Heuristic Approach for Optimal Location and Sizing of Distributed Generators in AC Distribution Networks

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Abstract: This paper addresses, from a heuristic point of view, the problem of the optimal location and sizing of distributed generators (DGs) in alternating-current distribution networks with radial topology. A master–slave optimization approach is followed to place and size the DGs. In the master stage a simple recursive seach method based on sequential searching is proposed. In the case of the slave algorithm, we present an emerging metaheuristic for solving the optimal power flow problem. This metaheuristic is called the vortex search algorithm. It works with a Gaussian distribution and a variable radius function for exploring and exploiting the solution space. Numerical simulations of 33- and 69-node test feeders show its efficiency, simplicity and robusteness in comparison to other methods in the literature.

Key Words: Distribution networks, distributed generators, heuristic approach, power loss minimization, optimal power flow, vortex search optimization.

1 Introduction

Electrical distribution networks are responsible for providing electricity service to end-users (i.e., industrial, commercial or residential users), and for satisfying quality, security and reliability criteria [1, 2]. For economic advantages to the utilities in terms of construction and protection, these grids typically have a radial structure, being operated under the alternating-current (AC) paradigm [3, 4]. This structure and the size of the distribution network make these grids responsible for high energy losses in the electricity service chain [5]. For this reason, all utilities focus their efforts on providing local solutions (at the distribution level) to mitigate the total power losses due to the energy distribution in their grids [6]. These solutions include the following:

- Optimal selection of conductors for the distribution networks based on power loss indicators [2, 5].
- Optimal location of capacitor banks for reducing active power losses based on reactive power compensation [7, 8, 9, 10].
- Optimal feeder reconfiguration for power loss reduction based on hourly demand variations [4, 11, 12]
- Optimal location of distributed generators (DGs) for power loss minimization [13, 1, 14].

These strategies allow minimizing the total energy losses of the network by placing devices (capacitors or GDs) as well as modifying the configuration of the grid (installing new conductors or reconfiguring the network through switches). In the specialized literature the efficiency of those strategies for power loss reduction and voltage profile improvement has been proved [15].

Here we are interested in studying, from a heuristic point of view, the problem of the location and sizing of DGs at distribution levels. This problem has been widely studied in the specialized literature on combinatorial approaches such as: genetic algorithms [13], particle swarm optimization [13], population based incremental leaning [1], teaching learning based optimizer [16, 17], tabu search algorithm [18], symbiotic organism search [14], krill herd algorithm [19, 20], bat algorithms [21, 22], bacterial foraging [23], and flower pollination approaches [7], etc. There are some reports of approaches based on branch & bound methods for solving the mixed integer nonlinear programming (MINLP) model that represents this problem, as reported in [24, 25] and [26]; however, these approaches use commercial optimization packages for solving the problem of the optimal placement and sizing of DGs in AC distribution networks, which might be inefficient in large scale distribution networks due to the dimension of the solution space and the processing times that will be required.

In the case of purely heuristic methods, the most common approach places the DGs based on loss sensitivity factors [1, 27] and voltage stability indices [26]; nevertheless, the numerical behavior of these methods remains questionable due to their poor performance when compared with metaheuristic searches. For this reason, they are commonly employed as conditioners for discrete optimization methods (i.e., genetic algorithms or particle swarm optimizers). Due to the lack of powerful heuristic methods for addressing this problem, here we propose an efficient and easily implementable heuristic algorithm for determining the locations and sizing of DGs in distribution networks.

Our proposal employs a master–slave optimization strategy based on a constructive search entrusted to the location stage in conjunction with the vortex search algorithm (VSA) that defines the optimal sizing of the DGs by solving the subsequent optimal power flow problem in the slave stage. Note that this optimization strategy has not been previously proposed in the specialized literature, since the VSA was recently developed [28, 29]; this consitutes a clear opportunity for investigation, to which this paper tries to contribute. In the Results section, the efficiency and accuracy of the proposed approach will be noted, since its results are comparable with those of powerful metaheuristics, with the advantage that our approach only requires a minimal exploration of the solution space to find an adequate solution.

