Stable Adaptive IIR System Identification using Particle Swarm Optimization

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Abstract: This paper presents an adaptive IIR system identification method using Particle Swarm Optimization (PSO). System identification is a method for estimating characteristic of an unknown system using the measured input and output signals. In PSO, potential solutions called particles are updated according to simple mathematical formulas of particle's positions and velocities. However, the IIR system identification methods using PSO have a problem that it is very difficult to get the global optimum solution when the adaptive filter becomes once unstable during system identification. Moreover, the standard PSO has a problem that it tends to converge to local optimal solution because of its strong directivity. In the proposed method, the particle's velocities are updated using plural better solutions in order to avoid the convergence to local optimal solution and the output signal of an unknown system is used as the feedback signal of the adaptive filter in order to achieve stable system identification. Some simulation results show that the proposed method has higher identification accuracy than conventional methods.

Key–Words: Adaptive IIR filters, System identification, Particle Swarm Optimization

1 Introduction

System identification is a method for estimating characteristic of an unknown system using the measured input and output signals. To control a system, it is very important to know its characteristic. Therefore, system identification is used in wide fields such as signal processing, control system, and modeling and communication system [1–5].

In general, system identification consists of an unknown system, adaptive filters to model characteristic of the unknown system, and adaptive algorithm to update adaptive filter coefficients. FIR (Finite impulse response) filters or IIR (Infinite impulse response) filters are used for the adaptive filters. The IIR filters can achieve characteristic equivalent to the FIR filters by using lower order, while it is necessary to consider stability because the IIR filters might or might not be stable depending on the values of the filter coefficients. Moreover, because the error surfaces in the IIR or nonlinear system identification have often multimodality, it is difficult to use a gradient-based algorithm like LMS (Least Mean Square) which is well suited for identification of linear static systems.

In recent years, system identification using the metaheuristic methods have been researched actively [6–10]. The metaheuristic method doesn't guarantee that optimal solutions can be found, but it can search directly and faster for non-linear problems. Among them, PSO (Particle swarm optimization) [11] is attracted attention as an optimization algorithm that can search optimal solutions of multimodal problems faster than other methods like GA (Genetic Algorithm) [12, 13]. PSO is based on social behavior of living things such as a flock of birds and uses information of a swarm and particles themselves. In PSO, potential solutions called particles are updated according to simple mathematical formulas of particle's positions and velocities. However, IIR system identification using PSO has two problems. One is that if the system (adaptive filter) becomes once unstable, the particles are likely to get trapped in the local minimum and never converge to the global optimum. Another is that the particles converge to a local optimal solution because of its high directivity to solutions [14]. These two facts cause low estimation accuracy.

This paper presents an IIR system identification method using PSO to improve identification accuracy of an unknown system. In the proposed method, the output signals of an unknown system are used as the feedback signals of the adaptive filter. As a result, the proposed method can carry out stable system identification because the feedback signals don't diverge even if the adaptive IIR filter becomes unstable. Moreover, for the filter coefficients update, the proposed method

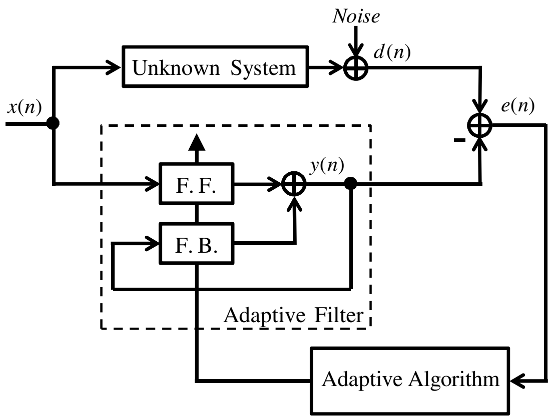

Figure 1: Block diagram of general IIR system identification

uses plural local solutions instead of the globally best solution. Therefore, strong directivity to solutions is regulated and convergence to the local solutions can be avoided.

This paper is organized as follows: in Section 2, conventional methods of IIR system identification using PSO are summarized. In Section 3, the improvement of IIR system identification method using PSO is proposed. To verify the effectiveness of the proposed method, several simulations are given in Section 4. Section 5 is the conclusions of this work.

