Mutual Inductance and Magnetic Force Calculations Between Thick Bitter Circular Coil of Rectangular Cross Section with Inverse Radial Current and Thin Wall Superconducting Solenoid with Constant Azimuthal Current

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Abstract— In this paper we calculate the mutual inductance and the magnetic force between the thick Bitter coil of rectangular cross section with the inverse radial current and the thin wall superconducting solenoid with the constant azimuthal current. The semi-analytical and the analytical expressions of these magnetic quantities are obtained over complete elliptic integrals of the first and second kind as well as Heuman's Lambda function. There is a simple integral which has to be solved numerically by some of numerical integrations. The results of this method are compared by those obtained by the modified filament method for the presented configuration. All results are in an excellent agreement.

Keywords— magnetic field, mutual inductance, magnetic force, Bitter coils, inverse radial current, azimuthal current.

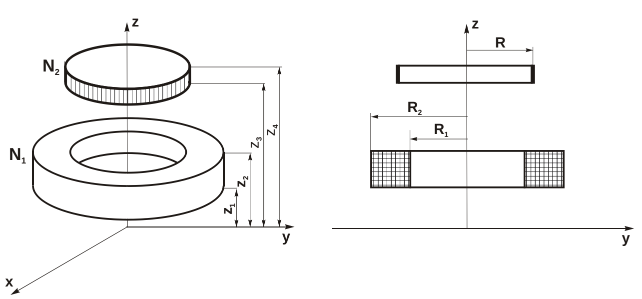
1 Introduction

High magnetic fields can be generated by a water-cooled magnet with Bitter coils or by hybrid magnet combined with an inner water-cooled magnet and an outset superconducting magnet. To have a strong and uniform magnetic field in the thin wall solenoid a high electric current is required but because of Joule effect and the resistance the coil will melt. This problem can be avoid where we can either cool the wire with water or use a superconducting wire. This is way the superconducting wall coil with the high uniform azimuthal current produces the strong uniform field. Otherwise, the high current density distribution of a Bitter coil in radial direction, however, is inversely proportional to the radius of Bitter coil, [1-5].

During a water-cooled magnet trip, the induced current in the superconducting coil changes as a function of decay time constant of the water-cooled magnet. The decay time constant is determined by the self-inductance and the resistance of the water-cooled magnet. Thus, an accurate estimation of the inductance of Bitter coils is of primary importance in the hybrid magnet design. In addition, in such coils strong electromagnetic forces develop in an axial direction, causing mechanical stresses upon the supporting structures. For the proper design of such structures, the electromagnetic forces between coils must be calculated. Therefore, measurement of the mutual inductance and the magnetic force are of extremely important for the design viewpoint, [6-15]. To optimize the support structure of the Bitter coil, it is vital to have accurate evaluation of the magnetic force between Bitter coils. Y.Ren at al., [16-17] and J.T.Conway, [18] calculated the mutual inductance and the magnetic force for the following two coil arrangements, one between an ordinary coil and a Bitter coil, both with the rectangular cross sections, and the other between two Bitter coils with rectangular cross sections. These studies aim to help engineers and physicists calculate the mutual inductance and the magnetic force for when the thick Bitter coil of rectangular cross section structures are used. They obtained these magnetic quantities expressed by two integrations [16-17] and by the integration of Bessel functions [18]. Recently, S. Babic and C. Akyel calculated the mutual inductance and the magnetic force calculation between some Bitter coils [19-22] where they obtained these quantities in the analytical and the semi-analytical form. In this paper we calculate the mutual inductance and the magnetic force between the thick Bitter coil of rectangular cross section with the inverse radial current and the thin superconducting wall solenoid with the constant azimuthal current. In the approach presented in this paper, semi-analytical and analytical forms are expressed as functions of complete elliptic integrals of the first and second kind, the Heuman's Lambda function and a simple integral with continuous kernel function and very convenient for numerical integration. The results of this calculations will be compared to those obtained by the improved filament method for concerned configuration. The results obtained by these two methods are in an excellent agreement.

