A Simplified Study for Reactive Power Management in Autonomous Microgrids

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Abstract: - Reactive power sharing is one of the main issues in the control and operation of an autonomous Microgrids (MGs), especially for MGs composed by power generation interfaced with the MG distribution by means of Power Electronic Converters (PEC). Among the various MG control techniques, the droop is the one that has been widely applied thanks to the possibility of achieving a communication-less regulation of the MG. However some issues arises as far as reactive power and voltage control is concerned. The aim of this study is to draw some analytical consideration of reactive power sharing in droop-controlled autonomous MGs in order to highlight the parameters and configuration that can lead to undesired operational conditions.

Key-Words: Microgrids, Reactive Power, Power Electronic Converters, Autonomous Microgrids, Voltage Control, Power Management.

1 Introduction

In the field of power systems, Microgrids (MGs) are one of the most challenging hot-topic of research due to their flexibility, capability of hosting renewable power generation [1-3] and easing the transition to smart grids [4]. Such an interest is witnessed by the relevant number of MG realizations and test-bed facility developed at Campus [5] and building level [6].

MGs can be operated into two main ways, grid connected or islanded [7]. With the only exception of geographically islanded MGs, grid connected, and islanded operation shall need to be both practicable allowing a seamless transition between the two. From a control architecture point of view the control aims for grid connected and islanded MGs are quite different [8]. In grid connected MGs primary control (frequency and voltage regulation) is usually absent since these quantities are provided by the main distribution grid [9]. In this configuration the MG control is provided at tertiary level, Energy Management System (EMS), providing suitable references for dispatchable production units [10, 11] while each Distributed Generation (DG) unit is provided with a proper component control system providing the DG unit response to the requests of the EMS [12].

For islanded MGs the hierarchical control architecture is more complex since below the EMS a primary and a secondary control levels are present in order to be capable to guaranty the MG power balance and the proper regulation of frequency and voltage [13-15]. As one may expect, this latter case is much more challenging, especially is a reduced or null communication layer wants to be considered between DG units. Focusing on autonomous MGs the most diffuse control technique applied for a proper management of primary control is the droop technique [16, 17]. It provides active power sharing among DG units avoiding any ICT needs. However, the droop control has still some drawbacks such as the necessity of secondary control to nullify frequency and voltage errors and the inability to achieve multiple goals. More in details one of the main issues for droop controlled islanded MGs is the management of reactive power sharing. This is due to the fact that if for the active power channel frequency is a global variable, and thus allows achieving a proper sharing of a load variation among the DG units, the reactive power droop control law is based on voltages that are different in the various point of the MG. This very often causes anomalous working assets of the MG in term of reactive power sharing that in some cases can also cause the negative effect of reactive power circulation.

In this context, this paper proposes an analytical analysis of reactive power sharing for an autonomous AC MG with the goal of highlighting the most impacting grid elements or parameters with respect to this issue considering both topological aspects of the MG distribution system and control parameters settings. The paper structure is the following: Section 2 provides a recall of the classical droop control strategy, pointing out the main differences between the active and the reactive power channels, Section 3 details the test-case MG considered for the analytical analysis provided in Section 4. Some conclusions are given in Section 5.
2 Recall about the classical droop control scheme for islanded MGs

Let us consider a generic MG composed by N power generating units, all interfaced with the MG distribution network by means of a Power Electronic Converter (PEC). The classical droop control scheme will provide for the generic i-th DG unit the following control laws \([13], [18]\):

\[
\begin{align*}
\omega_i - \omega_0 &= m_i (P_i - P_i^0) \\
V_i - V_0 &= n_i (Q_i - Q_i^0)
\end{align*}
\]  

(1) (2)

\(V_i^0\) and \(\omega_0\) being voltage and angular frequency references while \(Q_i^0\) and \(P_i^0\) are active and reactive power set points. \(P_i, Q_i, V_i, \omega_i\) are the controller measurements namely active power, reactive power, voltage and frequency.