2 Conventional PSO for adaptive system identification

A block diagram of general IIR system identification is shown in Fig. 1. In Fig. 1, $x(n)$ represents input signals, $d(n)$ and $y(n)$ represent output signals of an unknown system and an adaptive IIR filter, respectively. Moreover, F. F. and F. B. represent the parts of feedforward and feedback of the adaptive IIR filter. As shown in Fig. 1, the coefficients of an adaptive filter are updated so that error signals $e(n)$, which are difference between $d(n)$ and $y(n)$, are reduced. If the adaptive filter is a linear IIR filter, then the output signals are given by

$$y(n) = \sum_{m=0}^{M} b_m(n)x(n-m) - \sum_{m=1}^{L} a_m(n)y(n-m), \tag{1}$$

where $a_m(n)$ and $b_m(n)$ represent feedback and feedforward coefficients of the filter, respectively.

In optimization by PSO [11], potential solutions called particles compose a swarm and search particle's positions that have better fitness. For updating the particle's positions, each particle uses its own personal best (i. e. the best position it personally has found, Pbest) and the global best (i. e. the best position the swarm has found, Gbest). The updating algorithm of the adaptive IIR filter coefficients is shown as follows.

1) Initialization: In order to find an optimal solution, each particle has a position vector $\boldsymbol{P}^i(n)$ and a velocity vector $\boldsymbol{V}^i(n)$ as shown in eq.(2) and eq.(3).

$$\boldsymbol{P}^i(n) = \left\{p_0^i(n), p_1^i(n), \ldots, p_{S-1}^i(n)\right\}, \tag{2}$$

$$\boldsymbol{V}^i(n) = \left\{v_0^i(n), v_1^i(n), \ldots, v_{S-1}^i(n)\right\}, \tag{3}$$

where $i = 1, \ldots, N$ is an index of particles, $n = 0, \ldots, n_{max}$ is the number of iterations, and $S = M + L + 1$ is the number of the filter coefficients of the adaptive IIR filter. At $n = 0$, position vectors $\boldsymbol{P}^i(n)$ and velocity vectors $\boldsymbol{V}^i(n)$ are initialized by using random numbers drawn from a uniform distribution.

2) Calculating fitness: The fitness of each particle's current positon is calculated by

$$J^i(n) = \frac{1}{K}\sum_{k=0}^{K-1}\left[d(n-k) - y^i(n-k)\right]^2, \tag{4}$$

where K is the number of signals for calculating fitness, and $y^i(n)$ is the output signals of the adaptive IIR filter constructed by particle i. The output signal $y^i(n)$ is given by

$$y^i(n) = \sum_{j=0}^{M} p_j^i(n)x(n-j) - \sum_{j=1}^{L} p_{j+M}^i(n)y_{Gbest}(n-j). \tag{5}$$

As shown in (5), position vectors $\boldsymbol{P}^i(n)$ represent the coefficients of the adaptive filter in IIR system identification using PSO.

3) Updating Pbest and Gbest: Pbest positions of each particle $\boldsymbol{P}^i_{Pbest}$ are updated by comparing $J^i(n)$ with fitness of Pbest (J^i_{Pbest}) as follows:

$$\boldsymbol{P}^i_{Pbest} = \begin{cases} \boldsymbol{P}^i(n) & (J^i_{Pbest} > J^i(n)) \\ \boldsymbol{P}^i_{Pbest} & (otherwise) \end{cases} \tag{6}$$

When Pbest is updated, the Gbest position $\boldsymbol{P}_{Gbest}$ is updated by comparing $J^i(n)$ with fitness of Gbest (J_{Gbest}) as follows:

$$\boldsymbol{P}_{Gbest} = \begin{cases} \boldsymbol{P}^i(n) & (J_{Gbest} > J^i(n)) \\ \boldsymbol{P}_{Gbest} & (otherwise) \end{cases} \tag{7}$$

4) Updating position vectors and velocity vectors: Position vectors $\boldsymbol{P}^i(n)$ and velocity vectors $\boldsymbol{V}^i(n)$ are modified according to the following equations:

$$\boldsymbol{P}^i(n+1) = \boldsymbol{P}^i(n) + \boldsymbol{V}^i(n+1) \tag{8}$$

and

$$\begin{aligned}\boldsymbol{V}^i(n+1) = w\boldsymbol{V}^i(n) + \boldsymbol{\rho_1} \circ (\boldsymbol{P}^i_{Pbest} - \boldsymbol{P}^i(n)) \\ + \boldsymbol{\rho_2} \circ (\boldsymbol{P}_{Gbest} - \boldsymbol{P}^i(n)),\end{aligned} \tag{9}$$

where w is the inertia weight chosen in the interval $[0, 1]$, $\boldsymbol{\rho_1}$ is the Pbest acceleration coefficient, $\boldsymbol{\rho_2}$ is the Gbest acceleration coefficient, and $\circ$ is the Hadamard element-wise vector product.