2 Nomenclature

I_1 (A): Current imposed in the thick coil of rectangular cross section
I_2(A): Current imposed in the wall solenoid
N_1: Number of turns of the thick coil of rectangular cross section
N_2: Number of turns of the wall solenoid
R_1(m) and R_2(m): Inner and outer radius of the thick coil of rectangular cross section
R(m): Radius of the thin wall solenoid
z_1(m): Axial position to the bottom of the thick coil of rectangular cross section
z_2(m): Axial position to the top of the thick coil of rectangular cross section
z_3(m): Axial position to the bottom of the thin wall solenoid
z_4(m): Axial position to the top of the thin wall solenoid
M_B(H): Mutual inductance
F_B(N): Magnetic force between coils
J_1 and J_2: Current densities at the thick coil of rectangular cross section and the thin wall solenoid respectively
(r_{I}, z_I) and (r_{II}, z_{II}): Arbitrary radial and axial positions along the thick coil of rectangular cross section and the



thin wall solenoid respectively.

3 Basic Expressions

The thick Bitter circular coil of rectangular cross section with the inverse radial current and the thin superconducting wall solenoid with the constant azimuthal current are characterized by the following current densities [16-22] respectively:

Fig.1. Bitter thick coil of rectangular cross section and thin wall solenoid

$$J_1 = \frac{N_1 I_1}{\ln \frac{R_2}{R_1}} \frac{1}{r_I} \tag{1}$$

and

$$J_2 = \frac{N_2 I_2}{(z_4 - z_3)} \tag{2}$$

The mutual inductance and the magnetic force between coils in question (See Fig. 1) can be calculated from the following expressions which can be easily obtained from general expressions for these two important magnetic quantities [16-22]:

$$M_B = \frac{\mu_0 N_1 N_2 R \int\limits_0^{\pi} \int\limits_{R_1}^{R_2} \int\limits_{z_3}^{z_4} \int\limits_{z_1}^{z_2} \frac{\cos\theta \, \mathrm{d}r_I \mathrm{d}z_I \mathrm{d}z_{II} \mathrm{d}\theta}{r}}{(z_4 - z_3)(z_2 - z_1) \log \frac{R_2}{R_1}} \tag{3}$$

$$F_B = \frac{-\mu_0 N_1 N_2 I_1 I_2 R \int\limits_0^{\pi} \int\limits_{R_1}^{R_2} \int\limits_{R_3}^{R_4} \int\limits_{z_1}^{z_2} \frac{(z_{II} - z_I)\cos\theta \, \mathrm{d}r_I \mathrm{d}z_I \mathrm{d}z_{II} \mathrm{d}\theta}{r^3}}{(z_4 - z_3)(z_2 - z_1) \log \frac{R_2}{R_1}} \tag{4}$$

where

$$r = \sqrt{(z_{II} - z_I)^2 + r_I^2 + R^2 - 2 r_I R \cos\theta}$$

4 Calculation Method

Integrating in (3) and (4) over r_I , z_I, z_{II} and θ (substituting $\theta = \pi - \beta$) the mutual inductance and the magnetic force between previously mentioned coil combinations can be expressed respectively in an semi-analytical form as follow:

$$M_B = \frac{\mu_0 N_1 N_2}{(z_4 - z_3)(z_2 - z_1)\ln\frac{R_2}{R_1}} \sum_{n=1}^{n=8} (-1)^n T_n \tag{5}$$

$$F_B = \frac{\mu_0 N_1 N_2 I_1 I_2}{(z_4 - z_3)(z_2 - z_1)\ln\frac{R_2}{R_1}} \sum_{n=1}^{n=8} (-1)^{n-1} S_n \tag{6}$$

where

$$\rho_1 = \rho_4 = \rho_5 = \rho_8 = R_2$$
$$\rho_2 = \rho_3 = \rho_6 = \rho_7 = R_1$$
$$t_1 = t_2 = z_4 - z_1;\ t_3 = t_4 = z_4 - z_2;$$
$$t_5 = t_6 = z_3 - z_1;\ t_7 = t_8 = z_3 - z_2$$

