UFIR Filtering Under Uncertain One-Step Delayed and Missing Data

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Abstract—This paper develops the unbiased finite impulse response (UFIR) filter for wireless sensor network (WSN) systems whose measurements are affected by random delays and packet dropout due to inescapable failures in the transmission and sensors. The Bernoulli distribution is used to model delays in arrived measurement data with known transmission probability. The effectiveness of the UFIR filter is compared experimentally to the KF and game theory recursive H1 filter in terms of accuracy and robustness employing the GPS-measured vehicle coordinates transmitted with latency over WSN.

Key-Words: Delayed data, packet dropouts, Bernoulli distribution, unbiased FIR filter


1. Introduction

State estimation of measured quantities provided over wireless sensor networks (WSN) has attracted attention in recent decades due to the ability to create measurement environments over big areas for navigation, target tracking, manufacturing control, and communication [1]–[8]. A common weakness of such systems is associated with the influence of sensor uncertainties, random transmission delays, and packet dropouts causing estimation failures and deteriorating the performance significantly [9], [10].

To avoid large estimation errors caused by latency and packet dropout, various filtering algorithms have been developed during the last decades to address the phenomena separately [11]–[13], even though they practically occur jointly [14], [15]. The game theory $H_{\infty}$ filter is applied in [16]–[18] to network system considering different phenomena such as noise covariances, one-step delayed data, and multiplicative packed dropouts. Other techniques such as the recursive state estimation [19] and the Kalman filter (KF) [20], [21] have been applied to non-linear WSN-based systems. The particle filter [22] and robust $L_1$ filter, which minimizes the peak-to-peak errors, have been developed for delayed data in [23].

The Bernoulli distribution is most widely used to model latency and develop estimation algorithms [24], [25]. Even so, a big challenge still exists to design efficient algorithms under lost data. The Bernoulli distribution thus requires a modification to deal with the probability of lost data [26], [27]. Here, the zero-input procedure with a predictive algorithm is considered to include the packet dropouts to the observation model using the Bernoulli distribution.

A drawback of the traditional KF is the dependence on the incomplete knowledge about the system and environmental noise. Accordingly, the KF-based estimators often produce large errors when the conditions of optimality are not obeyed properly. This fact was recently pointed out in [28], where the KF and game theory $H_{\infty}$ filter [29] were compared for robustness to the unbiased finite impulse response (UFIR) filter [30], which ignores any information about zero-mean noise and initial values and is thus robust by design. For timestamped delays and missing data, the UFIR filter was originally designed in [31]. For systems with randomly delayed and missing data the UFIR approach still has not been applied that motivates our present work. In this paper, we develop the UFIR filter for systems with Bernoulli-distributed randomly-delayed and missing data. A better performance of the UFIR filter compared with the KF and $H_{\infty}$ is demonstrated under different scenarios of the tracking problem.

2. State-Space Model

As has been mentioned above, measurement data transmitted over a WSN typically arrive at a receiver with latency and packet dropouts originated from different sources. Understanding an importance of the mathematical model to design an efficient filtering algorithm, the following model using a Bernoulli distribution is presented to depict the real behavior of the transmitted data. We assume that each data are transmitted and received only once, hence one-step packet dropouts may happen. In Fig.1, we show typical scenarios with the transmitted/received data. It is assumed that data arrive at the correct time $t_1$ if received as $x_1^{(1)}$ and $x_1^{(1)}$. If at time $t_2$ data are received as $x_1^{(2)}$, then such data are one-step delayed with respect to time $t_1$ and this information will be lost at $t_1$. The observation can be modeled similarly to [35] as

$$z_n = \gamma_{0,n} y_n + (1 - \gamma_{0,n}) \left[ (1 - \gamma_{0,n-1}) \gamma_{1,n} y_{n-1} + (1 - (1 - \gamma_{0,n-1}) \gamma_{1,n}) \hat{z}_{n-1} \right],$$  \hspace{1cm} (1)$$

where a zero input compensation with prediction are using to substitute wrong and missing data. $z_n \in \mathbb{R}^M$ is the transmitted measurement vector at the processor, $\hat{z}_{n-1} \in \mathbb{R}^M$ is the predicted measurement, and $\hat{z}_{n-1}$ is the available estimate.

