$$\frac{\sigma_a}{\mu_a} = \frac{\sqrt{\int_{-\infty}^{+\infty} \left(a - k \cdot \mu_0\right)^2 \cdot \frac{1}{k} p_0\left(\frac{a}{k}\right) da}}{k \cdot \mu_0} \quad (7)$$

Then do the variable substitution $a=ka_0$ to the integral on the right side of Equation (7):

$$\frac{\sigma_a}{\mu_a} = \frac{\sqrt{\int_{-\infty}^{+\infty} \left(ka_0 - k\mu_0\right)^2 \cdot \frac{1}{k} p_0\left(a_0\right) d\left(ka_0\right)}}{k\mu_0}$$

$$= \frac{\sqrt{k^2} \cdot \sqrt{\int_{-\infty}^{+\infty} \left(a_0 - \mu_0\right)^2 \cdot p_0\left(a_0\right) da_0}}{k\mu_0} \quad (8)$$

Remember that the variables a and a_0 represent the amplitude value, which is non-negative. Therefore, k is also non-negative. Then Equation (8) can be rewritten as:

$$\frac{\sigma_a}{\mu_a} = \frac{\sqrt{\int_{-\infty}^{+\infty} \left(a_0 - \mu_0\right)^2 \cdot p_0\left(a_0\right) da_0}}{\mu_0} \quad (9)$$

Notice that the numerator of the right side of Equation (9) is just the standard deviation of a_0. Therefore,

$$\frac{\sigma_a}{\mu_a} = \frac{\sigma_0}{\mu_0} \quad (10)$$

Notice that the right side of Equation (10) is constant given the prototype distribution $p_0(a_0)$. Therefore, the standard deviation coefficient of a is consistent whatever the scaling factor k is, which is equal to that of the prototype variable a_0. This just accords well with the experimental results shown in Section 2. Therefore, the feature of "consistent standard deviation coefficient" supports the model proposed here.

4 Conclusion

In this paper, the statistic feature of unvoiced pronunciation in frequency domain is studied. The Study is focused on the short-time amplitude spectrum, and is based on the data obtained by STFT on signals of stable and sustaining unvoiced pronunciations. A new statistic feature named "consistent standard deviation coefficient" is discovered. This feature indicates strong associations between amplitude distributions of different frequency components. On the other hand, such association is also revealed by comparing the normalized amplitude histograms of every frequency components. A new model is proposed to representing such association. In this model, the random variables representing amplitude of every frequency component belong to the same pdf type, but they have different expectations. If the prototype pdf $p_0(a_0)$ is determined, the pdf of any frequency's amplitude a can be derived by $a=\mu a_0$, where μ is a's expectation. Moreover, by mathematical analysis, this model accords well with the feature of "consistent standard deviation coefficient". The results in the paper deepen the understanding of the stochastic features of unvoiced pronunciation, which is an important topic in speech signal analysis. In future work, the specific pdf type will be studied to suit the short-time amplitude spectrum data for unvoiced pronunciation. And other types of pronunciation like voiced phonemes will be also studied statistically for new possible features.

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