5) Termination conditions: The algorithm is stopped when $n = n_{max}$. Otherwise, it goes back to 2).

3 Proposed Method

3.1 Proposed IIR system identification

In the conventional system identification shown by Fig. 1, when the output signals $y(n)$ of an adaptive filter diverge once, after that, Gbest becomes difficult to be updated. This is because that the fitness shown in (4) also diverges. Therefore, in the proposed IIR system identification method, the output signals $d(n)$ of the unknown system is used as the feedback signals of the adaptive filter. As a result, Gbest can be updated and stable system identification can be carried out because the feedback signals don't diverge even if the adaptive IIR filter became unstable. The block diagram of the proposed IIR system identification is shown in Fig. 2. Then, the output signals of the adaptive filter are given by

$$y^i(n) = \sum_{j=0}^{M} p^i_j(n)x(n-j) - \sum_{j=1}^{L} p^i_{j+M}(n)d(n-j). \tag{10}$$

3.2 Proposed PSO

In the standard PSO, velocity vectors $\boldsymbol{V}^i(n)$ are updated by using position vectors of Gbest as shown in (9). However, this has a high probability that the particles converge to local solutions because of its high directivity to solutions. Therefore, we update the position vectors by using Pbest of other particle instead of Gbest. In the proposed PSO, velocity vectors $\boldsymbol{V}^i(n)$ are updated by

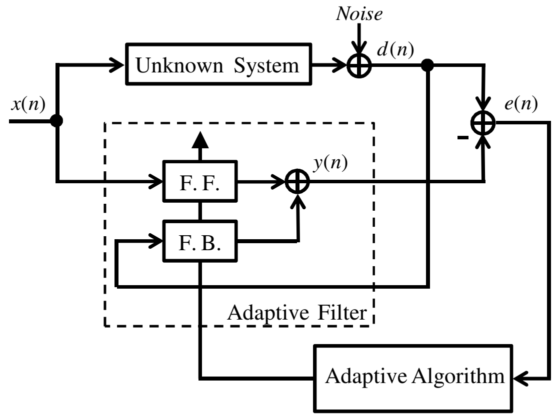

Figure 2: Block diagram of proposed IIR system identification

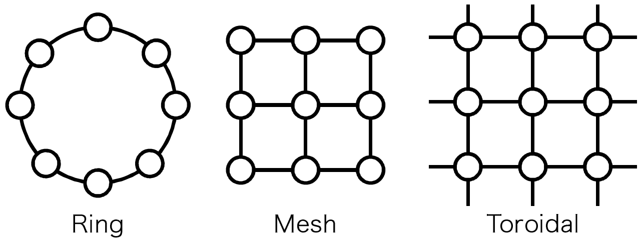

Figure 3: Structure of topologies

$$\begin{aligned}\boldsymbol{V}^i(n+1) = w\boldsymbol{V}^i(n) + \boldsymbol{\rho_1} \circ (\boldsymbol{P}^i_{Pbest} - \boldsymbol{P}^i(n)) \\ + \boldsymbol{\rho_2} \circ (\boldsymbol{P}^\alpha_{Pbest} - \boldsymbol{P}^i(n)),\end{aligned} \tag{11}$$

where α is uniformly distributed random integers in the interval $[0, R]$. That is, velocity vectors $\boldsymbol{V}^i(n)$ are updated by using Pbest chosen from top R for all particles. Therefore, the proposed PSO can search a solution in the wide areas more than the standard PSO.

4 Simulations

In this section, we show that the proposed method has higher identification accuracy than the conventional methods: conventional PSO (standard PSO [11]), FIPSO [15] and RegPSO [16]. Moreover, neighborhood topologies [17] shown in Fig. 3 is also applied to the proposed PSO, conventional PSO, and FIPSO. In all following simulations, white noise is used as the input signals and signal-to-noise rate (SNR) is 40dB, and the parameters used in simulations are shown in Table 1.