The remainder of this paper is organized as follows: Section 2 presents the mathematical formulation of the optimal location and sizing of DGs in AC radial distribution networks by presenting the corresponding mixed-integer nonlinear programming (MINLP) model. Section 3 presents the proposed heuristic algorithm by solving the location problem with a constructive search algorithm and solving the optimal power flow (OPF) problem in the sizing stage through a VSA. Section 4 presents the complete information of the two test feeders. Section 5 presents all the numerical results of our heuristic approach as well as a comparison with metaheuristic approaches reported in the specialized literature. Section 6 presents the main conclusions derived from this work as well as possible future research.

2 Mathematical formulation

The problem of the optimal location and sizing of DGs in an AC distribution networks can be represented as a nonlinear non-convex optimization model with continuous and discrete variables [25, 26], which leads to a MINLP model [1]. The objective function of the model as well as its set of constraints are presented below.

Objective function:

$$\min z = \sum_{i=1}^{n}\sum_{j=1}^{n} v_i v_j Y_{ij} \cos\left(\theta_{ij} + \delta_{ij}\right), \quad (1)$$

where z is the objective function variable associated to the total active power losses in all the branches of the network; v_i and v_j are the magnitude of the voltage profiles at nodes i and j, respectively; $\theta_{ij} = \theta_i - \theta_j$ represents the angular difference of the voltage profiles between nodes i and j; Y_{ij} and δ_{ij} are the magnitude and angle of the ij^{th} component of the admittance matrix that relates nodes i and j, respectively.

Set of constraints:

$$p_i^s + p_i^{dg} - p_i^d = \sum_{j=1}^{n} V_i V_j Y_{ij} \cos\left(\theta_{ij} + \delta_{ij}\right), \ \forall i \in \mathcal{N}, \quad (2)$$

$$q_i^s - q_i^d = \sum_{j=1}^{n} V_i V_j Y_{ij} \sin\left(\theta_{ij} + \delta_{ij}\right), \ \forall i \in \mathcal{N}, \quad (3)$$

$$v_{\min} \leq v_i \leq v_{\max}, \ \forall i \in \mathcal{N}, \quad (4)$$

$$p_{\min}^{dg} x_i \leq p_i^{dg} \leq p_{\max}^{dg} x_i, \ \forall i \in \mathcal{N}, \quad (5)$$

$$\sum_{i=1}^{n} x_i \leq N_{\max}^{dg}, \quad (6)$$

$$x_i \in \{0, 1\}, \ \forall i \in \mathcal{N}, \quad (7)$$

where p_i^s and p_i^{dg} are the power generation in the slack and the DG located at node i, while p_i^d denotes its active power consumption; q_i^s and q_i^d are the reactive power generation and consumption at node i; $v_{\min}$ and $v_{\max}$ are the minimum and maximum voltage bounds of the voltage profile allowed for the grid; $p_{\max}^{dg}$ and $p_{\min}^{dg}$ are the maximum and minimum power generation bounds on all the DGs considered for being positioned inside the network; $N_{\max}^{dg}$ represents the maximum number of generators available, while x_i is the binary variable associated to the placement or not of a DG at node i (e.g., $x_i = 1$ if the DG is located there and $x_i = 0$ otherwise.)

The interpretation of the mathematical model (1)–(7) is as follows: (1) represents the objective function of the problem, which is associated to the active power losses in all the branches of the network. Eqs. (2) and (3) are the active and reactive power balance equations in all the nodes of the grid; (4) presents the voltage regulation constraint per node; (5) defines the maximum power generation capabilities of the DGs in terms of the active power production per node. Eqs. (6) and (7) define the maximum number of DGs available and the binary nature of the decision variables.