$$T_n = \frac{\pi}{8}|t_n|\,\mathrm{sign}(\rho_n - R)(\rho_n^2 - 3R^2)[1 - \Lambda_0(\varepsilon_n, k_n)] - \frac{\pi}{12}|t_n|(t_n^2 - 3R^2)V_n + \frac{k_n t_n^2}{8\sqrt{R\rho_n}}\frac{(\rho_n - R)(\rho_n^2 - 3R^2)}{\rho_n + R}K(k_n) - \frac{k_n(t_n^2 - 3R^2)}{6\sqrt{R\rho_n}}[R^2 + t_n^2 - R\sqrt{R^2 + t_n^2}]K(k_n) + \frac{1}{18k_n^5\sqrt{R\rho_n}}[k_n^4(-58R^3\rho_n - 8R^4 - 8R^2t_n^2 - 36R^2\rho_n^2 - 3R\rho_n t_n^2 - 18R\rho_n^3) + k_n^2(64R^3\rho_n + 16R^4 + 16R^2t_n^2 + 72R^2\rho_n^2) - 64R^3\rho_n]E(k_n) + \frac{1}{72k_n^5\sqrt{R\rho_n}}[k_n^6(12t_n^4 - 36R^4 - 9\rho_n^2 t^2 - 6R\rho_n t_n^2 - 51R^2t_n^2 - 108R^2\rho_n^2 - 108R^3\rho_n - 36R\rho_n^3) + k_n^4(344R^3\rho_n + 64R^4 + 64R^2t_n^2 + 12R\rho_n t_n^2 + 288R^2\rho_n^2 + 72R\rho_n^3) + k_n^2(-384R^3\rho_n - 64R^4 - 64R^2t_n^2 - 288R^2\rho_n^2) + 256R^3\rho_n]K(k_n) + R^2 t_n I_{0n}$$

$$S_n = \frac{\pi}{8}\mathrm{sign}(t_n)\mathrm{sign}(\rho_n - R)(\rho_n^2 - 3R^2)[1 - \Lambda_0(\varepsilon_n, k_n)] - \frac{\pi}{4}\mathrm{sign}(t_n)(t_n^2 - R^2)V_n + \frac{3k_n t_n}{8\sqrt{R\rho_n}}[(R + \rho_n)^2 + t_n^2]E(k_n) + \frac{k_n t_n}{8\sqrt{R\rho_n}}[t_n^2 - 2R^2 - 4\rho_n^2 + \frac{(\rho_n - R)(\rho_n^2 - 3R^2)}{\rho_n + R} - \frac{4(t_n^2 - R^2)\sqrt{R^2 + t_n^2}}{\sqrt{R^2 + t_n^2} + R}]K(k_n) + R^2 I_{0n}$$

$$k_n^2 = \frac{4R\rho_n}{(R + \rho_n)^2 + t_n^2}\ ,\ h_n = \frac{4R\rho_n}{(R + \rho_n)^2}\ ,$$

$$m_n = \frac{2R}{\sqrt{R + t_n^2} + R} \le 1$$

$$\theta_{1n} = \arcsin\frac{|t_n|}{\sqrt{R^2 + t_n^2} + R}\ ,\qquad \theta_{2n} = \arcsin\sqrt{\frac{1 - m_n}{1 - k_n^2}},\ k_n^2 \le m_n,$$

$$\varepsilon_n = \arcsin\sqrt{\frac{1 - h_n}{1 - k_n^2}},\ k_n^2 \le h_n$$

$$V_n = 1 - \Lambda_0(\theta_{1n}, k_n) + \mathrm{sgn}(\sqrt{R^2 + t_n^2} - \rho_n)[1 - \Lambda_0(\theta_{2n}, k_n)]$$

$$J_{0n} = \int_0^{\pi/2} \sinh^{-1}\frac{t_n}{\sqrt{R^2 + \rho_n^2 + 2R\rho_n\cos 2\beta}}\,\mathrm{d}\beta$$

Singular cases

For $t_n = 0$ and $k_n^2 \neq 1$

$$T_{n0} = \frac{1}{18k_{n0}^5\sqrt{R\rho_n}}[k_{n0}^4(-58R^3\rho_n - 8R^4 - 36R^2\rho_n^2 - 18R\rho_n^3) + k_{n0}^2(64R^3\rho_n + 16R^4 + 72R^2\rho_n^2) - 64R^3\rho_n]E(k_{n0}) + \frac{1}{72k_{n0}^5\sqrt{R\rho_n}}[k_{n0}^6(36R^4 - 108R^2\rho_n^2 - 108R^3\rho_n - 36R\rho_n^3) + k_{n0}^4(344R^3\rho_n + 64R^4 + 288R^2\rho_n^2 + 72R\rho_n^3) + k_{n0}^2(-384R^3\rho_n - 64R^4 - 288R^2\rho_n^2) + 256R^3\rho_n]K(k_{n0}) \quad (7)$$

$$k_{n0}^2 = \frac{4R\rho_n}{(R+\rho_n)^2} = h_n$$

$$S_{n0} = 0 \quad (8)$$

For $t_n = 0$ and $k_n^2 = 1$

$$T_{n01} = -\frac{16}{9}R^3 \text{ or } -\frac{16}{9}\rho_n^3 \quad (9)$$

$$S_{n01} = 0 \quad (10)$$

$E(k)$ and $K(k)$ are complete integrals of the first and second kind and $\Lambda_0(\varepsilon,k)$ is Heuman's lambda function, [23-24].

5 Modified Filament Method

In [16-17] the mutual inductance is calculated between the Bitter coil of rectangular cross section and the superconducting coil with uniform current density by using the filament method. Here we give the modified formulas for the mutual inductance and the magnetic force for the thick Bitter coil of rectangular cross section with inverse radial current and the thin wall solenoid with the constant azimuthal current (See Fig. 2) using the filament method. Applying some modification in the mutual inductance calculation given in [16-17] we deduced the mutual inductance and the magnetic force between presented coils as follows:

$$M_{BF} = \frac{N_1N_2(R_4-R_3)\sum_{g=-K}^{g=K}\sum_{p=-m}^{p=m}\sum_{l=-n}^{l=n}\frac{M_F(g,p,l)}{r_{II}(l)}}{(2K+1)(2m+1)(2n+1)\ln\frac{R_4}{R_3}} \quad (11)$$

$$F_{BF} = \frac{N_1N_2I_1I_2(R_4-R_3)\sum_{g=-K}^{g=K}\sum_{p=-m}^{p=m}\sum_{l=-n}^{l=n}\frac{F_F(g,p,l)}{r_{II}(l)}}{(2K+1)(2m+1)(2n+1)\ln\frac{R_4}{R_3}} \quad (12)$$

where

$$M_F(g,p,l) = \frac{\mu_0\sqrt{Rr_{II}(l)}}{k(g,p,l)}[(2-k^2(g,p,l))K(k(g,p,l)) - 2E(k(g,p,l))]$$

$$F_F(g,p,l) = \frac{\mu_0 I_1 I_2 z(p)k}{4\sqrt{Rr_{II}(l)}}[\frac{2-k^2(g,p,l)}{1-k^2(g,p,l)}E(k(g,p,l)) - 2K(k(g,p,l))]$$

$$r_{II}(l) = R_{II} + \frac{h_{II}}{(2n+1)}l, \quad l = -n,\ldots,0,\ldots,n$$

$$R_{II} = \frac{R_3+R_4}{2}, \quad h_{II} = R_4 - R_3$$

$$z(g,p) = c - \frac{a}{(2K+1)}g + \frac{b}{(2m+1)}p$$

$$g = -K,\ldots,0,\ldots,K; \quad p = -m,\ldots,0,\ldots,m$$

$$k^2(g,p,l) = \frac{4Rr_{II}(l)}{(R+r_{II}(l))^2 + z^2(g,p)}$$

Expressions (11) and (12) will be used to confirm the validity of formulas (5) and (6).

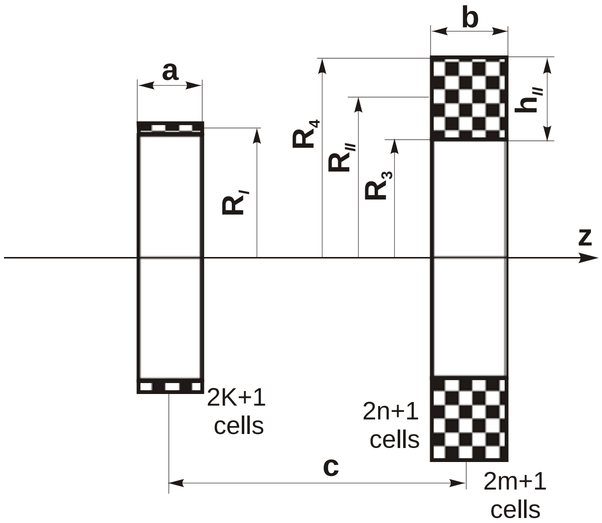


Fig. 2. Configuration of mesh matrix: Thin wall solenoid -Thick Bitter coil with rectangular cross section

6 Numerical Validation

6.1. Example 1.

To prove the new approach we give many examples which cover either the regular of the singular cases. In these examples all coils are with the unit currents. Also, the coil dimensions and the number of turns are given. For the comparative method (improved filament method) the number of subdivisions for each coil is given also. Obviously, the precision of the filament method depends on the number of subdivisions of coils. Increasing the number of the subdivisions of coils will increase the computational time. Our goal is to verify the results of the presented semi-analytical method so that we will fix the number of subdivisions of coils ($N = n = m = 50$) in the following examples without taking into consideration the computational time in the calculations.

First coil: $R_1 = 0.3$ m, $R_2 = 0.4$ m, $z_1 = -0.4$ m, $z_2 = 0.4$ m, $N_1 = 100$.
Second coil: $R = 0.5$ m, $z_3 = -0.2$ m, $z_4 = 0.2$ m, $N_2 = 100$.

Applying the presented method (5) and (6) we obtain:

$$M_B = \mathbf{3.9026}415 \text{ mH}$$

$$F_B = \mathbf{0} \text{ N}$$

By using the modified filament method (11) and (12) we obtain:

$$M_{BF} = \mathbf{3.9026}558 \text{ mH}$$

$$F_{BF} = \mathbf{0} \text{ N}$$

6.2. Example 2.

First coil: $R_1 = 0.3$ m, $R_2 = 0.4$ m, $z_1 = -0.2$ m, $z_2 = 0.2$ m, $N_1 = 100$.
Second coil: $R = 0.5$ m, $z_3 = 0.4$ m, $z_4 = 0.8$ m, $N_2 = 100$.

Applying the presented method (5) and (6) we obtain:

$$M_B = \mathbf{1.2320}891 \text{mH}$$

$$F_B = \mathbf{-3.714}5291 \text{ mN}$$

By using the modified filament method (11) and (12) we obtain:

$$M_{BF} = \mathbf{1.2320}724 \text{ mH}$$

$$F_{BF} = \mathbf{-3.714}46853 \text{ mN}$$

6.3. Example 3.

First coil: $R_1 = 0.3$ m, $R_2 = 0.4$ m, $z_1 = -0.2$ m, $z_2 = 0.2$ m, $N_1 = 100$.
Second coil: $R = 0.3$ m, $z_3 = 0.4$ m, $z_4 = 0.8$ m, $N_2 = 100$.

Applying the presented method (5) and (6) we obtain:

$$M_B = \mathbf{653.8}1529 \mu\text{H}$$

$$F_B = \mathbf{-2.473}9581 \text{ mN}$$

By using the modified filament method (11) and (12) we obtain:

$$M_{BF} = \mathbf{653.8}0025 \mu\text{H}$$

$$F_{BF} = \mathbf{-\ 2.473}8708 \text{ mN}$$

6.4. Example 4.

First coil: $R_1 = 0.3$ m, $R_2 = 0.4$ m, $z_1 = -0.2$ m, $z_2 = 0.2$ m, $N_1 = 100$.
Second coil: $R = 0.3$ m, $z_3 = 0.2$ m, $z_4 = 0.8$ m, $N_2 = 100$.

This is the singular case.
Applying the presented method (5), (6), (7) (8), (9) and (10) we obtain:

$$M_B = \mathbf{1.1581}969 \text{mH}$$

$$F_B = \mathbf{-\ 5.34}90210 \text{ mN}$$

By using the modified filament method (11) and (12) we obtain:

$$M_{BF} = \mathbf{1.1581}287 \text{ mH}$$

$$F_{BF} = -\mathbf{5.34}84753 \text{ mN}$$

6.5. Example 5.

First coil: $R_1 = 0.3$ m, $R_2 = 0.4$ m, $z_1 = -0.2$ m, $z_2 = 0.2$ m, $N_1=100$.
Second coil: $R = 0.3$ m, $z_3 = 0$ m, $z_4 = 0.8$ m, $N_2=100$.

This is the singular case.
Applying the presented method (5), (6), (7) (8), (9) and (10) we obtain:

$$M_B = \mathbf{2.042}3685 \text{ mH}$$

$$F_B = -\mathbf{6.14}91443 \text{ mN}$$

By using the modified filament method (11) and (12) we obtain:

$$M_{BF} = \mathbf{2.042}447774531412 \text{ mH}$$

$$F_{BF} = -\mathbf{6.14}8900903658425 \text{ mN}$$

6.6. Example 7.

First coil: $R_1 = 1$ m, $R_2 = 3$ m, $z_1 = 1$ m, $z_2 = 2$ m, $N_1=100$.
Second coil: $R = 3$ m, $z_3 = 2$ m, $z_4 = 3$ m, $N_2=100$.

This is the singular case.
Applying the presented method (5), (6), (7) (8), (9) and (10) we obtain:

$$M_B = \mathbf{21.49}2989 \text{ mH}$$

$$F_B = -\mathbf{11.951}579 \text{ mN}$$

By using the modified filament method (11) and (12) we obtain:

$$M_{BF} = \mathbf{21.49}3041 \text{ mH}$$

$$F_{BF} = -\mathbf{11.951}458 \text{ mN}$$

6.7. Example 8.
First coil: $R_1 = 1$ m, $R_2 = 3$ m, $z_1 = 1$ m, $z_2 = 2$ m, $N_1=100$.
Second coil: $R = 1$ m, $z_3 = 0$ m, $z_4 = 1$ m, $N_2=100$.

This is the singular case.
Applying the presented method (5), (6), (7) (8), (9) and (10) we obtain:

$$M_B = \mathbf{6.9589}585 \text{ mH}$$

$$F_B = \mathbf{6.37}30223 \text{ mN}$$

By using the modified filament method (10) and (11) we obtain:

$$M_{BF} = \mathbf{6.9589}276 \text{ mH}$$

$$F_{BF} = \mathbf{6.37}27538 \text{ mN}$$

6.8. Example 9.

First coil: $R_1 = 2$ m, $R_2 = 4$ m, $z_1 = 0$ m, $z_2 = 2$ m, $N_1=100$.
Second coil: $R = 4$ m, $z_3 = 1$ m, $z_4 = 3$ m, $N_2=100$.

This is the singular case.
Applying the presented method (5), (6), (7) (8), (9) and (10) we obtain:

$$M_B = \mathbf{43.886}289 \text{ mH}$$

$$F_B = -\mathbf{13.01}2766 \text{mN}$$

By using the modified filament method (10) and (11) we obtain:

$$M_{BF} = \mathbf{43.886}310 \text{ mH}$$

$$F_{BF} = -\mathbf{13.01}3218 \text{ mN}$$

As we can see all results obtained by two different approaches are in an excellent agreement. We give more examples which represent the singular cases so that all cases either regular or singular were covered. We kept the eight significant figures and the bold digits illustrate the significant digits with the same accuracy in both calculation.

7 Conclusion

The new accurate mutual inductance and magnetic force formulas for the system of the thick Bitter coaxial coil of rectangular cross section with the inverse radial current and the thin superconducting wall solenoid with the constant azimuthal current in air are derived and presented in this paper. All expressions either for the mutual inductance or the magnetic force is obtained in the semi-analytical form expressed over complete elliptic integrals of the first and second kind and the Heuman's Lambda function. There is a simple integral which has not the analytical solution, and it is solved numerically by the Gaussian numerical integration. All singular case are obtained in the close form. Also we gave in this paper the improved formulas for the mutual inductance and the magnetic force between treated coils by using the modified filament method. All results obtained by two presented methods are in an excellent agreement.

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REFERENCES

[1] F. Bitter, "The Design of Powerful Electromagnets Part II. The Magnetizing Coil, " Rev. Sci. Instrum., 1936, 7, (12), pp.482-489.

[2] Y. Sakai, K. Inoue, H. Maeda, "High-strength and high-conductivity Cu-Ag alloy sheets: new promising conductor for high-field Bitter coils, " *IEEE Trans. Mag.*,1994, 30, (4), pp. 2114-2117.

[3] Y. Nakagawa, K. Noto, A. Hoshi, K. Watanabe, S. Miura, G. Kido, Y. Muto, "High field laboratory for superconducting materials, Institute for Materials Research, Tohoku University," Physica B: Condensed Matter, Vol. 155, No. 1–3, pp. 69-73, 1989.

[4] Y. Sakai, K. Inoue,H. Maeda, "High-strength and high-conductivity Cu-Ag alloy sheets: new promising conductor for high-field Bitter coils," IEEE Trans. Mag., Vol. 30, No.4, pp. 2114-2117, 1994.

[5] S. Babic, C. Akyel, V. Y. Ren and W. Chen, "Magnetic force calculation between circular coils of rectangular cross section with parallel axes for superconducting magnet" Progress in Electromagnetics Research B, Vol 37, 275-288, 2012.

[6] S. Babic, C. Akyel, S. J. Salon, S. Kincic, "New expressions for calculating the magnetic field created by radial current in massive disks, " *IEEE Trans. Mag.*, Vol. 38, No. 2, pp. 497-500, Mars, 2002.