Table 1: Parameters of simulations

The number of the maximum iteration n_{max}	3000
The number of signals for calculating fitness K	50
Inertia weight w	0.8
The Pbest acceleration coefficient ρ_1	[0,1]
The Gbest acceleration coefficient ρ_2	[0,1]
The acceleration coefficient of FIPSO	[0,4]
The velocity clamping percentage of RegPSO	0.5
The regrouping factor of RegPSO	0.00011

4.1 Simulation 1

First, it is shown that the proposed IIR system identification method shown in Fig. 2 has more higher system identification accuracy than the conventional method described in Fig. 1. In this simulation, population size $N = 4$, and initial search range of RegPSO is $[-1.3, 1.3]$.

Output signals of the unknown system are given by

$$d(n) = 0.05x(n) - 0.4x(n-1) - 1.1314d(n-1) + 0.25d(n-2). \quad (12)$$

This was taken from [10]. The filter order of the adaptive IIR filter is $L = 2$ and $M = 1$. Here, the position vectors $\boldsymbol{P}^i(n)$ and velocity vectors $\boldsymbol{V}^i(n)$ are updated using eq.(8) and eq.(9) for all methods.

Convergence characteristics of each system identification method are shown in Fig. 4 and table 2. In Fig. 4, "Gbest feedback" is the results by the conventional system identification method in Fig. 1 and "system feedback" is the results by the proposed system identification method in Fig. 2. Moreover, the circles represent MSE of each system identification result and crossbars represent average MSE of 10 system identification results.

From Fig. 4 and table 2, it is shown that the proposed system identification method has higher estimation accuracy than the conventional system identification method in all topologies. This is because that Gbest of the conventional system identification method is not updated almost after the system (adaptive filter) became once unstable. Therefore, the conventional method converges to the local minimum solution at the early iteration steps. On the other hand, Gbest of the proposed system identification method is continuously updated.

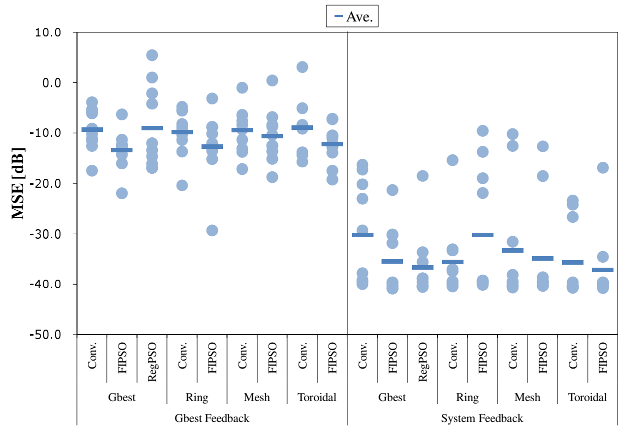

Figure 4: MSE of Gbest feedback and system feedback in Sim. 1

4.2 Simulation 2

Next, it is shown that proposed PSO described in Sec.3.2 avoids convergence to the local solutions. In this simulation, population size $N = 9$, initial search range of RegPSO is $[-0.3, 0.3]$, and the selected numbers R for Gbest, Ring, Mesh, Toroidal are $6, 3, 5, 4$ respectively. Moreover, in this simulation, system identification shown in Fig. 2 was used for all PSO.

Output signals of the unknown system are given by

$$\begin{aligned} d(n) &= 0.1084x(n) + 0.5419x(n-1) \\ &+ 1.0837x(n-2) + 1.0837x(n-3) \\ &- 0.5419x(n-4) + 0.1084x(n-5) \\ &- 0.9853d(n-1) + 0.9738d(n-2) \\ &+ 0.3864d(n-3) + 0.1112d(n-4) \\ &+ 0.0113d(n-5) \end{aligned} \quad (13)$$