Remark 1 *The mathematical model (1)–(7) can be solved by implementing master–slave optimization approaches in order to decouple the location problem (a discrete problem) from the sizing problem (a continuous problem).*

For this purpose, we propose a master–slave optimization approach for addressing this problem as follows: in the master stage a constructive algorithm based on recursive optimal power flow solutions, and in the slave stage each OPF is solved by implementing a vortex search algorithm. This proposed optimization approach is presented in the following section.

3 Heuristic optimization approach

Here, we present the master–slave approach for solving the problem of the optimal location and sizing of DGs in a radial AC distribution networks by decoupling this problem into two subproblems: the master problem to determine the locations of the generators and the slave problem determines their optimal sizes. The main characteristics of the master and slave problems are studied below.

3.1 Master stage: Constructive algorithm

In this algorithm, the determination of the number of DGs available to be placed in the AC grid is defined by a constructive simple algorithm based on recursive optimal power flow solutions (addressed in detail in the slave stage). For implementing this constructive algorithm, suppose that the AC grid is composed by $n-1$ candidate nodes (the slack node is excluded, since it makes no physical sense to locate a DG at this node); in addition, suppose the the number $N^{dg}_{\max}$ of generators available is greater than or equal to 1. With this in mind, the next steps are:

1. Begin by placing the first DG in node 2 ($i = 1$) and determine its optimal size (minimum power losses) by solving the resulting OPF model; then store the node and its final power losses (z) in a rectangular matrix with the following structure: $R(i,:) = [i+1, z]$. Increase i by $i = i+1$ and repeat this process for all the nodes.

2. Order all the solutions contained in R in ascending order with respect to the second column; then select $R(1,1)$ as the optimal location for the first DG.

3. Fix the location of this generator in the system and return to step 1 to place the second generator, and so on. Note that if a node was previously selected, then, move over $R(k,1)$ ($k = 2, ..., n$) until finding a node which is not contained in the set of selected nodes. The process finishes when the number of nodes selected is equal to $N^{dg}_{\max}$.

Remark 2 *The number of recursive solutions for solving the problem of the optimal location and sizing of DGs in AC distribution networks is equal to* $N^{dg}_{\max} \times (n-1)$*, which only depends on the number of candidate nodes as well as the number of DGs available.*

It is important to mention that the proposed constructive algorithm (heuristic approach) can be used to reduce the solution space of the problem, which allows speeding up the convergence of metaheuristic techniques such as genetic algorithms [13], tabu search algorithms [18], or any discrete optimization approach.

Remark 3 *Note that the master stage must define which components of the binary vector* x *are activated, i.e., this stage defines which* x_i *take the value* 1 *and which 0. This implies that the location problem is solved in this stage.*

3.2 Slave stage: Vortex search algorithm

The vortex search algorithm (VSA) is a physically inspired metaheuristic optimization technique for single-based numerical optimization of nonlinear non-convex functions [28]. This technique was inspired by the vortical behavior of stirred fluids [29]. VSA provides an adequate balance between the the exploration and exploitation of the solution space by modeling its search behavior on a vortex pattern by using an adaptive step size adjustment scheme [30]. In the exploration step, the VSA uses a wide region of the solution space to increase the globality of its search; once the algorithm attains a near sub-optimal solution its search works in an exploitative manner to tune the solution to an optimal value.

Here, the VSA is entrusted with solving each OPF problem defined by the master stage. For this purpose, it concentrates on solving the active and reactive power balance equations, which are the conventional power flow equations of AC grids. The unknown variables in these equations are the voltage profiles v_i and v_i, the angular voltages θ_i and θ_j, the active and reactive power generation in the slack node p^s_i and q^s_i, as well as the active power generation in the DGs p^{dg}_i. Note that if we use a conventional Gauss–Seidel, Newton–Raphson, or any sweep power flow method, then all of the power generation in the DGs must be known in order to solve the power flow problem[1]. Hence, to solve the sizing problem, a metaheuristic approach is needed to determine all the active power outputs in the DGs, and is presented in the following steps.

3.2.1 Initial population

The generation of the initial population in the VSA starts by defining the initial center of a Gaussian distribution. To do this, we define μ_0 as follows:

$$\mu_0 = \frac{p_{\min} + p_{\max}}{2}, \tag{8}$$

where $p_{\min}$ and $p_{\max}$ are vectors that contain the minimum and maximum bounds of power generation of the DGs available for sizing. Note that μ_0 will permit distributing all the solutions around the physical center of the solution space.

The second component of the Gaussian distribution for generating the initial population is the covariance matrix Σ, which is defined here only as the variance matrix, as recommended in [28]. Suppose that the variance matrix takes the following form:

$$\Sigma = diag\left(\sigma^2\right), \tag{9}$$

where σ represents the standard deviation[2], which in the VSA algorithm is related with the adaptive radius (i.e., $r_0 = \sigma_0$ for the first iteration). Its initial value can be determined as follows:

$$\sigma_0 = \frac{\max\left\{p_{\max}\right\} - \min\left\{p_{\min}\right\}}{2}, \tag{10}$$

It is important to mention that σ in the VSA method represents the radius of the hypersphere where the Gaussian distribution will generate all the individuals, which implies that this parameter represents the external bound of the population. Now, to generate the population, a Gaussian distribution is defined.

$$p_0 = \left((2\pi)^d \left|\Sigma\right|\right)^{-\frac{1}{2}} e^{\left\{-\frac{1}{2}(y-\mu)^T\Sigma^{-1}(y-\mu)\right\}} \tag{11}$$

Here, p_0 represents the initial population, y is a vector of random variables, and d is the dimension of the population. Note that this initial population is dependent on Σ (i.e., it indirectly depends on σ_0) and the initial center μ_0, which will be updated after each iteration (t), i.e., these parameters are initialized as σ_t and μ_t for $t = 0$.

To guarantee that the initial population be feasible, each individual inside of p_t ($t = 0$) should be reviewed in order to guarantee that its power generation lies between $p_{\min}$ and $p_{\max}$. Once this review has been made, the initial populationin is contained inside p_t.

3.2.2 Calculation of the new center

When the entire population has been created, then all of the individuals are passed though the conventional power flow analysis in order to determine which of them produces the lowest power losses. Suppose that p_t^k ($k = 1, 2, ..., d$) is the best individual (e.g., it produces the minimal power losses in the population). Then this position is assigned as the new center of the hypersphere, i.e.,

$$\mu_{t+1} = p_t^k, \tag{12}$$

Remark 4 *Note that the VSA moves the center of the hypersphere where all the candidate solutions will be located in order to intensify the exploration about the most promising solution found up to the iteration t.*

3.2.3 Adaptive radius calculation

Once the center of the hypersphere that will contain all the candidate solutions has been determined, the radius of the hypersphere is decreased to exploit the promising region of the solution space, where it is presumed that the optimal solution of the OPF is located. In the specialized literature, this reduction is typically made through incomplete inverse gamma functions [28, 29] even though such a calculation is complex, since it uses factorials. Here, we propose an alternative calculation using exponential functions as follows:

$$r_{t+1} = \sigma_0 \left(1 - \frac{t}{t_{\max}}\right) e^{\left(-a\frac{t}{t_{\max}}\right)}, \tag{13}$$

where a is a constant parameter that governs the speed of reduction of the radius of the hypersphere that represents the solution space. After numerical simulations, we recommend that a be set to 6 for an adequate balance between the exploration and the exploitation of the solution space. Note that $t_{\max}$ is the maximum number of iterations assigned for the VSA to solve the OPF problem.

[1] In the power flow analysis, the slack power balance equations are not included, which implies that determining p_i^s and q_i^s is not necessary, since these are free variables and absorb the power deficits of the network.