[7] B. Azzerboni, G.A. Saraceno and E. Cardelli, "Three-dimensional calculation of the magnetic field created by current-carrying massive disks, " IEEE Trans. on Mag., Vol. 34, No 5, pp. 2601 – 2604, Sept. 1998.

[8] Babic, S., Milojkovic, S., Andjelic, Z., Krstajic, B, Salon, J.S., "Analytical calculation of the 3D magnetostatic field of a toroidal conductor with rectangular cross section," IEEE Trans. Mag., 24, (2), 1988, pp 3162-3164.

[9] C. Akyel, S. I. Babic, S. Kincic and J. P. Lagacé, " Magnetic Force Calculation of Some Circular Coaxial Coils in Air, " *Journal of Electromagnetic Waves and Applications*, Vol. 21, No.9, 1273-1283, 2007.

[10] K.-B. Kim E. Levi Z. Zabar L. Birenbaum, "Restoring force between two non-coaxial circular coils, " IEEE Trans. Magn., Vol. 32, No. 2, pp. 478-484, Nov. 1997.

[11] E.P. Furlani, "A formula for the levitation force between magnetic disks, " IEEE Trans. Magn., Vol. 29, No. 6, pp. 4165 – 4169, Mar. 1993.

[12] J.T.Conway, " Non coaxial force and inductance calculations for bitter coils and coils with uniform radial current distributions, " Applied Superconductivity and Electromagnetic devices (ASEMD) International Conference, Melbourne, Australia, pp. 61-64.

[13] Lang, M., "Fast calculation method for the forces and stiffness's of permanent-magnet bearings," 8th International Symposium on Magnetic Bearing, 533–537, 2002.

[14] Selvaggi, J. P., et al., "Computation of the external magnetic field, near-field or far-field from a circular cylindrical magnetic source using toroidal functions" IEEE Trans. Magn., Vol. 43, No. 4, 1153–1156, 2007.

[15] A. Shiri A. Shoulaie, "A new methodology for magnetic force calculations between planar spiral coils," PIER, vol. 95 pp. 39-57, 2009.

[16] Y. Ren, F. Wang, G.Kuang, W. Chen, Y. Tan, J. Zhu and P. He, "Mutual Inductance and Force Calculations between Coaxial Bitter Coils and Superconducting Coils with Rectangular Cross Section, " Jou. Supercond. Nov. Magn., 2010, DOI 10.1007/s10948-010-1086-0.

[17] Y. Ren, G.Kuang, and W. Chen, "Inductance of Bitter Coil with Rectangular Cross-Section, " Jou. Supercond. Nov. Magn., 2013, 26:2159–2163, DOI 10.1007/s10948-012-1816-6.

[18] J. T. Conway, "Inductance Calculations for Noncoaxial Coils using Bessel Functions, " *IEEE Trans. Magn.*, vol 43, n 3, pp 1023-1034, 2007.

[19] Babic S. and Akyel C., "Mutual Inductance and Magnetic Force Calculations for Bitter Disk Coils (Pancakes)" IET Science, Measurement & Technology, Vol.10, Issue 8, 2016, pp. 972-976.

[20] S. Babic S. and C. Akyel, "Mutual Inductance and Magnetic Force Calculations for Bitter Disk Coil (Pancake) with Nonlinear Radial Current and Filamentary Circular Coil with Azimuthal Current, "Hindawi Publishing Corporation Advances in Electrical Engineering Volume 2016, Article ID 3654021, 6 pages http://dx.doi.org/10.1155/2016/3654021.

[21] Babic S. and Akyel C., "Mutual Inductance and Magnetic Force Calculations between two Thick Coaxial Bitter Coils of Rectangular Cross Section, " IET Electric Power Applications, Vol. 11, Issue 3, 2017, pp. 441-446.

[22] Babic S. and Akyel C.," Mutual Inductance and Magnetic Force Calculations between Thick Coaxial Bitter Coil of Rectangular Cross Section with Inverse Radial Current and Filamentary Circular Coil with Constan Azimuthal Current, "IET Electric Power Applications, Vol. 11, Issue 9, 2017, pp. 1596 – 1600.

[23] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions*, National Bureau of

Standards Applied Mathematics, Washington DC, December 1972, Series 55, p. 595.
[24] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals*, Series *and Products*, New York and London, Academic Press Inc., 1965.