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TABLE I
DATA RECEIVED WITH PACKET DROPOUTS USING A ZERO-INPUT COMPENSATOR

<table>
<thead>
<tr>
<th>n</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_0$</td>
<td>-</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$\theta_1$</td>
<td>0</td>
<td>-</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\theta_2$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0</td>
<td>-</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
| $z(n)$ | y(1) | z(2) | y(3) | z(5) | y(6) | y(5) | y(7) | y(9) | z(10) |...

3. State-Space Model Transformation

To simplify the mathematical derivations, the observation equation can be rewritten as

$$z_n = \alpha_{0,n} y_n + \alpha_{1,n} y_{n-1} + \alpha_{2,n} \tilde{z}_n,$$  \hspace{1cm} (5)

where, taking into account that $\alpha_{0,n} + \alpha_{1,n} + \alpha_{2,n} = 1$, the auxiliary coefficients can be defined by

$$\alpha_{0,n} = \gamma_{0,n},$$ \hspace{1cm} (6)
$$\alpha_{1,n} = (1 - \gamma_{0,n}) (1 - \gamma_{0,n-1}) \gamma_{1,n},$$ \hspace{1cm} (7)
$$\alpha_{2,n} = (1 - \gamma_{0,n}) (1 - (1 - \gamma_{0,n-1}) \gamma_{1,n},$$ \hspace{1cm} (8)

for which the properties of the Bernoulli distribution with $i = 0, 1, 2$ are denoted as

$$E\{\alpha_i(\alpha_i)\} = \bar{\alpha}_i,$$
$$E\{(1 - \alpha_i)(1 - \alpha_i)\} = 1 - \bar{\alpha}_i,$$ \hspace{1cm} (9)

To represent the $k_n$-step delayed state $x_{n-k_n}$ via the current state $x_n$, we reorganize (1) to have the backward-in-time solution [36]:

$$x_n - k_n = F^{-k_n} \left( x_n - \sum_{i=0}^{k_n-1} F^i w_{n-i} \right).$$ \hspace{1cm} (10)

Assuming that $k_n = 1$ and substituting the delayed state $x_{n-1} = F^{-1} (x_n - w_n)$ and $y_n$ (1) into (5), we obtain the new observation equation, which has no latency:

$$z_n = \alpha_{0,n} (H x_n + v_n) + \alpha_{1,n} (H x_{n-1} + v_{n-1}) + \alpha_{2,n} (H F x_{n-1}),$$
$$= \alpha_{0,n} (H x_n + v_n) + \alpha_{1,n} (H (F^{-1} (x_n - w_n)) + v_{n-1}) + \alpha_{2,n} (H F (F^{-1} (x_n - w_n))),$$
$$= \alpha_{0,n} (H + \alpha_{1,n} H F^{-1} + \alpha_{2,n} H) x_n + \alpha_{0,n} v_n + \alpha_{1,n} v_{n-1} - (\alpha_{1,n} H F^{-1} + \alpha_{2,n} H) w_n,$$
$$= \tilde{H}_n x_n + \tilde{v}_n,$$ \hspace{1cm} (11)

where the modified observation matrix $\tilde{H}_n$ and white Gaussian noise vector $\tilde{v}_n$ are given by

$$\tilde{H}_n = (\alpha_{0,n} + \alpha_{2,n}) H + \alpha_{1,n} H F^{-1},$$ \hspace{1cm} (12)
$$\tilde{v}_n = \alpha_{0,n} v_n + \alpha_{1,n} v_{n-1} - (\alpha_{1,n} H F^{-1} + \alpha_{2,n} H) w_n,$$ \hspace{1cm} (13)

and the noise covariances $\tilde{R}_n = E\{\tilde{v}_n \tilde{v}_n^T\}$ and $Q_n = E\{w_n w_n^T\}$ are given with

$$\tilde{R}_n = \tilde{\alpha}_{0,n} R_n + \tilde{\alpha}_{1,n} R_{n-1} + \tilde{\alpha}_{1,n} H F^{-1} Q H^T F^{-1} + \tilde{\alpha}_{2,n} Q H^T.$$ \hspace{1cm} (14)
Note that the noise vector $\hat{v}_n$ and $w_n$ are time-correlated in (13). Since an optimal estimate is guaranteed under the independence of noise vectors, the de-correlation is required as will be shown next.

### 3.1. De-Correlation of $w_n$ and $\hat{v}_n$

To de-correlate the noise vectors $v_n$ and $w_n$, the Lagrange multiplier method can be used as proposed in [32], [33]. The approach suggests that the state equation must be subjected to a de-correlating condition as

$$x_n = Fx_{n-1} + w_n + \Lambda_n(z_n - \tilde{H}_n x_n - \hat{v}_n)$$

$$(15)$$

where $z_n$ is a vector of real data, $A_n = F - \Lambda_n \tilde{H}_n F$, $w_n = \Lambda_n z_n$, and $\hat{v}_n = (I - \Lambda_n \tilde{H}_n)w_n - \Lambda_n \bar{v}_n$. Our desire is to have a noise vector $\hat{v}_n$ such that $\hat{v}_n \sim \mathcal{N}(0, Q_v) \in \mathbb{R}^K$ has the covariance $Q_v = E\{\hat{v}_n \hat{v}_n^T\}$ defined by

$$Q_v = E\{(I - \alpha_{0,n} \Lambda_n H)(I - \alpha_{0,n} \Lambda_n H)^T + \alpha_{0,n} \Lambda_n R_n \Lambda_n^T + \alpha_{1,n} \Lambda_n R_{n-1} \Lambda_n^T\}$$

The Lagrange multiplier $\Lambda_n$ can now be derived to satisfy the desired property of $0 = E\{\hat{v}_n \hat{v}_n^T\}$

$$= E\{[(I - \Lambda_n \tilde{H}_n)w_n - \Lambda_n \bar{v}_n]\hat{v}_n^T\}$$

$$= E\{[(I - \alpha_{0,n} \Lambda_n H)n - \alpha_{0,n} \Lambda_n v_n - \alpha_{1,n} \Lambda_n v_{n-1}]$$

$$[\alpha_{0} \hat{v}_n + \alpha_{1,n} \hat{v}_{n-1} - (\alpha_1 H_n F_n^{-1} + \alpha_2 H_n w_n]^T\}$$

and further transformations yield

$$\Lambda_n = -Q_v(\alpha_{1,n} H_n F_n^{-1} + \alpha_{2,n} H_n)^T(\alpha_{0,n} R_n + \alpha_{1,n} R_{n-1})^{-1}.$$  

(17)

Provided the de-correlation of noise vectors $\hat{v}_n$ and $\bar{v}_n$, the estimation algorithms can be developed accordingly.

### 4. Filters Design for Delayed and Missing Data

In this section, the UFIR filter will be developed for systems represented with an uncertain observations model (12). Due to the ability of the UFIR filter to ignore zero mean noise [32] and because time-correlation does not produce bias errors, this filter can be applied straightforwardly. On the contrary, the KF and $H_{\infty}$ filter require the noise de-correlation.

#### 4.1. Batch UFIR Filter

The UFIR filter is the convolution-based structure that satisfies the unbiasedness condition $E\{x_n\} = E\{\hat{x}_n\}$ to ensure the unbiasedness. The batch UFIR filter produces an estimate over an averaging horizon $[m, n]$ of $N$ points, where $m = n - N + 1$ and the horizon length is required to be optimal as $N_{opt}$ to minimize the mean square error (MSE). This filter does not require any information about zero mean noise and initial values [34].

To design the batch UFIR filter, we consider the following model expanded on $[m, n]$ [34],

$$X_{m,n} = A_N x_m + B_N W_{m,n},$$

$$Y_{m,n} = H_{m,n} x_m + G_{m,n} W_{m,n} + V_{m,n},$$

(19)

where $X_{m,n} = [x_m^T x_{m+1}^T \ldots x_n^T]^T$ and $Y_{m,n} = [y_m^T y_{m+1}^T \ldots y_n^T]^T$ are extended vectors and the extended matrices are defined as

$$A_N = [I F^T \ldots F^{N-1} T]^T,$$

(20)

$$B_N = \begin{bmatrix} I \\ F \\ \vdots \\ F^{N-2} \\ F^{N-3} \end{bmatrix},$$

(21)

$$H_{m,n} = \begin{bmatrix} \tilde{H}_{m+1} F \\ \vdots \\ H_n F^{N-1} \end{bmatrix},$$

(22)

$G_{m,n} = H_{m,n} D_{m,n}$, $H_{m,n} = \text{diag}(H_m, H_{m+1} \ldots H_n)$, and matrix $H_n$ is specified with (14).

The batch UFIR filtering estimate is given by [34]

$$\hat{x}_n = F^{N-1}(H_{m,n} H_m)^{-1} H_{m,n} Y_{m,n}$$

(24)

$$Y_{m,n}$$

and the latency-dependent matrix $\tilde{H}_n$ is given by (14).

#### 4.2. Iterative UFIR Algorithm Using Recursions

Given the coefficients $\alpha_{0,n}$, $\alpha_{1,n}$, and $\alpha_{2,n}$ and the initial estimate at $s = n - N + K$ computed in a short batch form (23c) over $[m, s]$, the iterative UFIR filter computes estimates recursively on a horizon $[m, n]$ for the initial estimate at $s = n - N + K$. A pseudo code of the iterative UFIR filtering algorithm for delayed and missing data is listed as Algorithm 1.

#### 5. Applications

In this section, a numerical example of the maneuvering vehicle tracking is considered to test the performances of the UFIR, Kalman, and $H_{\infty}$ filtering algorithms under the randomly delayed and missing measurement data. The purpose is to investigate advantages and disadvantages of the algorithms...
under the same operation conditions. We consider a vehicle trajectory measured using a GPS reader in the Cook County of Illinois and available from the University of Illinois at Chicago [37]. The GPS-based vehicle trajectory in the local north and east coordinates is shown in Fig. 2.

![Vehicle trajectory](image)

Fig. 2. GPS-measured vehicle trajectory in the local north (y) and east (x) coordinates.

### 5.1. Tracking State-Space Model

Because information about the vehicle and its measurement is limited and no noise statistics are provided, we specify the vehicle trajectory and the process model as in the following.

- The state model (2) is described to have two states in each of the directions, \( K = 4 \), the state vector \( x_n = [x_{1n}, x_{2n}, x_{3n}, x_{4n}]^T \) has the components \( x_{1n} = x_n, x_{2n} = x_n' \), and matrices are

\[
F = \begin{bmatrix}
1 & \tau & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & \tau \\
0 & 0 & 0 & 1
\end{bmatrix},
B = \begin{bmatrix}
\frac{\tau^2}{4} & 0 & 0 \\
\frac{\tau^2}{4} & 1 & 0 \\
0 & 0 & \frac{\tau^2}{4} \\
0 & 0 & 0 & \frac{\tau^2}{4}
\end{bmatrix}
\]

- The observation is represented with (1) with the probabilities \( \gamma_0 = 0.7 \) and \( \gamma_1 = 0.8 \) and observation matrix

\[
H = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
\]

- The system noise and observation noise are supposed to be zero mean and white Gaussian. Supposing that the vehicle has an average speed of about 15 m/s, we assign the velocity noise standard deviation as \( \sigma_{v_w} = 1.5 \) m/s and neglect noise in the distance, \( \sigma_{1_w} = 0 \) m. The GPS service guarantees an error of smaller than 15 meters with the probability of 95\% in the 2\sigma sense. We thus set \( \sigma_v = 3.75 \) m and define the noise covariances as

\[
Q = \sigma_{v_w}^2 \begin{bmatrix}
\frac{\tau^2}{4} & 0 & 0 \\
\frac{\tau^2}{4} & 1 & 0 \\
0 & 0 & \frac{\tau^2}{4} \\
0 & 0 & 0 & \frac{\tau^2}{4}
\end{bmatrix},
R = \begin{bmatrix}
\sigma_v^2 & 0 \\
0 & \sigma_\gamma^2
\end{bmatrix}
\]