[2] The operator $diag\left(\text{arg}\right)$ generates a square matrix with its diagonal equal to the argument (arg).

Finally, the new population (see Eq. (11)) is generated by using the new center μ_{t+1} and the adaptive radius r_{t+1}.

3.3 General comments

The proposed constructive algorithm in the master stage for determining the optimal location of the DGs will always find the same solution, since it makes a recursive evaluation per node to determine the most sensitive set of nodes for placing all the power sources in terms of power loss minimization. This heuristic approach will reach global or local optimum solutions, since it depends depends of the number of generators available and the topology of the network under study. Note that if there is only one available DG, our heuristic approach will always attain the global solution. This is because in that scenario all the solution space of the problem is evaluated for the recursive approach.

On the other hand, the VSA algorithm is a powerful combinatorial optimizer that also guarantees uniqueness of the solution, since its exploration and exploitation approaches, based on adaptive hyperspheres, allows moving its center to promising regions of the solution space, while its decreasing radius permits exploiting that region.

Remark 5 *Note that the proposed master–slave optimization approach has not previously been reported in the specialized literature, and is the main contribution of the present paper. In addition, it will be used as a solution space reduction strategy for powerful metaheuristic optimization approaches in the discrete domain.*

4 Test systems

This section presents the electrical configuration as well as the information about the test system related to the radial distribution systems employed for validating the constructive heuristic optimizer for optimal location and sizing of DGs in AC grids. The first test feeder is a 33-node test system and the second test system is a 69-node test feeder. The complete information of these test systems is presented below.

4.1 The 33-node test feeder

The configuration of this test system is composed of 33 nodes and 32 branches with 12.66 kV of operating voltage. The slack node is located at node 1, and its configuration is presented in Fig. 1. This feeder has 3715 kW and 2300 kVAr of total active and reactive power demand. The initial active power losses of this system are 210.9876 kW. For this test system, the possibility of installing three DGs is considered since this is the most classical reported solution in the specialized literature due to its complexity in comparison with one or two DGs [1]. Each DG is restricted to the range from 0 kW to 2500 kW[3]. In addition, we consider as voltage and power base values 12.66 kV and 1000 kW, respectively.

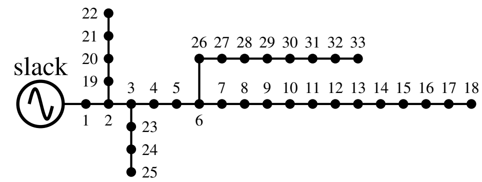

Figure 1: Electrical configuration for the 33-node test system

All the information about the branches as well as the load consumptions of the 33-node test feeder have been listed in Table 1.

Table 1: Parameters of the 33-node test feeder

Node i	**Node** j	R_{ij} [Ω]	X_{ij} [Ω]	P_j [kW]	Q_j [kW]
1	2	0.0922	0.0477	100	60
2	3	0.4930	0.2511	90	40
3	4	0.3660	0.1864	120	80
4	5	0.3811	0.1941	60	30
5	6	0.8190	0.7070	60	20
6	7	0.1872	0.6188	200	100
7	8	1.7114	1.2351	200	100
8	9	1.0300	0.7400	60	20
9	10	1.0400	0.7400	60	20
10	11	0.1966	0.0650	45	30
11	12	0.3744	0.1238	60	35
12	13	1.4680	1.1550	60	35
13	14	0.5416	0.7129	120	80
14	15	0.5910	0.5260	60	10
15	16	0.7463	0.5450	60	20
16	17	1.2890	1.7210	60	20
17	18	0.7320	0.5740	90	40
2	19	0.1640	0.1565	90	40
19	20	1.5042	1.3554	90	40
20	21	0.4095	0.4784	90	40
21	22	0.7089	0.9373	90	40
3	23	0.4512	0.3083	90	50
23	24	0.8980	0.7091	420	200
24	25	0.8960	0.7011	420	200
6	26	0.2030	0.1034	60	25
26	27	0.2842	0.1447	60	25
27	28	1.0590	0.9337	60	20
28	29	0.8042	0.7006	120	70
29	30	0.5075	0.2585	200	600
30	31	0.9744	0.9630	150	70
31	32	0.3105	0.3619	210	100
32	33	0.3410	0.5302	60	40