Usually, droop coefficients \(m_i\) and \(n_i\) are chosen according to the DG unit rated power, \(A_{iR}\) such that:

\[
m_i A_{ik} = m_j A_{jk} = \ldots = m_k A_{rk} \quad \text{and} \quad n_i A_{ik} = n_j A_{jk} = \ldots = n_k A_{rk}
\]  

(3) (4)

From (3) and (4) one can see that only one droop coefficient for the active power channel and one for the reactive power channel can be chosen independently. It is also well known that imposing (3) the active power and frequency channel guarantee a suitable active power sharing of the load request among all the MGs DG units, i.e.:

\[
P_i = \frac{P^0_i}{\sum_{i=1}^{N} \frac{1}{m_i}}
\]  

(5)

The same result is not so straightforward, in principle even possible, for the load reactive power sharing. This is a consequence of the fact that for the reactive power channel the local DG unit voltage is not a global variable (as it is for the frequency) and this led to the necessity to provide a deeper investigation about reactive power sharing for the droop reactive power channel.

3 Problem Formulation

The problem formulation is led providing a mathematical representation of the MG in order to point out the weakness of reactive power sharing for the tradition droop scheme. Since the aim of the article is to conduct an analytical assessment the MG layout need to be kept simple, as depicted in Fig. 1. As one can notice the layout includes two DG units provided with their dedicated PEC connected to the same bus (PCC) where also the load is connected.

\[
\begin{align*}
V_1 - V^0_1 &= n_1 (Q_1^0 - Q_1) \\
V_2 - V^0_2 &= n_2 (Q_2^0 - Q_2) \\
Q_1 &= \frac{V_1 - V_2}{X_1} \\
Q_2 &= \frac{V_2 - V_1}{X_2} \\
Q_1 + Q_2 &= Q_L
\end{align*}
\]  

(10)

Fig. 1: Test-case MG layout

In general, one can recall the classical power flow equations form to nodes to evaluate the active power form between the i-th DG PEC and the PCC as:

\[
P_i = \frac{V}{Z_i} \left[ V R_i - V_{REC} R \cos(\delta_i - \delta_{REC}) + V_{REC} X_i \sin(\delta_i - \delta_{REC}) \right]
\]  

(6)

While for reactive power flow one can write:

\[
Q = \frac{V}{Z_i} \left[ V X_i - V_{REC} R \sin(\delta_i - \delta_{REC}) + V_{REC} X_i \cos(\delta_i - \delta_{REC}) \right]
\]  

(7)

Equations (6) and (7) highlight that the active and reactive power flows strongly depend on voltages and phase angles. However is one assumes that the PCC voltage is the system phase reference, that the MG impedances are mainly reactive and that all phases are small (6) and (7) turn into:

\[
P_i = \frac{V^0_{PCC} \delta_i}{X_i}
\]  

(8)

\[
Q = \frac{(V_i - V_{PCC}) V_i}{X_i}
\]  

(9)

the simplified system (8)-(9) highlights the notorious relation between voltage and reactive power and between frequency and active power in inductive power systems. This allow us to study the reactive power case avoiding considering the active power channel keeping a sufficient degree of precision. Combining equation (9) and (2) and considering the MG layout of Fig. 1 one can obtain the mathematical model of the reactive power vs. voltage part of the MG as follows:
$Q_i$ being the PCC reactive power and $V_L$ is the voltage at PCC. It is clear that system (10) is an algebraic non-linear system with seven equations. However, system (10) can be further simplified considering only voltages. After a simple algebraic manipulations one can get:

$$\begin{align*}
X_V V_1 V_10 &= n X Q 010 - n V 2 L 1 V 2 L 1 \\
X_V V_2 V_20 &= n X Q 20 - n V 2 2 2 L 2 V 2 L 2 \\
V V L V L 1 V 2 L 1 &= \frac{n V 2 2 1 V 2 L 1}{X 1} - \frac{n V 2 2 2 V 2 L 2}{X 2} \\
V V L V 2 L 2 &= \frac{n V 2 2 1 V 2 L 1}{X 1} - \frac{n V 2 2 2 V 2 L 2}{X 2} = Q L
\end{align*}$$