- The tuning factor of the UFIR filter (\( N_{opt} \)) is determined to minimize the MSE in the UFIR filter by solving the minimization problem

\[
N_{opt} = \arg \min_N [\text{tr} P_n(N)],
\]

where \( P_n \) is the error covariance. For the trajectory shown in Fig. 1, the optimal horizon was found to be \( N_{opt} = 5 \).

- The optimum tuning factor \( \theta_{opt} \) for the \( H_\infty \) filter is determined by minimizing the MSE. The value \( \theta_{opt} \) needs to be kept accurately in view of a high sensitivity of the \( H_\infty \) filter to \( \theta \). Otherwise, this filter may diverge [28], [36]. In this paper, we apply \( \theta_{opt} \approx 0.02 \) found experimentally to the entire trajectory.

The estimates produced by the properly tuned UFIR filter, KF, and \( H_\infty \) filter are sketched in Fig 3. It follows that all estimates are consistent with poorly distinguishable differences despite data failures. The prediction option is used when some data are lost as shown in Fig 3. Effect of the missing measurement phenomenon is illustrated in Fig 4 and it is seen that the estimation errors grow due to fast changes in the trajectory. Even so, the UFIR filter demonstrates an ability to converge faster to the regular mode than other filters due to the inherently bounded input bounded output (BIBO) stability.

### 5.2. Effect of Probabilistic Errors in \( \gamma_0 \)

Assuming that measurements are transmitted with the probabilities \( \gamma_0 = 0.7 \) and \( \gamma_1 = 0.8 \) at each \( n \), we next investigate the estimation errors caused by errors in \( \gamma_0 \). The aim is to
We thus assume that $\overline{\tau}_0$ varies from 0.1 to 0.9 and sketch the RMSEs as functions of $\overline{\tau}_0$ in Fig 5. What follows now is that the decrease in $\overline{\tau}_0$ results in growing errors, which means that the minimum tracking errors can be obtained only if the data transmission probability is equal to that in the algorithms. The KF errors range lower than in the UFIR filter for all $\gamma$. The $H_\infty$ improves the performance when $\gamma > 0.4$, but produces rapidly growing large errors when $\gamma < 0.3$ and diverges when $\gamma < 0.1$.

5.3. Effect of Errors in Noise Covariances

The noise statistics and error matrices are typically not well defined in object tracking. To investigate effects of the relevant errors on the estimator performance, we next introduce a scalar scaling error factor $\beta$ [36] and substitute in the algorithms $Q = \bar{Q}$ with $Q/\beta^2$ and $R = \bar{R}$ with $\beta^2 R$.

The RMSEs produced for the UFIR filter, KF, and $H_\infty$ filter under $\beta \neq 1$ are displayed in Fig 6 as functions of $\beta$. Inherently, the UFIR filter performance is $\beta$-invariant, while the KF and $H_\infty$ filter have different sensitivities to $\beta$ that affect their performances. It is especially true for the $H_\infty$ filter, which diverges beyond the interval of $0.5 < \beta < 1.0$.

6. Conclusions

The UFIR filter, KF, and game theory $H_\infty$ filter developed in this paper for randomly delayed binary Bernoulli-distributed data with packet dropouts have demonstrated a better performance than the standard filters. The effect was achieved by transforming the discrete-time state-space model to have no latency and extending the transformed model on a horizon of $N$ past data points. The problem with missing data was solved using a prediction option. An experimental example of GPS-based vehicle tracking has demonstrated that the UFIR filter generally outperforms both the KF and $H_\infty$ filter and is not prone to divergency.
References


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