4.2 The 69-node test feeder

The configuration of this test system consists of 69 nodes and 68 branches with 12.66 kV of operating voltage. The slack node is located at node 1, and its configuration is depicted in Fig. 2. This feeder has 3890.7 kW and 2693.6 kVAr of total active and reactive power demand. The initial active power losses of

[3] These bounds were selected to guarantee equal conditions for the comparison with techniques reported in the literature.

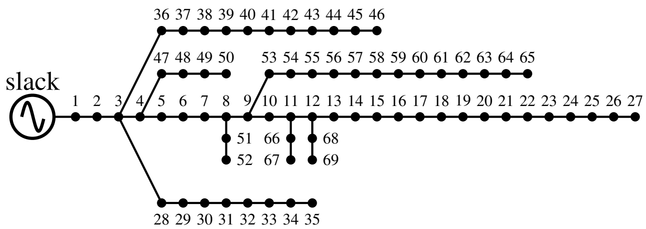

Figure 2: Electrical configuration for the 69-node test system

this system are 225.0718 kW. For this test system, we also consider the possibility of installing three DGs, each of them restricted to the range from 0 kW to 200 kW. In addition, we also consider as voltage and power base values 12.66 kV and 1000 kW, respectively.

All the information about the branches as well as the load consumptions of the 69-node test feeder are presented in Table 2.

Table 2: Parameters of the 69-node test feeder

Node i	**Node** j	R_{ij} [Ω]	X_{ij} [Ω]	P_j [kW]	Q_j [kW]
1	2	0.0005	0.0012	0	0
2	3	0.0005	0.0012	0	0
3	4	0.0015	0.0036	0	0
4	5	0.0251	0.0294	0	0
5	6	0.3660	0.1864	2.6	2.2
6	7	0.3811	0.1941	40.4	30
7	8	0.0922	0.0470	75	54
8	9	0.0493	0.0251	30	22
9	10	0.8190	0.2707	28	19
10	11	0.1872	0.0619	145	104
11	12	0.7114	0.2351	145	104
12	13	1.0300	0.3400	8	5
13	14	1.0440	0.3450	8	5
14	15	1.0580	0.3496	0	0
15	16	0.1966	0.0650	45	30
16	17	0.3744	0.1238	60	35
17	18	0.0047	0.0016	60	35
18	19	0.3276	0.1083	0	0
19	20	0.2106	0.0690	1	0.6
20	21	0.3416	0.1129	114	81
21	22	0.0140	0.0046	5	3.5
22	23	0.1591	0.0526	0	0
23	24	0.3463	0.1145	28	20
24	25	0.7488	0.2475	0	0
25	26	0.3089	0.1021	14	10
26	27	0.1732	0.0572	14	10
3	28	0.0044	0.0108	26	18.6
28	29	0.0640	0.1565	26	18.6
29	30	0.3978	0.1315	0	0
30	31	0.0702	0.0232	0	0
31	32	0.3510	0.1160	0	0
32	33	0.8390	0.2816	10	10
33	34	1.7080	0.5646	14	14
34	35	1.4740	0.4873	4	4
3	36	0.0044	0.0108	26	18.55
36	37	0.0640	0.1565	26	18.55
37	38	0.1053	0.1230	0	0
38	39	0.0304	0.0355	24	17
39	40	0.0018	0.0021	24	17
40	41	0.7283	0.8509	102	1
41	42	0.3100	0.3623	0	0
42	43	0.0410	0.0478	6	4.3
43	44	0.0092	0.0116	0	0
44	45	0.1089	0.1373	39.22	26.3
45	46	0.0009	0.0012	39.22	26.3
4	47	0.0034	0.0084	0	0
47	48	0.0851	0.2083	79	56.4
48	49	0.2898	0.7091	384.7	274.5
49	50	0.0822	0.2011	384.7	274.5
8	51	0.0928	0.0473	40.5	28.3
51	52	0.3319	0.1140	3.6	2.7
9	53	0.1740	0.0886	4.35	3.5
53	54	0.2030	0.1034	26.4	19
54	55	0.2842	0.1447	24	17.2
55	56	0.2813	0.1433	0	0
56	57	1.5900	0.5337	0	0
57	58	0.7837	0.2630	0	0
58	59	0.3042	0.1006	100	72
59	60	0.3861	0.1172	0	0
60	61	0.5075	0.2585	1244	888
61	62	0.0974	0.0496	32	23
62	63	0.1450	0.0738	0	0
63	64	0.7105	0.3619	227	162
64	65	1.0410	0.5302	59	42
11	66	0.2012	0.0611	18	13
66	67	0.0047	0.0014	18	13
12	68	0.7394	0.2444	28	20
68	69	0.0047	0.0016	28	20

