the feedback controller parameters. The coefficients of polynomial (24) were chosen as $d_1=-1.6, d_2=0.64$.

Polynomial $R\left(z^{-1}\right)$ of the feed forward control part

$$G_R(z)=\frac{R\left(z^{-1}\right)}{P\left(z^{-1}\right)} \qquad (26)$$

was computed by solving the polynomial equation

$$D_w(z^{-1})S(z^{-1})+B(z^{-1})R(z^{-1})=D(z^{-1}) \qquad (27)$$

where $S\left(z^{-1}\right)$ is the auxiliary polynomial. For a step change of the reference value $w(k)$, $D_w\left(z^{-1}\right)=1-z^{-1}$, and it is then possible to solve equation (27) by substituting $z=1$.

$$r_0=\frac{D(1)}{B(1)}=\frac{1+d_1+d_2}{\hat{b}_1+\hat{b}_2} \qquad (28)$$

From Fig. 8 it is obvious that the controller is given by the equation

$$P(z^{-1})K(z^{-1})u(k)=R(z^{-1})w(k)-Q(z^{-1})y(k) \qquad (29)$$

and the CE control law is then

$$\begin{aligned} u_{CE}(k)&=r_0w(k)-q_0y(k)-q_1y(k-1) \\ &-q_2y(k-2)+(1-p_1)u(k-1)+p_1u(k-2) \end{aligned} \qquad (30)$$

A sampling period of T_0 = 0.05 s was used in all experiments, and initial parameter estimates were set without using *a priori* information about the controlled system:
$\hat{a}_1(0)=0.1$, $\hat{a}_2(0)=0.2$, $\hat{b}_1(0)=0.3$, $\hat{b}_2(0)=0.4$ and the initial value of the covariance matrix $C(0)=10^9I$ was used.

The first controller used was the dual controller described in the previous sections. This controller is further referenced as *dual*. The individual vectors and parameters in equations (15) and (17) have the form

$$\boldsymbol{\Phi}_0^T(k)=\left[u(k),y(k),-y(k-1)\right];$$

$$\boldsymbol{c}_{b_1\boldsymbol{\Theta}_0}^T=\left[c_{12}(k),c_{13}(k),c_{14}(k)\right]; \; c_{b_1}(k)=c_{11}(k)$$

(see the covariance matrix (11)); the selectable parameter chosen for equation (3) was $\eta=30$.

The other three controllers were taken from the Self-tuning Controllers Simulink Library [5], [38], [39]. The controller structure *pp2chp* was used. These controllers only differed in the algorithm for recursive identification.

The first one used the pure Recursive Least Squares Method (RLSM) [40] and is referenced as *lsm*.

The second one contained RLSM with an exponential forgetting factor [41] and is referenced as *ef*.

The third one used RLSM, extended to include the technique directional (adaptive) forgetting [5], [42], [43] and is referenced as *adf*. In this case the vector of parameter estimates is updated according to the recursive relation

$$\hat{\boldsymbol{\Theta}}(k)=\hat{\boldsymbol{\Theta}}(k-1)+\frac{\boldsymbol{C}(k-1)\boldsymbol{\Phi}(k)}{1+\xi(k)}\hat{e}(k) \qquad (31)$$

where

$$\hat{\boldsymbol{\Theta}}^T(k)=\left[\hat{a}_1,\hat{a}_2,\hat{b}_1,\hat{b}_2\right]$$

$$\boldsymbol{\Phi}^T(k)=\left[-y(k-1),-y(k-2),u(k-1),u(k-2)\right] \qquad (32)$$

are the vector of parameter estimates and the data (regression) vector.

The expression

$$\xi(k)=\boldsymbol{\Phi}^T(k)\boldsymbol{C}(k-1)\boldsymbol{\Phi}(k) \qquad (33)$$

is an auxiliary scalar, and

$$\hat{e}(k)=y(k)-\hat{\boldsymbol{\Theta}}^T(k-1)\boldsymbol{\Phi}(k) \qquad (34)$$

is the prediction error. If $\xi(k) > 0$, the square covariance matrix is updated by the relation

$$\boldsymbol{C}(k) = \boldsymbol{C}(k-1) - \frac{\boldsymbol{C}(k-1)\boldsymbol{\Phi}(k)\boldsymbol{\Phi}^T(k)\boldsymbol{C}(k-1)}{\varepsilon^{-1}(k) + \xi(k)} \quad (35)$$

where

$$\varepsilon(k) = \varphi(k) - \frac{1-\varphi(k)}{\xi(k-1)} \quad (36)$$

If $\xi(k-1) = 0$, then

$$\boldsymbol{C}(k) = \boldsymbol{C}(k-1). \quad (37)$$

The value of adaptive directional forgetting $\varphi(k)$ is then calculated for each sampling period as

$$[\varphi(k)]^{-1} = 1 + (1+\rho)\left[\ln(1+\xi(k))\right] + \left[\frac{(\upsilon(k)+1)\eta(k)}{1+\xi(k)+\eta(k)} - 1\right]\frac{\xi(k)}{1+\xi(k)} \quad (38)$$

where

$$\eta(k) = \frac{\hat{e}^2(k)}{\lambda(k)}; \qquad \upsilon(k) = \varphi(k-1)\left[(\upsilon(k-1)+1\right]$$

$$\lambda(k) = \varphi(k-1)\left[\lambda(k-1) + \frac{\hat{e}^2(k)}{1+\xi(k)}\right] \quad (39)$$

are auxiliary variables.

