Distributed UFIR Filtering with Applications to Environmental Monitoring

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Abstract—Environmental monitoring requires an analysis of large and reliable amount of data collected through node stations distributed over a very wide area. Equipments used in such stations are often expensive that limits the amount of sensing stations to be deployed. The technology known as Wireless Sensor Networks (WNS) is a viable option to deliver low-cost sensor information. However, electromagnetic interference, damaged sensors, and the landscape itself often cause the network to suffer from faulty links as well as missing and corrupted data. Therefore robust estimators are required to mitigate such effects. In this sense, the unbiased finite impulse response (UFIR) filter is used as a robust estimator for applications over WSN, especially when the process statistics are unknown. In this paper, we investigate the robustness of the distributed UFIR (dUFIR) filter with optimal consensus on estimates against missing and incorrect data. The dUFIR algorithm is tested in two different scenarios of very unstable WSN using real data. It is shown that the dUFIR filter is more suitable for real life applications requiring the robustness against missing and corrupted measurements under the unknown noise statistics.

Key-Words: WSN, robust estimation, distributed estimation, missing data, environmental monitoring.

1. Introduction

The interest to environmental monitoring has grown in recent decades due to climate change and serious natural disasters. However, to provide a reliable monitoring a large amount of data is required for environmental scientist to be able to provide useful information about the behavior of physical variables in question, provide forecasting of such behaviors, and emit or validate recommendations that will lead to new legislation [1]–[3]. The collection of data is often performed manually at a local scale, which sometimes is a difficult task due to extreme environmental events. Also, harsh whether may affect the sensing stations causing a significant data loss.

The wireless sensor networks (WNS) are well suited for environmental data acquisition [4]–[7] and allow the implementation of distributed methods, which are known to be more robust than centralized approaches. The robustness can also be improved using the unbiased finite impulse response (UFIR) filtering approach, which is effective in harsh environments, where electromagnetic interference, damaged sensors, or the landscape itself cause the network to suffer from faulty links and missing data. In many cases, optimal estimation is required along with adequate sensor fusing [8]–[11] to be robust against missing data, model errors (mismodeling), and incomplete information about noise statistics.

On the other hand, the restrictions of WSN caused by limited battery life and processing power put a stress on the development of the algorithms to ensure that these limited resources are exploited efficiently. In this sense, distributed filtering helps to improve battery life by minimizing the computational burden while performing real time estimation [12], [13]. Under the distributed filtering approach, each node is tasked to estimate \( \mathbf{Q} \) and a consensus protocol is implemented to average the estimates, measurements, or information [14], so that all nodes agree in a common value called the group decision value [15].

The low computational burden and optimal estimation make the Kalman filter (KF) a very popular sensor fusion technique [16]. Based on the KF approach, many authors have addressed the consensus problem in WSNs. A KF-based structure proposed in [17] requires each node to locally aggregate data and the covariance matrices taken from the neighbors and, in a posterior step, compute estimates using a KF with a consensus term. In [18], the KF has been developed as a fusion technique for local estimation and a consensus matrix. In [19], a KF-based algorithm has been presented to address an issue with missing data. Let us notice that the KF optimality is guaranteed only under the complete knowledge of the Gaussian noise statistics, adequate process modeling, and initial conditions [20]–[23]. Otherwise, the performance of the KF may drastically degrade and become unacceptable for real world WSN applications [24], [25].

It has been proven in [26]–[29] that a better robustness can be achieved by using filters operating on finite data horizons. Under such an assumption, a moving average estimator has been designed in [30] for weak observability. A consensus finite-horizon \( H_\infty \) approach was developed in [31] under missing data. In [32], an unbiased finite impulse response (UFIR) filter was developed for consensus on measurements. Although this filter has demonstrated a better robustness than the KF for WSN, it was designed under the condition that all of the sensors measure the same state at the same time. In [24], the UFIR structure has been developed for consensus on estimates, but the consensus factor was obtained through a previous analysis and without a mathematical background. In [33], a distributed UFIR (dUFIR) filter has been developed...
and tested over WSN for a rapid maneuvering object to show a better performance than the KF and $H_\infty$ filter.

Let us notice that in real life the whether monitoring often suffers from missing or false data and uncertain noise. The issue is illustrated in Fig 1, where the real temperature data are taken from a whether station. In the first 300 samples, one watches for noisy measurements with missing data and around $k = 400$, the sensor generates incorrect measurement of $-1C$.

The issue of missing data can be addressed using the KF and UFIR approaches. In [35], an extended KF was modified with this aim and in [25] a UFIR filter was developed, proving to be more robust in uncertain noise environments. For WSN, a KF was developed in [36] to address intermittent observations and, in [37], a UFIR filter developed under delayed and missing data. A version of the dUFIR filter with a prediction option was developed and tested in [38] to provide a better robustness. However, the data reconstruction capabilities of the algorithm were not shown under the missing data.

## 2. Dufir Filter for Wsn Under Missing Data

Consider dynamics of a quantity $Q$ measured over a distributed WSN and represent it with the following discrete $K$-states space equations,

$$ x_k = F_kx_{k-1} + B_kw_k, $$

$$ y_k^{(i)} = H_k^{(i)}(x_k + v_k^{(i)}), $$

$$ y_k = y_k x_k + v_k, $$

$$ \text{where } x_k \in \mathbb{R}^K, u_k \in \mathbb{R}^M, F_k \in \mathbb{R}^{K \times K}, E_k \in \mathbb{R}^{K \times M}, \text{and } B_k \in \mathbb{R}^{K \times L}. \text{ The } i, i \in [1, n], \text{ is a part of the WSN regarded as an undirected graph } G = (V, \mathcal{E}), \text{ where each vertex } v^{(i)} \in V \text{ is a node and each link is an edge of a set } \mathcal{E}, \text{ for } i \in I = \{1, \ldots, n\} \text{ and } n = |V| \text{ with } J \text{ inclusive neighbors.}$$

Each node measures $x_k$ by $y_k^{(i)} \in \mathbb{R}^p, p \leq K$, with $H_k^{(i)} \in \mathbb{R}^p \times K$. Local data $y_k^{(i)}$ are united in the observation vector $y_k = \left[y_{k}^{(i)} \ldots y_{k}^{(n)}\right]^T \in \mathbb{R}^{Jp}$ with $H_k = \left[H_k^{(1)} \ldots H_k^{(n)}\right]^T \in \mathbb{R}^{Jp \times K}$. Noise vectors $w_k \in \mathbb{R}^L$ and $v_k = \left[v_{k}^{(1)} \ldots v_{k}^{(n)}\right]^T \in \mathbb{R}^{Jp}$ are zero mean, not obligatorily white Gaussian, uncorrelated, and with the covariances $Q_k = E\{w_kw_k^T\} \in \mathbb{R}^{L \times L}, R_k = \text{diag}(R_k^{(1)} \ldots R_k^{(n)} \ldots) T \in \mathbb{R}^{Jp \times Jp}$, and $R_k^{(i)} = E\{v_{k}^{(i)} v_{k}^{(i)T}\}$. A binary variable $\gamma_k$ serves as an indicator of whether a measurement exist ($\gamma_k = 1$) or not ($\gamma_k = 0$), in which case the measurement prediction $y_k^{(i)}$ (2) is used by substituting $x_{k-1}$ with the estimate.

## 2.1. Predictive Distributed UFIR Filter

To obtain optimum estimates and achieve a consensus on estimates, we formulate the distributed estimate as

$$ \hat{x}_k = \hat{x}_k + \lambda_k^{\text{opt}} \Sigma_k, $$

where the centralized and individual estimates, $\hat{x}_k$ and $\hat{x}_k^{(i)}$ respectively, are obtained through

$$ \hat{x}_k = K_{m,k}Y_{m,k}, $$

$$ \hat{x}_k^{(i)} = K_{m,k}^{(i)}Y_{m,k}^{(i)}, $$

and $\Sigma_k = \sum_{i} (\hat{x}_k^{(i)} - \hat{x}_k^{(i)})$ is a consensus protocol that minimizes the disagreement between the first-order neighbors [17]. A consensus factor $\lambda_k^{\text{opt}}$ is chosen such that the root mean squared error (RMSE) is minimized by

$$ \lambda_k^{\text{opt}} = \arg \min \{ \text{tr} P(\lambda_k) \} $$

with $P(\lambda_k) = E\{(x - \hat{x}^{ic})(x - \hat{x}^{ic})^T\}$ as the relevant error covariance.

## 2.2. Batch Distributed UFIR Filter Design

To determine gains $K_{m,k}$ and $K_{m,k}^{(i)}$, we express the model equations (1)–(4) in the extended state space form over horizon $N$ as described in [24], [39],

$$ X_{m,k} = A_{m,k}x_m + D_{m,k}w_m, $$

$$ Y_{m,k} = C_{m,k}x_m + M_{m,k}w_m + v_m, $$

$$ Y_{m,k}^{(i)} = C_{m,k}^{(i)}x_m + M_{m,k}^{(i)}w_m + v_{m}^{(i)}, $$

$$ \text{where } X_{m,k} = \left[\begin{array}{c} \mbox{m,k} \mbox{m,k} x_{m+1} \ldots x_{k} \end{array}\right]^{T}, $$

$$ Y_{m,k} = \left[\begin{array}{c} \mbox{m,k} \mbox{m,k} y_{m+1} \ldots y_{k} \end{array}\right], $$

$$ W_{m,k} = \left[\begin{array}{c} w_{m+1} \ldots w_{k} \end{array}\right]^{T}, $$

$$ V_{m,k} = \left[\begin{array}{c} v_{m+1} \ldots v_{k} \end{array}\right]^{T}, $$

$$ Y_{m,k}^{(i)} = \left[\begin{array}{c} \mbox{m,k} \mbox{m,k} y_{m+1}^{(i)} \ldots y_{k}^{(i)} \end{array}\right]^{T}, $$

and the extended matrices are

$$ A_{m,k} = \left[\begin{array}{c} I \left[F_{m,k}^{(i)}(\mathcal{F}_{m,k}^{(i)}\ldots)\right]^{T} \end{array}\right], $$

$$ C_{m,k} = \bar{C}_{m,k}A_{m,k}, \text{ and } \bar{C}_{m,k} = \bar{C}_{m,k}^{(i)}A_{m,k}, $$

$$ M_{m,k} = \bar{M}_{m,k}D_{m,k}, \text{ where } \bar{M}_{m,k} = \mbox{diag}(H_m \ldots H_k), $$

$$ \bar{C}_{m,k} = \mbox{diag}(H_m^{(i)} \ldots H_k^{(i)}), $$

$$ \bar{F}_{m,k}^{(i)} = \left\{ \begin{array}{c} 1, \quad g = r + 1 \vspace{0.5cm} 0, \quad g > r + 1 \end{array} \right. $$

Fig. 1. Missing data and uncertain (colored) noise in temperature measurement data taken from [34].
Referring to [24], equation (5) can now be rewritten as
\[ \hat{x}_k = K_{m,k} y_{m,k} + J \lambda_k^{opt} K_{m,k} y_{m,k} - J \lambda_k^{opt} K_{m,k}^{(i)} y_{m,k}. \] (17)

Since we are interested in a robust UFIR filter that ignores the initial values, the unbiasedness condition must hold for the distributed, centralized and individual estimates,
\[ E\{\hat{x}_k\} = E\{\hat{x}_k\} = E\{\hat{x}_k^{(i)}\} = E\{x_k\} \] (18)

where
\[ x_k = \mathcal{F}_{m,k}^{-1} x_{m,k} + \mathcal{D}_{m,k} \mathbf{W}_{m,k} \] (19)

with \( \mathcal{D}_{m,k} = [\mathcal{F}_{m,k}^{-1} B_m \mathcal{F}_{m,k}^{-1} B_{m-1} \ldots B_{k-1} B_k]. \) The corresponding gains are defined by
\[ K_{m,k} = G_k C_{m,k}^T, \] (20)
\[ K_{m,k}^{(i)} = G^{(i)} C_{m,k}^{(i)} C_{m,k}^{(i)}^T, \] (21)

where \( G_k = (C_{m,k}^T C_{m,k})^{-1} \) and \( G^{(i)} = (C_{m,k}^{(i)} C_{m,k}^{(i)})^{-1}. \)

In real world applications, the nodes of the WSN may be unable to implement equation (17) due to large-dimension matrices and operations involved into the limited memory resources of the smart sensors. Therefore, below we develop an iterative form of (17) which fits better with the WSNs resources.

### 2.3. Optimum \( \lambda_k \)

The final expression of (8) is obtained by following [38]. The batch form of \( \lambda_k^{opt} \) is
\[ \lambda_k^{opt} = \frac{1}{J} (K_{m,k} \bar{R}_{m,k} \bar{R}_{m,k}^T - G_k G_{k-1}^{(i)} K_{m,k}) \times \bar{R}_{m,k} (K_{m,k} \bar{R}_{m,k} \bar{R}_{m,k}^T - 2 G_k G_{k-1}^{(i)}) \times K_{m,k} (K_{m,k} \bar{R}_{m,k} \bar{R}_{m,k}^T + K_{m,k}^{(i)} \bar{R}_{m,k} \bar{R}_{m,k}^T)^{-1}. \] (22)

where
\[ \bar{R}_{m,k} = E\{v_{m,k} v_{m,k}^T\} = \text{diag}(R_m \ldots R_k), \]
\[ \bar{R}_{m,k}^{(i)} = E\{v_{m,k}^{(i)} v_{m,k}^{(i)T}\} = \text{diag}(R_m^{(i)} \ldots R_k^{(i)}), \]
\[ \bar{R}_{m,k} = E\{v_{m,k} v_{m,k}^T\} = \text{diag}(R_m \ldots R_k). \]

If, for some particular application, the network and the process dynamics are both time invariant, \( \lambda_k^{opt} \) is also time invariant, to mean that equation (22) can be computed beforehand and embedded into the nodes.

### 2.4. Iterative Distributed UFIR filter

An iterative algorithm for the centralized estimates \( \hat{x}_k \) can be derived following the procedure described in [39], including a variable \( l \) that starts at \( l = k - N + K + 1 \) and ending in \( l = k \). The recursions are given by
\[ G_l = [H_l^T H_l + (A_l G_{l-1} A_l^T)^{-1}]^{-1}, \] (23)
\[ \hat{x}_l = A_l \hat{x}_{l-1}, \] (24)
\[ \hat{x}_l = \hat{x}_l^+ + G_l H_l^T (y_l - H_l \hat{x}_l^-). \] (25)

The initial values \( G_{l-1} \) and \( \hat{x}_{l-1} \) are computed at \( s = k - N + K \) in batch forms as
\[ G_s = (C_{m,s}^T C_{m,s})^{-1}, \] (26)
\[ \hat{x}_s = G_s C_{m,s}^T y_m. \] (27)

The individual estimates \( \hat{x}_k^{(i)} \) are provided by
\[ G_l^{(i)} = [H_l^{(i)} H_l^{(i)} + (A_l G_{l-1} A_l^{(i)})^{-1}]^{-1}, \] (28)
\[ \hat{x}_l^{(i)} = A_l \hat{x}_{l-1}^{(i)}, \] (29)
\[ \hat{x}_l^{(i)} = \hat{x}_l^{(i)} + G_l^{(i)} H_l^{(i)} (y_l - H_l \hat{x}_l^{(i)}) \] (30)

with the initial values
\[ G_s^{(i)} = (C_{m,s}^{(i)} C_{m,s}^{(i)})^{-1}, \] (31)
\[ \hat{x}_s^{(i)} = G_s^{(i)} C_{m,s}^{(i)} y_m. \] (32)

A pseudo code of the designed iterative dUFIR algorithm with consensus on estimates is listed as Algorithm 1.

**Algorithm 1: Iterative dUFIR Filtering Algorithm**

**Data:** \( y_k, R_k, R_k, \lambda_k^{opt} \)

**Result:** \( \hat{x}_k \)

1. begin
2. for \( k = N - 1 : \infty \) do
3. \( m = k - N + 1, \) \( s = m + K - 1; \)
4. \( G_s = (H_m^T H_m)^{-1}; \)
5. \( G_s^{(i)} = (H_m^{(i)} H_m^{(i)})^{-1}; \)
6. if \( \gamma_k = 0 \) then
7. \( y_k^{(i)} = H_k^T F_k \hat{x}_k; \)
8. end if
9. \( \hat{x}_s = G_s H_m^T Y_m; \)
10. end for
11. for \( l = s - 1 : k \) do
12. \( \hat{x}_l^- = \hat{x}_{l-1}^+; \)
13. \( \hat{x}_l^{(i)} = F_l \hat{x}_{l-1}^{(i)}; \)
14. \( G_l = [H_l^T H_l + (F_l G_{l-1} F_l^T)^{-1}]^{-1}; \)
15. \( G_l^{(i)} = [H_l^{(i)} H_l^{(i)} + (F_l G_{l-1} F_l^{(i)})^{-1}]^{-1}; \)
16. \( \hat{x}_l = \hat{x}_l^- + G_l H_l^T (y_l - H_l \hat{x}_l^-); \)
17. \( \hat{x}_l^{(i)} = \hat{x}_l^{(i)} + G_l^{(i)} H_l^{(i)} (y_l - H_l^{(i)} \hat{x}_l^{(i)}). \)
18. end for
19. \( \hat{x}_k^+ = (I + J \lambda_k^{opt}) \hat{x}_k - J \lambda_k^{opt} \hat{x}_k; \)
20. end
21. end
22. † First data \( y_0, y_1, \ldots, y_{N-1} \) must be available.

### 3. Applications to Environment Monitoring

We consider temperature measurements provided in 2007 at the Grand-St-Bernard pass at 2400 m between Switzerland and Italy as part of the Sensorscope project, which aims to develop a large-scale distributed environmental measurement system centered on a wireless sensor network. Measurements were recorded individually by low-cost sensing stations and
are available from [34]. In this work, we consider only the stations shown in Fig. 4 and Fig. 7, depicted as red dots. Measurements were performed each two seconds during two months. For each sensor, the average of the measurements was computed each hour along with the error variance. In Fig 2, we show the resulting standard deviations for each sensor, where stations 2 and 9 demonstrate large temperature deviations for unknown reasons.

![Temperature standard deviations observed in the stations.](image)

Fig. 2. Temperature standard deviations observed in the stations. 

The individual one-hour average temperature measurements are sketched in Fig. 3. Here, we observe similar behaviors in all stations. However, some stations present large gaps of information and a very unstable performance. It is important to notice that stations 2, 20 and 9 also conduct incorrect measurements of −1°C that cannot be regarded as missing data.

![Temperature measurements conducted by 11 stations.](image)

Fig. 3. Temperature measurements conducted by 11 stations.

### 3.1. Tuning dUFIR Algorithm

To apply the dUFIR algorithm, we use model (1)–(4) with the following matrices [25],

\[ A = \begin{bmatrix} 1 & \tau \\ 0 & 1 \end{bmatrix}, \quad H^{(i)} = \begin{bmatrix} 1 & 0 \end{bmatrix}, \]

where \( \tau = 1 \) and \( B = I \). As stated by (22), to compute \( \lambda_k^{opt} \) we need individual variances of the sensors, but this information is not available in the data set. Furthermore, it is unclear if all sensors are of the same manufacturer. In Fig 2, we observe that the standard deviation behave similarly for eleven sensors and we take the average and determine an estimated variance for each sensor. The results are shown in Table I and the optimum horizon was measured to be \( N^{opt} = 37 \) [27].

### 3.2. Network with 350 m Link Range

To test the algorithm, we simulate a WSN for a maximum link distance of 350 m. The resulting topology is sketched in Fig. 4. The estimation results by Algorithm 1 are shown in Fig. 5 for three sensing stations. Here, noise reduction is observed in all stations and yet large gaps are bridged over in the 2nd station (Fig. 5 b) and 9th station (Fig. 5 c).

A key difference between the 9th and 2nd stations is observed in the range of 540 < \( k \) < 780. While measurements are completely lost in the 2nd station, a false measurement of \(-1\)°C is recorded in the 9th station. The algorithm employs the prediction option only when missing data are detected and it considers a wrong measurement of \(-1\)°C as true. However, due to the distributive nature of the dUFIR filter, the estimates of the 9th station do not get away from the remaining stations. This can be seen in Fig. 6a, where we show estimates of all of the stations. In Fig. 6b, the estimate variances are considered as an indicator of disagreements between the nodes. It can also be seen that much less disagreements are observed when the measurements are correct.

### 3.3. Network with 200 m Link Range

Performance of the dUFIR Algorithm 1 depends on the amount of redundant available information. When the number of the links decreases, the disagreements and the estimation errors increase. In Fig. 7, we consider the same base stations but with a restriction link range of 200 m. In this case, a smaller number of the links are available and the 9th station
TABLE I

<table>
<thead>
<tr>
<th>Station</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.13</td>
</tr>
<tr>
<td>11</td>
<td>0.17</td>
</tr>
<tr>
<td>12</td>
<td>0.18</td>
</tr>
<tr>
<td>13</td>
<td>0.20</td>
</tr>
<tr>
<td>14</td>
<td>0.13</td>
</tr>
<tr>
<td>15</td>
<td>0.17</td>
</tr>
<tr>
<td>2</td>
<td>0.15</td>
</tr>
<tr>
<td>20</td>
<td>0.15</td>
</tr>
<tr>
<td>3</td>
<td>0.16</td>
</tr>
<tr>
<td>8</td>
<td>0.11</td>
</tr>
<tr>
<td>9</td>
<td>0.19</td>
</tr>
</tbody>
</table>

Fig. 5. Temperature measurements and estimates: a) 10th station, b) 9th station, and c) 2nd station.

Fig. 6. Temperature measurements and estimates: a) all stations and b) disagreement between estimates.

Fig. 7. Simulated network links between the sensing stations for the range of 200 m.

Fig. 8. Temperature measurements and estimates: a) all stations and b) disagreement between the estimates.

has a single link rather than three links in the previous case (Fig 4). Due to a lack of the redundant information and an inability to process wrong data as missing, estimates by the 9th station deviate from those by other stations (Fig 8a). Here, one can also see the effect of the 9th station on the performance of the 13th station. Under such circumstances, the consensus and prediction capabilities of the Algorithm 1 are not able to compensate incorrect data in the 9th station that results in growing disagreement between the estimates (Fig. 8b).
4. Conclusions

In this paper, we have applied the developed dUFIR filtering algorithm to real temperature measurements with missing and incorrect data. We have discussed the dUFIR filter performance in two feasible scenarios of different numbers of the links and confirmed that under the allowed minimum three links the dUFIR filter produces acceptable estimates and provides a good data reconstruction. Given that the dUFIR filter does not require any information about the process statistics, it thus better suites the real life WSN architectures, where the noise statistics are either unavailable or known very approximately. This can be stated as a great advantage against the KF-based algorithms.

References