5 Computational validation

To solve the general MINLP model that represents the problem of the optimal location and sizing of DGs in radial distribution systems, we implemented the proposed master–slave heuristic optimizer in a desk computer INTEL(R) Core(TM) $i5-3550$, 3.50 GHz, 8 GB RAM with 64-bit Windows 7 Professional.

To demonstrate the robustness and efficiency of the proposed approach for placing and sizing DGs in distribution networks, we compare our results with the solutions previously reported in [14] and [27]. In addition, we assume that all the DGs are operated with a unity power factor, as recommended in [1].

In the results section we call our master–slave optimizer CHVSA, which means the constructive heuristic vortex search algorithm.

5.1 The 33-node test feeder

In Table 3 are presented a list of solutions provided by [14] for the 33-node test feeder with the corresponding locations, sizes, and power losses when three DGs have been considered.

Note that when the CHVSA is applied for the optimal location and sizing of the DGs, it reaches a final power loss of 78.45 kW, which is near the optimal solution reported in [20], which is 75.41 kW when the KHA is implemented, and also closer to the solution reported by [31], 76.91 kW when the REPSO method is applied. This result implies that the proposed CHVSA method is suitable for efficiently designing DGs, with low computational effort and easy implementation. In addition, for being a heuristic approach, it is clear from the results in Table 3 that CHVSA is efficient, as well as better than classical techniques such as loss sensitivity factor particle swarm optimizers [27] or genetic algorithms [33].

An additional important point about CHVSA is that if only one generator is located, then the optimal solution is node 6, and in the case of two DGs, the optimal locations are nodes 6 and 14. Besides, their maximum sizes can be easily determined with the VSA approach studied in Section 3.

Table 3: Location and dispatch of the DGs in the 33-node test feeder

Method	**Power generation** [p.u] (**Node**)			z [kW]
GA [13]	1.5000 (11)	0.4228 (29)	1.0714 (30)	106.30
PSO [13]	1.1768 (8)	0.9816 (13)	0.9297 (32)	105.35
TLBO [17]	0.8847 (9)	0.8953 (18)	1.1958 (31)	104.00
REPSO [31]	1.2274 (6)	0.6068 (14)	0.6870 (31)	76.91
HSA [32]	0.5927 (16)	0.2133 (17)	0.1913 (18)	135.69
SOS [14]	2.2066 (6)	0.2000 (28)	0.7167 (29)	104.19
LSFSA [27]	1.1124 (6)	0.4874 (18)	0.8679 (30)	82.03
KHA [20]	0.8107 (13)	0.8368 (25)	0.8410 (30)	75.41
CHVSA	1.1846 (6)	0.6468 (14)	0.6881 (31)	**78.45**

5.2 The 69-node test feeder

In Table 4 are presented a list of solutions provided by [14] for the 69-node test feeder with the corresponding locations, sizes, and resulting power losses, when three DGs are considered.

