$$f = \mathrm{sgn}[\sum_{i=0}^{l} \lambda_i y_i (z^T z_i) + b] \tag{19}$$

From the functional theory, a non-negative symmetrical function $K(u,v)$ uniquely defines a Hilbert space H, where K is the rebuild kernel in the space H:

$$K(u,v) = \sum_i \alpha \varphi_i(u) \varphi_i(v) \tag{20}$$

This stands for an internal product of a characteristic space:

$$z_i^T z = \Phi(x_i)^T \Phi(x) = K(x_i, x) \tag{21}$$

Then the decision function can be written as:

$$f = \mathrm{sgn}[\sum_{i=1}^{l} \lambda_i y_i K(x_i, x) + b] \tag{22}$$

The development of a SVM classification model depends on the selection of kernel function. There are four typical kernels that can be used in SVM models. These include linear, polynomial, radial basis function (RBF) and sigmoid function, as described below.

The linear kernel function is defined as

$$K(x_i, x_j) = x_i^T x_j \tag{23}$$

The polynomial kernel function is defined as

$$K(x_i, x_j) = (\gamma x_i^T x_j + coefficient)^{\deg ree} \tag{24}$$

The RBF kernel function is defined as

$$K(x_i, x_j) = \exp(-\gamma \mid x_i - x_j \mid^2 \tag{25}$$

The sigmoid kernel function is defined as

$$K(x_i, x_j) = \tanh(\gamma x_i^T x_j + coefficient) \tag{26}$$

Many real-world data sets involve multi-class problem. Since SVM is inherently binary classifiers, the binary SVM is needed to extend to be multi-class SVM for multi-class problem. Currently, there are two types of approaches for building multi-class SVM. One is the “single machine” approach, which attempts to construct multi-class SVM by solving a single optimization problem. The other is the “divide and conquer” approach, which decomposes the multi-class problem into several binary sub-problems, and builds a standard SVM for each. The most popular decomposing strategy is probably the “one-against-all”. The “one-against-all” approach consists of building one SVM per class and aims to distinguish the samples in a single class from the samples in all remaining classes. Another popular decomposing strategy is the “one-against-one”. The “one-against-one” approach builds one SVM for each pair of classes. When applied to a test point, each classification gives one vote to the winning class and the point is labeled with the class having most votes. In practice, the “one-against-one” approach is more effective than the “one-against-all” approach due to its computation simplicity and comparable performance.

7 Experiments

In this section, we perform facial expression recognition experiments on two popular facial databases, i.e., the JAFFE database and the Cohn-Kanade database, and present experimental results and analysis.

7.1 Experiment setup

For the extraction of the Gabor wavelets representation and LBP, the pre-processing procedure of facial images is given as follows.

Following the setting in [12, 14], we normalized the eye distance of face images to a fixed distance of 55 pixels once the centers of two eyes were located. Generally, it is observed that the width of a face is roughly two times of the distance, and the height is roughly three times. Therefore, based on the normalized value of the eye distance, a resized image of 110 × 150 pixels was cropped from original image. To locate the centers of two eyes, automatic face registration was performed by using a robust real-time face detector based on a set of rectangle Harr-wavelet features [34]. From the results of automatic face detection including face location, face width and face height, two square bounding boxes for left eye and right eye were automatically constructed by using the geometry of a typical up-right face, which has been widely utilized to find a proper spatial arrangement of facial features [35]. Then, the approximate center locations of two eyes can be automatically worked out in terms of the centers of two square bounding boxes for left eye and right eye.

Fig.5 shows the detailed process of two eyes location and the final cropped image from the Cohn-Kanade database. No further alignment of facial features such as alignment of mouth was performed. Additionally, there was no attempt made to remove illumination changes due to LBP's gray-scale invariance.

The cropped facial images of 110 × 150 pixels contain facial main components such as mouth, eyes, brows and noses. The LBP operator is applied to the whole region of the cropped facial images. For better uniform-LBP feature extraction, two parameters, *i.e.*, the LBP operator and the number of regions divided, need to be optimized. Similar to the setting in [12, 14], we selected the 59-bin operator $LBP_{P,R}^{u2}$, and divided the 110 × 150 pixels face images into 18 × 21 pixels regions, giving a good trade-off between recognition performance and feature vector length. Thus face images were divided into 42 (6 × 7) regions, and represented by the LBP histograms with the length of 2478 (59 × 42).

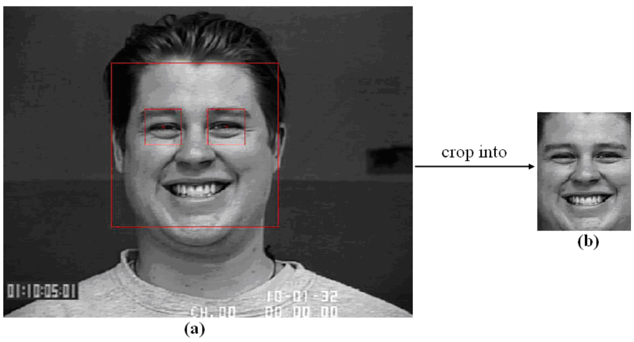

Fig.5 (a) Two eyes location of an original image from the Cohn-Kanade database, (b) The final cropped image of 110 × 150 pixels.

To reduce the feature length of the Gabor wavelets representations, principal component analysis (PCA) [36] is used for dimensionality reduction. The reduced feature dimension is confined to the range of [0, 100] with an interval of 10.

A 10-fold cross validation scheme is employed in 7-class facial expression recognition experiments, and the average recognition results are reported. In detail, the data sets are split randomly into ten groups of roughly equal numbers of subjects. Nine groups are used as the training data to train a classifier, while the remaining group is used as the testing data. The above process is repeated ten times for each group in turn to be omitted from the training process. Finally, the average recognition results on the testing data are reported.

As a representation ANN, RBFNN is used for its computational simplicity. For the KNN classifier, we set K to be 1 for its satisfying performance. We employ the LIBSVM package, available at http://www.csie.ntu.edu.tw /~cjlin/libsvm, to perform the SVM algorithm with the linear kernel function, one-against-one for multi-class problems. The experiment platform is Intel CPU 2.10 GHz, 1G RAM memory, MATLAB 7.0.1 (R14).

7.2 Experiments on the JAFFE database

When using the LBP features for facial expression recognition, the recognition results of different classification methods on the JAFFE database, including ANN, KNN, SVM and SRC, are given in Table 1. It can be seen from Table 1 that SRC obtains the highest accuracy of 84.76%, outperforming the other used methods. The recognition accuracies using the extracted LBP features for the other used methods, are 68.09% for ANN, 80.95% for KNN and 79.88% for SVM.

When using the Gabor wavelets representations for facial expression recognition, the recognition results of different classification methods along with reduced dimension of the Gabor wavelets representations are presented in Fig.6. Table 2 gives the best accuracy of different classification methods with the corresponding reduced dimension of the Gabor wavelets representations. The results in Table 2 and Fig.6 reveal that SRC achieves an accuracy of 88.57% at best with 60 reduced dimension of the Gabor wavelets representations, outperforming the other used methods. This confirms the validity and high performance of SRC for facial expression recognition.

Table 1 Comparison of recognition results for different classification methods with the LBP features on the JAFFE database

Methods	Accuracy (%)
ANN	68.09
KNN	80.95
SVM	79.88
SRC	84.76

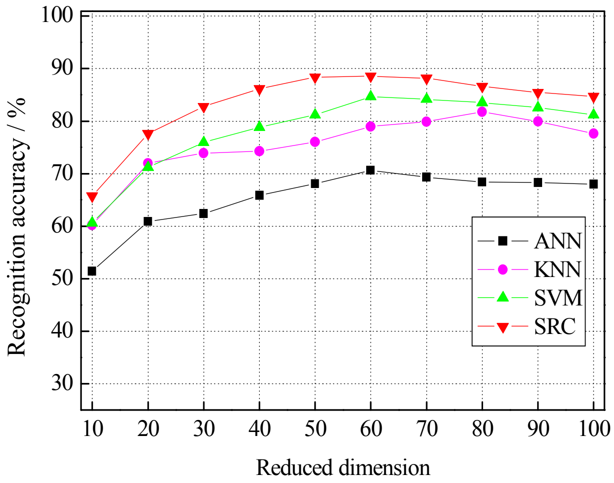

Fig.6 Recognition results on the JAFFE database for different classification methods with the reduced dimension of the Gabor wavelets representations

Table 2 Best results on the JAFFE database for different methods with corresponding reduced dimension of the Gabor wavelets representations

Methods	Dimension	Accuracy (%)
ANN	60	70.64
KNN	80	81.76
SVM	60	84.65
SRC	60	88.57

To further explore the recognition accuracy per expression, Table 3 and 4 separately present the confusion matrix of 7-class facial expression recognition results with the LBP features and the Gabor wavelets representations. The bold values in Table 3 and 4 represent the recognition accuracy of each expression. From Table 3 and 4, we can observe that three expressions, i.e., anger, joy and neutral, are classified with an accuracy of around 90%, while other four expressions, sad, surprise, disgust and fear, are discriminated with relatively low accuracy (less than 90%).

In our work, the obtained recognition accuracy of SRC (i.e., 84.76% with the LBP features, and 88.57% with the Gabor wavelets representations) for 7-class facial expression recognition on the JAFFE database is highly competitive, compared with previously reported results on the JAFFE database. In [12], similar to our experimental settings, they reported the best accuracy of 81% with SVM and LBP features.

In [37], they extracted the local texture information by applying LBP to facial feature points and obtained an accuracy of 83%.with the nearest neighbour classifier. In [6], by using the Gabor wavelets representations and learning vector quantization (LVQ), they achieved an accuracy of 87.51%. In our previously published work [14], based on LBP and local Fisher discriminant analysis (LFDA), we obtained the best recognition accuracy of 90.7%, outperforming the reported accuracy in this work. Nevertheless, LFDA is used to extract the low-dimensional discriminative embedded data representations from the extracted high-dimensional LBP features with striking performance improvement on facial expression recognition tasks.

7.3 Experiments on the Cohn-Kanade database

Table 5 presents the recognition results of different classification methods with the LBP features on the Cohn-Kanade database. Fig.7 gives the recognition results of different classification methods with the reduced dimension of the Gabor wavelets representations. Table 6 provides the best accuracy for different classification methods with the corresponding reduced dimension.

As shown in Fig.7 and Table 5-6, we can see that SRC still performs best among all used methods for facial expression recognition. In detail, SRC gives an accuracy of 97.14% with the LBP features, and 98.09% with 50 reduced dimension of the Gabor wavelets representations. This indicates the effectiveness of SRC for facial expression recognition, again.

Table 3 Confusion matrix of 7-class facial expression results with the LBP features on the JAFFE database

	Anger (%)	Joy (%)	Sad (%)	Surprise (%)	Disgust (%)	Fear (%)	Neutral (%)
Anger	**93.33**	0	6.67	0	0	0	0

Joy	0	**100**	0	0	0	0	0
Sad	3.22	3.22	**74.19**	3.22	3.22	6.48	6.45
Surprise	0	3.45	3.45	**82.76**	0	10.34	0
Disgust	10.35	0	6.89	0	**82.76**	0	0
Fear	0	0	12.52	3.12	9.37	**71.87**	3.12
Neutral	3.45	0	0	6.89	0	0	**89.66**

Table 4 Confusion matrix of 7-class facial expression results with the Gabor wavelets representations on the JAFFE database

	Anger (%)	Joy (%)	Sad (%)	Surprise (%)	Disgust (%)	Fear (%)	Neutral (%)
Anger	**96.55**	0	3.45	0	0	0	0
Joy	0	**90.32**	9.68	0	0	0	0
Sad	6.45	3.22	**87.10**	0	3.23	0	0
Surprise	0	3.45	0	**86.21**	0	6.89	3.45
Disgust	7.14	0	0	0	**82.14**	10.72	0
Fear	0	0	9.37	6.25	6.25	**78.13**	0
Neutral	0	0	0	0	0	0	**100**

Table 5 Comparison of recognition results for different classification methods with the LBP features on the Cohn-Kanade database

Methods	Accuracy (%)
ANN	93.45
KNN	96.22
SVM	95.24
SRC	97.14

Table 6 Best results on the Cohn-Kanade database for different methods with corresponding reduced dimension of the Gabor wavelets representations

Methods	Dimension	Accuracy (%)
ANN	70	94.86
KNN	60	97.14
SVM	80	96.16
SRC	50	98.09

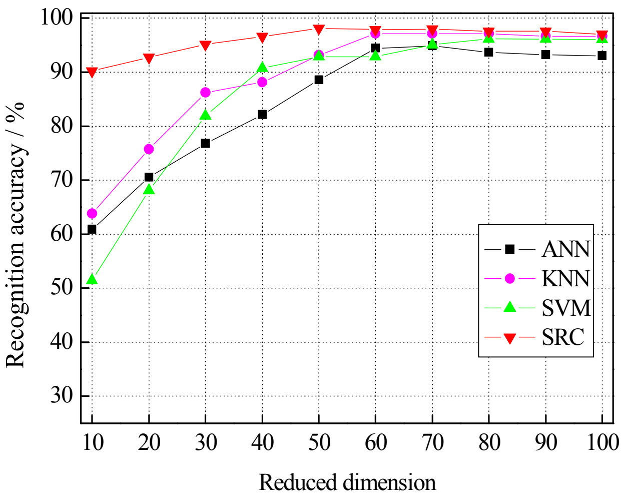

Fig.7 Recognition results on the Cohn-Kanade database for different classification methods with the reduced dimension of the Gabor wavelets representations

Table 7 and 8 presents the confusion matrix of 7-class expression recognition results with the LBP features and the Gabor wavelets representations, respectively. As shown in Table 7-8, it can be seen that most of seven expressions are identified very well with an accuracy of 100%.

Now we compare our reported results (i.e., 97.14% with the LBP features, and 98.09% with the Gabor wavelets representations) with previously reported results on the Cohn-Kanade database. In [12], they obtained a 7-class recognition accuracy of 91.4% at best with LBP features and SVM. In [38], they obtained the highest accuracy of 93.4% with SVM on 7-class tasks, but they used an improved LBP features called local directional pattern (LDP).

Table 7 Confusion matrix of 7-class facial expression results with the LBP features on the Cohn-Kanade database

	Anger (%)	Joy (%)	Sad (%)	Surprise (%)	Disgust (%)	Fear (%)	Neutral (%)
Anger	**90**	0	0	0	0	0	10
Joy	0	**100**	0	0	0	0	0
Sad	3.33	0	**90**	0	0	0	6.67
Surprise	0	0	0	**100**	0	0	0
Disgust	0	0	0	0	**100**	0	0
Fear	0	0	0	0	0	**100**	0
Neutral	0	0	0	0	0	0	**100**

Table 8 Confusion matrix of 7-class facial expression results with the Gabor wavelets representations on the Cohn-Kanade database

	Anger (%)	Joy (%)	Sad (%)	Surprise (%)	Disgust (%)	Fear (%)	Neutral (%)
Anger	**100**	0	0	0	0	0	0
Joy	0	**100**	0	0	0	0	0
Sad	0	0	**100**	0	0	0	0
Surprise	0	0	0	**96.67**	0	0	3.33

Disgust	10	0	0	0	**90**	0	0
Fear	0	0	0	0	0	**100**	0
Neutral	0	0	0	0	0	0	**100**

8 Conclusions

Automatic facial expression recognition has increasingly attracted attention duo to its important applications to human computer interaction. Designing a good classifier is a crucial step for any successful facial expression recognition system. In this paper, we presented a new method of facial expression recognition via the sparse representation classifier (SRC). The experiment results on the JAFFE database and the Cohn-Kanade database show that the SRC method obtains the promising performance on facial expression recognition tasks due to its good classification property. In our future work, it's an interesting task to employ the SRC technique to develop a real-time facial expression recognition system for natural human-computer interaction. In addition, it's also interesting to investigate the performance of the SRC technique to predict the behavior of financial time series [39].

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