Table 2: Average MSE in Sim. 1

Topology	Gbest			Ring		Mesh		Toroidal	
Type of PSO	Conv.	FIPSO	RegPSO	Conv.	FIPSO	Conv.	FIPSO	Conv.	FIPSO
Gbest F.B.	-9.3	-13.4	-9.0	-9.7	-12.6	-9.4	-10.6	-8.9	-12.2
System F.B.	-30.2	-35.5	-36.6	-35.5	-30.2	-33.3	-34.9	-35.7	-37.1
Difference	-20.9	-22.1	-27.7	-25.8	-17.5	-23.8	-24.3	-26.7	-25.0

Table 3: Average MSE in Sim. 2

	Sim. 2			
Type of PSO	Conv.	FIPSO	RegPSO	Prop.
Gbest	-28.1	-26.9	-33.5	**-36.8**
Ring	-29.9	-29.9	/	**-35.4**
Mesh	-30.3	-33.7	/	**-36.2**
Toroidal	-27.7	-31.2	/	**-37.3**

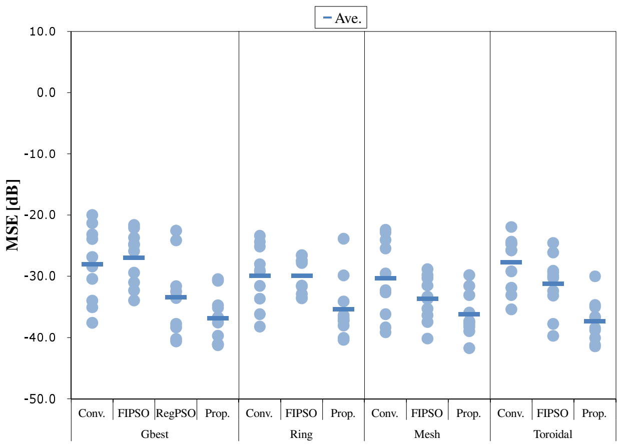

Figure 5: MSE of conventional methods and the proposed method in Sim. 2

This is the same in [18]. The filter order of the adaptive IIR filter is $L = 5$ and $M = 5$.

Convergence characteristics of each PSO are shown in Fig. 5 and Table 3. Moreover, convergence characteristics of PSO with each topology are shown in Figs. 6-9. As shown in Fig. 5, it is shown that proposed PSO has higher estimation accuracy than PSO, FIPSO and RegPSO for all topogies. Moreover, it is shown from Figs.6-9 that the conventional PSO, FIPSO and RegPSO converge to the local minimum solutions in the early iteration steps, after that, Gbest is not updated almost.

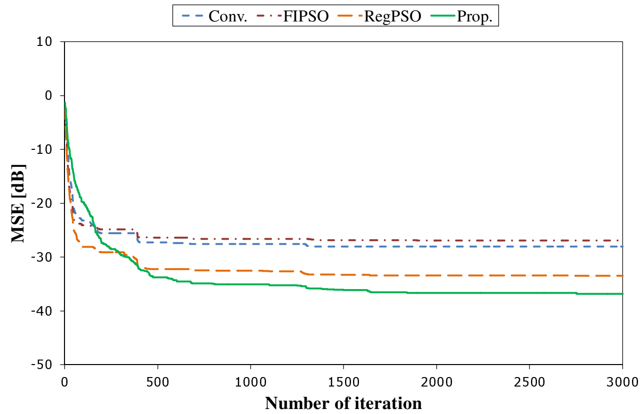

Figure 6: Convergence characteristics of Gbest in Sim. 2

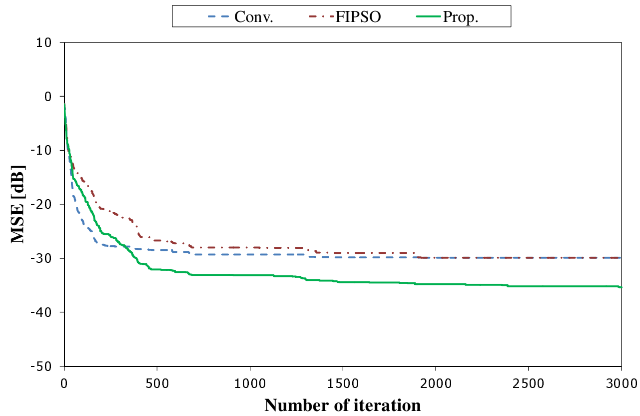

Figure 7: Convergence characteristics of Ring in Sim. 2

4.3 Simulation 3

Finally, it is shown that availability of the proposed PSO is clarified in a nonlinear system. In this simulation, population size $N = 4$, initial search range of RegPSO is $[-1.2, 1.2]$, and the selected numbers R of Gbest, Ring, Mesh, Toroidal are $4, 3, 3, 3$ respectively.