From the results in Table 4 it is clear that our proposed CHVSA is the best solution for the optimal location of three DGs in the 69-node test feeder by reaching a final power loss of 69.55 kW followed pretty closely by the best result reported in the specialized literature, that of the KHA approach, which obtains 69.56 kW. This result implies that our heuristic approach is the most efficient method in terms of power losses of all the classical approaches used in the specialized literature, as can be seen in Table 4.

In the case of the optimal location of only one or two DGs, the selected nodes are 61 or 17 and 61. Remember that their sizes are easily calculated by solving an OPF with the VSA reported in Section 3.

5.3 Additional comments

It is important to highlight the following aspects of the proposed CHVSA approach.

- The results provided in Tables 3 and 4 are reproducible by any person if the VSA algorithm is parametrized with ten individuals and 1000 iterations, supposing that the maximum capability of the DGs for both systems are 2000 kW.

- The number of evaluations required by our method is $N_{\max}^{dg} \times (n-1)$, which in the case of the 33-node test feeder is 96 evaluations, and for the 69-test feeder, 207. This is important since the dimension of the solution space for these systems are 4960 and 50116, respectively. This entails that our approach only evaluates 1.94% of the solution space in the case of the 33-node test feeder and 0.41% in the case of the 69-node test feeder.

- The numerical results achieved by the proposed CHVSA method were checked by solving the exact nonlinear non-convex model (1)–(7) in the general algebraic modeling system (GAMS) with the solver BONMIN when the constructive approach reported in Section 3 is followed. This implies that our approach is comparable with commercial optimizing packages and classical metaheuristic approaches as reported in Tables 3 and 4.

Table 4: Location and dispatch of the DGs in the 69-node test feeder

Method	**Power generation** [p.u] (**Node**)			z [kW]
GA [13]	0.9297 (21)	1.0752 (62)	0.9925 (64)	89.00
PSO [13]	0.9925 (17)	1.1998 (61)	0.7956 (63)	83.20
TLBO [17]	0.7574 (25)	1.0188 (60)	1.1784 (63)	81.00
HSA [32]	1.6283 (63)	0.1416 (64)	0.0149 (65)	86.66
SOS [14]	0.2588 (57)	0.2000 (58)	1.5247 (61)	82.08
LSFSA [27]	0.4962 (18)	0.3113 (60)	1.7354 (65)	77.10
KHA [20]	0.4962 (12)	0.3113 (22)	1.7354 (61)	69.56
CHVSA	0.5284 (11)	0.3794 (17)	1.7186 (61)	**69.55**

6 Conclusion

In the present paper, a master–slave heuristic optimization approach for solving the problem of the optimal location and dimensioning of distributed generators in AC radial distribution networks was proposed. In the master stage a recursive constructive algorithm for solving the location problem was presented; this recursive approach allows identifying the most sensitive nodes in terms of power injection and power loss reduction; these nodes are considered as the solution of the location problem. In the slave stage the vortex search algorithm was employed to solve the optimal power flow problem in conjunction with a classical sweep power flow method in order to obtain the optimal sizes of the generators. The combination of the master and slave optimization strategies provided the proposed CHVSA approach with results comparable with those from metaheuristic methods reported in the specialized literature, such as genetic algorithms, particle swarm optimizers, loss sensitivity factors, and krill herd algorithms, among others.

In future work, it would be possible to use the proposed heuristic search for solving the problem of the optimal location of capacitor banks in distribution networks for optimal power loss reduction considering different demand scenarios.

Acknowledgements: This work was supported in part by the Administrative Department of Science, Technology and Innovation of Colombia (COLCIENCIAS) through the National Scholarship Program under Grant 727-2015 and in part by the Universidad Tecnológica de Bolívar under Project C2019P020.

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