System (11) is still non-linear but it has been reduced to a three equation three unknown quantities being $V_j$, $V_2$ and $V_L$. Another simplification of system (11) consist into a linearization of the system, which implies choosing a working point to centre the linearization process. Calling $V^*_j$, $V^*_2$, $V^*_L$ the voltages of the chosen working point and $Q^*_j$ its load reactive power one can re-write system (11) as follows:

$$\begin{bmatrix}
A & 0 & -1 \\
0 & B & -1 \\
1 & C & -D
\end{bmatrix}
\begin{bmatrix}
\Delta V_j \\
\Delta V_2 \\
\Delta V_L
\end{bmatrix}
= \begin{bmatrix}
0 \\
0 \\
E
\end{bmatrix}$$

being:

$$A = \frac{X_V V 10}{n V 2 2 1} + \frac{X Q 010}{V 2 2 1} + 1 - \frac{X 1}{X 2}$$

$$B = \frac{X_V V 20}{n V 2 2 2} + \frac{X Q 20}{V 2 2 2} + 1$$

$$C = -\frac{X 1}{X 2}$$

$$D = \frac{X 1}{X 2} - \frac{Q L X 1}{V 2 2 1} + 1$$

$$E = \frac{X 1}{V_L} \Delta Q_L$$

### 3 Problem Solution

The solution proposed in the present paper is done considering the numerical solution of the MG layout provided in Fig. 1. For this reason, the MG numerical parameter need to be defined. They are provided in Table 1.

**Table 1: MG and droop controllers relevant data**

<table>
<thead>
<tr>
<th>$A_{1R}$</th>
<th>100 kVA</th>
<th>$n_2$</th>
<th>$6 \times 10^{-4}$ V/Var</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{2R}$</td>
<td>50 kVA</td>
<td>$V_{10}$</td>
<td>400 V</td>
</tr>
</tbody>
</table>

#### 3.1 Evaluation of modelling approximations

In order to assess to error introduced by the considered linearization a sensitivity analysis has been lead calculating the results of systems (11) and (12) for many values of the reactive power load from 0 kVAr to 150 kVAr. The linearization error for the inverter and DG voltages is assessed by comparing of the voltages as depicted in Fig. 2. As one can see the impact of such approximation is very good and can be considered acceptable for the present study.

**Fig. 2: Voltage comparison between linear and non-linear models.**

#### 3.2 MG parameters impact on the reactive power sharing

As shown by (10) the reactances connecting each DGs to the PCC have a direct influence on the MG reactive power sharing. One possibility to quantify this impact is solving the linearized system of (12) considering one of the two reactances as a variable parament. Fig. 3 and Fig. 4 show the reactive power production and the voltages respectively as a function of reactive power demand and the parameter $X_2$ (increasing value).

As a result, one can verify that a bigger connection reactance causes a reduced participation in reactive power sharing. More in details, it is possible defining the reactive power sharing error $Q_{err\%}$ as:

$$Q_{err\%} = \frac{Q - Q_{\text{desired}}}{Q_{\text{desired}}} \times 100$$

![Graph showing comparison between linear and non-linear models.](image-url)
Fig. 3: Reactive powers as a function of $X_1$ and $X_2$.

Fig. 4: Voltages as a function of $X_1$ and $X_2$. Where $Q_{\text{desired}}$ is the desired reactive power production of $i$-th DG unit, defined as:

$$Q_{\text{desired}} = \sum_{i=1}^{N} A_{R} Q_{L}$$

(19)

Considering (18) one can plot the variation of the reactive power sharing error as a function of the load and of the two connection impedances as reported in Fig. 5.

Fig. 5: Reactive power sharing errors with variable connection impedance.

In order to test the performances in reactive power sharing in a more accurate approach the test case MG has been implemented in Simscape considering detailed model of the MG including inductances and capacitors dynamics, inner control loops of PEC and, of course, the coupling between active and reactive power in the power system.

The simulation performed providing no lead active power variations and two reactive power load variations: (i) from 0 kVAr to 50 kVAr and (ii) from 50 kVAr to 100 kVAr. Examining Fig. 6 and Fig. 7 one can verify that the simulations results confirm the solution of the linearized system (12) which properly describes the reactive power sharing problem among PECs.