Output signals of the unknown system are given by

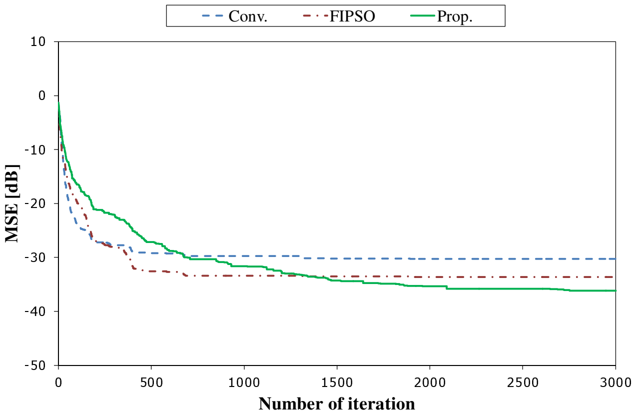

Figure 8: Convergence characteristics of Mesh in Sim. 2

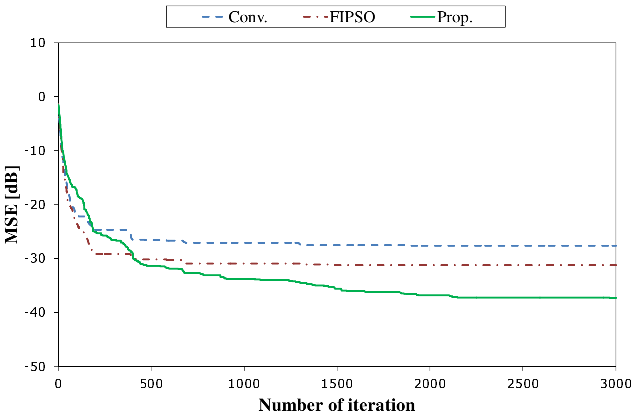

Figure 9: Convergence characteristics of Toroidal in Sim. 2

$$d(n) = x(n) + 0.04x^2(n) + 0.1x^3(n) - 0.3d(n-1) + 0.002d^2(n-1). \quad (14)$$

As the model of the adaptive filter, we use the folloing equation:

$$y(n) = p_0^i x(n) + p_1^i x^2(n) + p_3^i x^3(n) \; p_4^i d(n-1) + p_5^i d^2(n-1) \quad (15)$$

Convergence characteristics are shown in Fig. 10 and Table 4. Figs. 11-14 show the convergence characteristics for each topology. From these figures and table, it is shown that proposed PSO has higher estimation accuracy than the conventional PSO, FIPSO and Reg-PSO in all topogies. Therefore, the proposed method is also useful for nonlinear system identification.

Table 4: Average MSE in Sim. 3

	Sim. 3			
Type of PSO	Conv.	FIPSO	RegPSO	Prop.
Gbest	-28.1	-26.9	-33.5	**-36.8**
Ring	-29.9	-29.9	/	**-35.4**
Mesh	-30.3	-33.7	/	**-36.2**

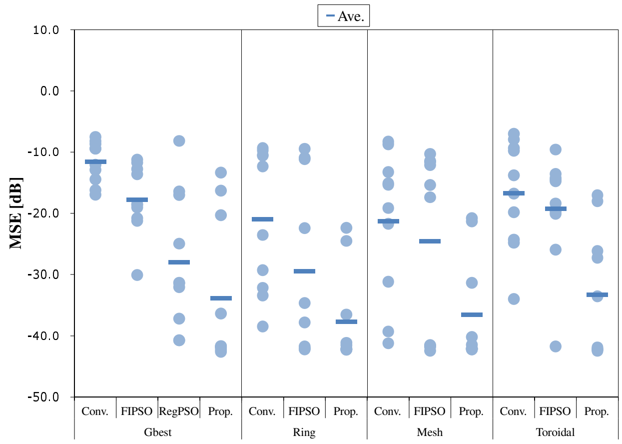

Figure 10: MSE of conventional methods and the proposed method in Sim. 3

5 Conclusion

In this paper, new system identification method using Particle Swarm Optimization (PSO) was proposed. In the proposed system identification method, the output signals of the unknown system are used as the feedback signals of the adaptive filter. As a result, the proposed method can continuously update Gbest, even if the adaptive filter became unstable once. Moreover, by using random selected Pbest instead of Gbest for updating the velocity of particle, it can be avoided convergence to the local minimum solutions. In the simulations, it was confirmed that the proposed method has higher system identification accuracy than the conventional methods.

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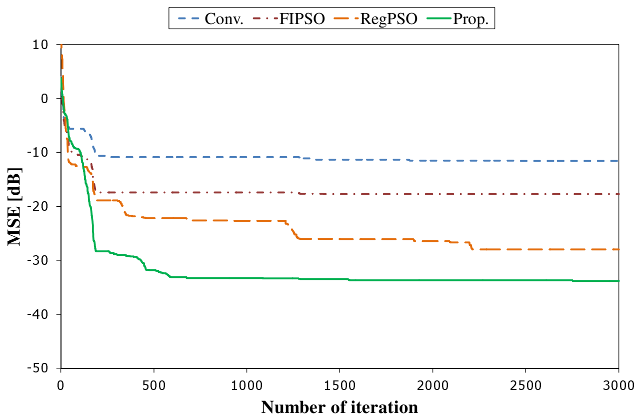

Figure 11: Convergence characteristics of Gbest in Sim. 3

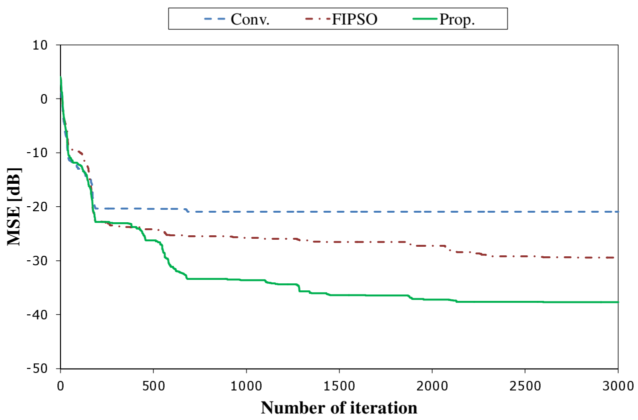

Figure 12: Convergence characteristics of Ring in Sim. 3

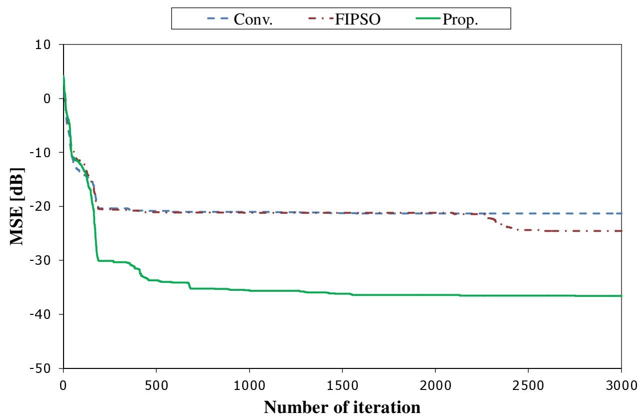

Figure 13: Convergence characteristics of Mesh in Sim. 3

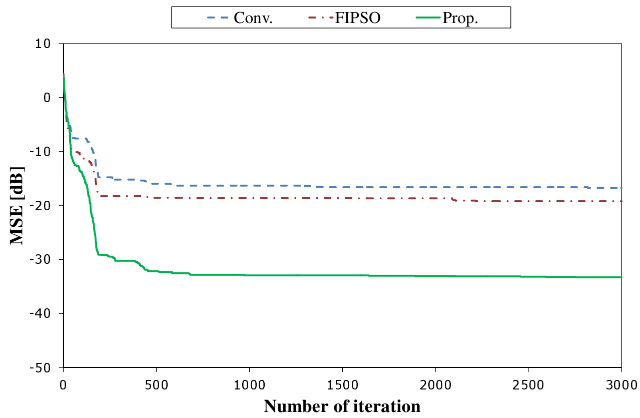

Figure 14: Convergence characteristics of Toroidal in Sim. 3

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