The closed loop stability is one of the problems that are not satisfactory solved for the STC. Usage of a suitable recursive identification method and related achievement of unbiased and convergent parameter estimates are very important to ensure the closed loop stability. The problem of unbiasedness and convergence of parameter estimates of STC has been considered as an important and very difficult mathematical problem. This problem was not investigated for a long time because of the difficulties connected with nonlinearity and complexity of the adaptive control laws. If the unbiasedness and convergence of parameter estimates is guaranteed, STCs operate without problem. It is also to be taken into consideration that for processes described with adequate lower order model the large tolerance exists for the model inaccuracy. In case of the control of the laboratory model DR300, the second order model is used which contributes to better stability of the closed loop. Because the pole assignment method is used in the control part of the control loop, stability can be provided by an appropriate selection of the coefficients of the desired polynomial $D(z^{-1})$ (24).

General problem of the convergence and stability of adaptive dual controllers is investigated and stability conditions are derived in [44].

4.1 Controllers' Performance

The control performance of the DR300 system controlled by controller *dual* is shown in Fig. 9. It can be seen that the control process quickly became stable and the parameters of the model reached values sufficient for the asymptotic tracking of the reference signal.

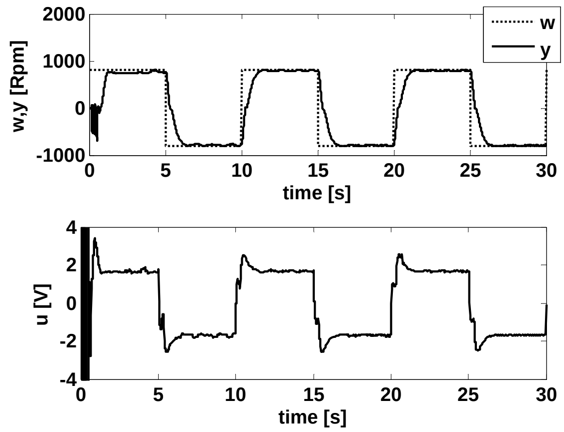

Fig. 9. Control of DR300 using *dual* controller

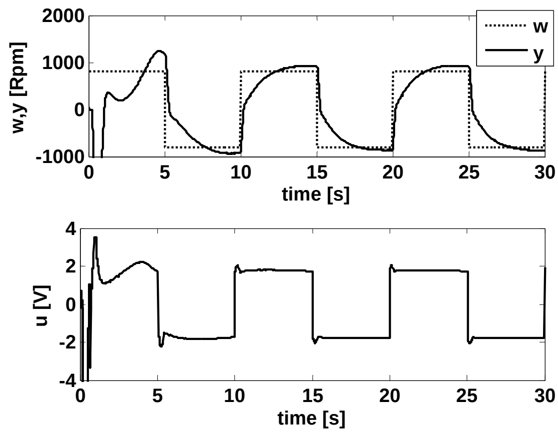

Fig. 10. Control of DR300 using *lsm* controller

The *lsm* controller used pure RLSM for identification of the controlled system. All input–output pairs affect the parameters of the model by the same weight. As can be seen in Fig. 10, the *lsm* controller is not able to cope with the DR300 control problem as accurately as the *dual* controller.

The identification based on the RLSM *with exponential forgetting* is used in the *ef* controller.

The forgetting coefficient of $\varphi = 0.95$ was used for the control of the DR300 system. The performance of the control loop signals is shown in Fig. 11. It can be seen that after about 3 s the parameters produced by recursive identification became good enough to make the control loop stable with asymptotic tracking of the reference signal. Oscillations of both controller and process signal output occur when the reference signal changes. This behaviour is caused by the different model parameters for the positive and negative values of process signal output (see the static characteristic in Fig. 7).

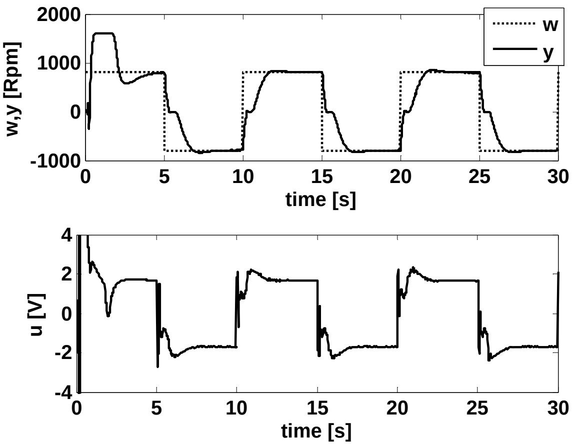

Fig. 11. Control of DR300 using *ef* controller

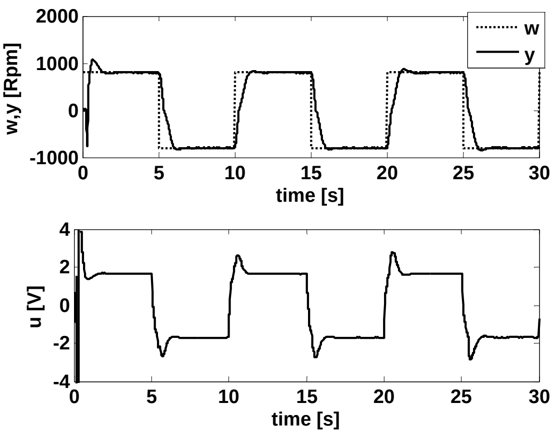

Fig. 12. Control of DR300 using *adf* controller

4.2 Comparison of Control Performance Using Summing Criteria

The performances of the individual controllers were compared not only by investigating graphs of controller performance and process output signals, but also mathematical criteria. Four criteria were used to compare the control results obtained by individual controllers

$$S_{e2} = \frac{1}{b-a+1}\sum_{k=a}^{b}\left[w(k) - y(k)\right]^2 \tag{40}$$

$$S_{ea} = \frac{1}{b-a+1}\sum_{k=a}^{b}\left|w(k) - y(k)\right| \tag{41}$$

$$S_{u2} = \frac{1}{b-a}\sum_{k=a}^{b-1}\left[u(k+1) - u(k)\right]^2 \tag{42}$$

$$S_{ua} = \frac{1}{b-a}\sum_{k=a}^{b-1}\left|u(k+1) - u(k)\right| \tag{43}$$

Values of individual criteria are shown in Table 1.

Criteria S_{e2} and S_{ea} are based on control error. The sum of squares of control error and the sum of absolute values of control error were used to obtain S_{e2} and S_{ea} respectively. These criteria represent the accuracy of the control process. Criteria S_{u2} and S_{ua} are based on changes to the control signal. The sum of squares of the control sequence and the sum of absolute values of the control sequence were used to obtain S_{u2} and S_{ua} respectively. These criteria represent demands for actuators. Values of a and b were selected to cover the whole control process except the first 3 s.

Table 1. Values of criteria for the control results

controller	$S_{e2}\cdot10^{-3}$	S_{ea}	$S_{u2}\cdot10^{4}$	$S_{ua}\cdot10^{4}$
dual	127.3	138.6	102.2	167.7
lsm	158.7	242.3	281.5	105.7
ef	161.2	198.7	506.1	247.5
adf	134.3	130.2	53.6	115.7

The best performance according to the S_{e2} criterion was given by the *dual* controller, while using the S_{ea} criterion leads to the *adf* controller having the best performance. The accuracy of the *dual* and the *adf* controllers is significantly better then the accuracy obtained by the *lsm* or *ef* controllers. This result is valid for both S_{e2} and S_{ea} criteria. The best results according to the S_{u2} criterion were obtained with the *adf* controller, while the *lsm* controller gave the lowest value of the S_{ua} criterion. Despite this result, the *lsm* controller is not suitable for control of the DR300 system because of the unsatisfactory accuracy of the control process. Higher values of the S_{u2} and the S_{ua} criterions for the *dual* controller comparing to the *adf* controller are causer by excitation of the controlled system incorporated in the bicriterial dual approach. This excitation leads to better identification of the controlled system and subsequently to lower value of the S_{e2} criterion for the *dual* controller.

4 Conclusion

Dual control using the bicriterial approach was verified and compared with some other standard adaptive control approaches in real-time conditions by controlling a laboratory model. Examples of control of a highly non-linear system with a dead zone – the DR300 Speed Control – were shown. The primary aim of this work was not to control the DR300 system but to use it as a demonstration example of the more general class of non-linear systems. Even though the non-linear system was modelled by a linear model, real-time experiments demonstrated very good performance of the dual controller. It should be emphasized that initial parameter estimates were set without using *a priori* information about the controlled system. Usage of converged parameter estimates as an initial setting of the next control process would lead to better performance in the initial phase of the control courses.

Dual bicriterial control is a suitable and promising approach for the control of non-linear systems, time-varying systems, or systems with unknown parameters.

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