Fig. 6: Reactive power time profiles.

Fig. 7: Comparison between simplified and detailed simulation for reactive power sharing.

3.3 Control parameter effect on the reactive power sharing

Another important aspect to be considered is the effect of the droop control parameters on the reactive power sharing problem. In particular examining equation (2) one can notice that parameter $Q_{\ell 0}$ is something changed in accordance to secondary regulation or by the EMS [19], and thus it should be considered as a possible interference in the reactive power sharing problem. In particular, always making reference to the simplified MG layout of Fig. 1 it is
possible to provide an analytical quantification of this effect. Linearizing system (11) around a zero reactive power load working point one get that $V_1^*, V_2^*$ and $V_L^*$ are all equal. By means of simple mathematical manipulations one can get:

$$\begin{align*}
\left[\begin{array}{c}
1 + \frac{n_1}{X_1} \\
1 + \frac{n_2}{X_2}
\end{array}\right]^{-1} \left[\begin{array}{c}
1 + \frac{n_3}{X_2} \\
1 + \frac{n_4}{X_1}
\end{array}\right] &= \left[\begin{array}{c}
\frac{V_1 - V_{20} + n_1Q_{10} - n_2Q_{20}}{Q_L}
\end{array}\right]
\end{align*}$$

(20)

where $a_1 = V_1 - V_L$ and $a_2 = V_2 - V_L$. The solution of (20) can be calculated as:

$$\begin{align*}
a_1 &= \frac{X_2}{X_1 + X_2 + n_1 + n_2} \left[\frac{V_1 - V_{20} + n_1Q_{10} - n_2Q_{20} + X_2 + n_2}{X_1} - \frac{V_1 - V_{20} + n_1Q_{10} - n_2Q_{20} + X_1 + n_1}{X_2}\right]
\end{align*}$$

(21)

This means that the sign of $a_1$ and $a_2$ are directly related to $Q_{i0}$. Since the sign of $a_1$ and $a_2$ is also the sign of the corresponding DG unit reactive power this implies that the choice of $Q_{i0}$ has an effect on the possible issue of reactive power circulation. For a given value of reactive power demand $Q_L$ one can have two possible scenarios:

1. $n_1Q_{10} + V_{10} = n_2Q_{20} + V_{20}$ which implies that $\text{sing}(a_1) = \text{sing}(a_2)$.
2. $n_1Q_{10} + V_{10} \neq n_2Q_{20} + V_{20}$ which implies that $\text{sing}(a_1) \neq \text{sing}(a_2)$.

This second case implies that one DG injects reactive power while the other DG absorbs reactive power, creating a circulating current in the MG. This issue is highlighted by means of a simulation where $Q_{10}$ is changed to 25 kVAr. As one can see from Fig. 8 for a load lower than 10 kVAr there is reactive power circulation.

![Fig. 8: Reactive power time profile: $Q_{10} = 25$ kVAr.](image)

For a reactive power load value lower than 50 kVAr the smaller DG ($DG_2$) produces more reactive power than the bigger one ($DG_1$) causing a higher reactive power sharing error. For sake of completeness the comparison between simplified problem solution and detailed Simscape simulation is shown in Fig. 10.

![Fig. 9: Reactive power sharing errors.](image)

![Fig. 10: Comparison between simplified solution and Simscape simulation: $Q_{10} = 25$ kVAr.](image)

4 Conclusion

This paper proposed an analytical evaluation of the problem of reactive power sharing in islanded MGs composed by DG units interfaced with the MG distribution network by means of PECs. The proposed model allowed to draw some analytical evaluations considering variations of the MG parameters and of the droop controllers parameters. The proposed approach, for its simplicity can be computed by a simple programing implementation. However, the reliability of achieved results have been confirmed by means of detailed simulations done in Simscape environment. As a final result it was possible pointing out that reactive power sharing issues in droop-controlled MGs are mainly related to
interconnecting impedances and by reactive power set points of the droop control law. These parameters need to be taken into deep consideration in the design of the MG control in order to avoid undesired reactive power circulation.

References: