Sensorless Speed High Gain Observer Based on Field-Oriented Control for the Asynchronous Machine

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Abstract: A state observer is proposed for asynchronous machine; with this observer it possible to observe rotor flux and rotating speed. The gain of this observer involves a design function that has to satisfy some mild conditions which are given. Of particular interest, at the powerful of Field-Oriented Control with the new observer. The control algorithm is studied through simulations and applied in flux and speed profiles , and is shown to be very efficient.

Key–Words: Sensorless Speed; High Gain Observer; Field-Oriented Control; Asynchronous Machine.

1 Introduction

The removal of the mechanical speed sensors offers an economic interest and may improve the reliability in the fields of low power applications.

This article has as a principal objective to study the technique of determination mechanical speed and rotor flux of the asynchronous machine without velocity sensor.

The robustness, the low cost, the performances and the maintainability make the advantage of the asynchronous machine in many industrial applications or general public. Joint progress of the power electronics and numerical electronics makes it possible today to approach the controlling of axis at variable speed in applications low powers. Jointly with these technological projections, the scientific community developed various approaches of order to control in real time the flux and the speed of the electric machines.

That it is the vectorial control, the scalar control or DTC control , to control the speed of the load it is necessary to measure this one by means of a mechanical sensor. For economic reasons and/or of safety of operation, certain applications force to be freed some.
The information speed must then be rebuilt starting from the electric quantities. Multiple studies were undertaken, and without claim of exhaustiveness, we can distinguish several approaches (see [1], [2], [3]).

The control speed sensorless must however have performances which do not deviate too much from those that we would have had with a mechanical sensor. It is thus significant, during the development of an approach of velocity measurement without sensor to lay the stress on the static precise details and dynamics of this one according to the point of operation of the machine.

The article is organized in four sections:

1. Dynamic model of asynchronous machine;
2. Sensorless speed high gain observer;
3. Synthesis of Field-Oriented Control;
4. Results and simulations.
5. Conclusion

2 Dynamic Model of Asynchronous Machine

In this study, the model of the motor rests on the following hypothesis [4]:

- The fluxes and the currents are proportional by the intermediary of inductances and the mutual.
- The losses iron are neglected .
- The air-gap is constant (squirrel-cage rotor).
- The homopolar components are null.

It results from these assumptions that the various mutual between rotor and stator can be expressed like functions sinusoidal of the rotor position.

Its vector state is composed by the stator currents, rotor fluxes and speed, as follows:

$$\left\{\begin{array}{lcl} \dot{i}_s & = & -\gamma i_s + KA(\Omega)\psi_r + \frac{1}{\sigma L_s} u_s \\ \dot{\psi}_r & = & \frac{M}{T_r} i_s - A(\Omega)\psi_r \\ \dot{\Omega} & = & \frac{p}{J}\frac{M}{L_r} i_s^T \mathcal{J}_2 \psi_r - \frac{1}{J} T_L \\ \dot{T}_L & = & \varepsilon_{T_l} \end{array}\right. \quad (1)$$

The states variables accessible to measurement are the stator currents $i = \begin{bmatrix} i_{s\alpha} & i_{s\beta} \end{bmatrix}^T$ but to in no case rotor flux $\psi = \begin{bmatrix} \psi_{s\alpha} & \psi_{s\beta} \end{bmatrix}^T$ and possibly rotating speed. The source of energy $u = \begin{bmatrix} u_{s\alpha} & u_{s\beta} \end{bmatrix}^T$.

The parameters are defined as follows:

$$\begin{array}{ll} T_r = \dfrac{L_r}{R_r} & ; \quad \sigma = 1 - \dfrac{M^2}{L_s L_r} \\ K = \dfrac{M}{\sigma L_s L_r} & ; \quad \gamma = \dfrac{R_s}{\sigma L_s} + \dfrac{R_r M^2}{\sigma L_s L_r} \\ \mathcal{I}_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} & ; \quad \mathcal{J}_2 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \\ A(\Omega) = \frac{1}{T_r}\mathcal{I}_2 - p\Omega\mathcal{J}_2 & . \end{array}$$

In these equations:

- L_s : Stator inductance cyclic ,
- L_r : Rotor inductance cyclic ,
- M : Cyclic mutual inductance between stator and rotor
- R_s : Stator resistance,
- R_r : Rotor resistance,
- σ : Scattering coefficient,
- T_r : Time constant of the rotor dynamics,
- J : Rotor inertia,
- T_l : Resistive torque,
- p : Pole pair motor,
- $\mathcal{I}_2$: is the 2-dimensional identity matrix,
- $\mathcal{J}_2$: is a skew - symmetric matrix.

We need to transform system (1) to the triangular form. One will introduce the change of variable according to:

$$\left\{ \begin{array}{l} z_1 = i_s \\ z_2 = \left(\frac{1}{T_r}\mathcal{I}_2 - p\Omega\mathcal{J}_2\right)\psi_r \\ z_3 = \begin{bmatrix} \Omega & T_L \end{bmatrix}^T = \begin{bmatrix} z_{31} & z_{32} \end{bmatrix}^T \end{array} \right. \quad (2)$$

Using this transformation and a time derivative of these states, we can rewrite from model (1), a following model :

$$\left\{ \begin{array}{lcl} \dot{z}_1 & = & -\gamma z_1 + K z_2 + \frac{1}{\sigma L_s} u_s \\ \dot{z}_2 & = & \frac{1}{T_r}\left(\frac{M}{T_r} z_1 - z_2\right) - p z_{31}\mathcal{J}_2\left(\frac{M}{T_r} z_1 - z_2\right) \\ & & -pd\omega\mathcal{J}_2\psi_r + \frac{p}{J} z_{32}\mathcal{J}_2\psi_r \\ \dot{z}_3 & = & \begin{bmatrix} \frac{p}{J}\frac{M}{L_r} z_1^T\mathcal{J}_2\psi_r + \varepsilon_\Omega & \varepsilon_{T_l} \end{bmatrix}^T \\ y & = & z_1 \end{array} \right. \quad (3)$$

With $d\omega = \frac{p}{J}\frac{M}{L_r} z_1^T\mathcal{J}_2\psi_r$; and $\varepsilon_\Omega = -\frac{1}{J}T_L$.

3 Sensorless Speed High Gain Observer

Consider the nonlinear uniformly observable class of systems as the following form :

$$\left\{ \begin{array}{lcl} \dot{z} & = & f(u,z) + \bar{\varepsilon}(t) \\ y & = & Cz = z_1 \end{array} \right. \quad (4)$$

with

$$z = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}; \bar{\varepsilon}(t) = \begin{bmatrix} 0 \\ 0 \\ \varepsilon_3(t) \end{bmatrix}; \quad \varepsilon_3(t) = \begin{bmatrix} \varepsilon_\Omega \\ \varepsilon_{T_l} \end{bmatrix};$$

$C = \begin{bmatrix} \mathcal{I}_2 & 0 & 0 \end{bmatrix}; \ u = u_s$.

$$f(z,u) = \begin{bmatrix} f_1(u,z_1,z_2) \\ f_2(u,z_1,z_2,z_3) \\ f_3(u,z) \end{bmatrix} =$$

$$\begin{bmatrix} -\gamma z_1 + K z_2 + \frac{1}{\sigma L_s} u_s \\ \begin{pmatrix} \frac{1}{T_r}\left(\frac{M}{T_r} z_1 - z_2\right) - p z_{31}\mathcal{J}_2\left(\frac{M}{T_r} z_1 - z_2\right) \\ -pd\omega\mathcal{J}_2\psi_r + \frac{p}{J} z_{32}\mathcal{J}_2\psi_r \end{pmatrix} \\ \begin{bmatrix} \frac{p}{J}\frac{M}{L_r} z_1^T\mathcal{J}_2\psi_r & 0 \end{bmatrix}^T \end{bmatrix};$$

Our objective consists in designing state observers for system (4). We pose the following hypothesis (see. [7]).

$\mathcal{H}1$ **:** There are four positive constants α and β such as:

$$0 \quad < \quad \alpha^2 \leq \left(\frac{\partial f_1(u,z)}{\partial z_2}\right)^T \frac{\partial f_1(u,z)}{\partial z_2} \leq \beta^2 \quad (5)$$

$$0 \quad < \quad \alpha^2 \leq \left(\frac{\partial f_2(u,z)}{\partial z_3}\right)^T \frac{\partial f_2(u,z)}{\partial z_3} \leq \beta^2 \quad (6)$$

$\mathcal{H}2$ **:** The function $\varepsilon_3(t)$ is uniformly bounded by $\delta > 0$.

When $\varepsilon_3(t) = 0$, system (4) is identical to that considered in [5] and it characterizes a sub-class of locally $\mathbb{U}$-uniformly observable systems. In [6], the authors considered a sub-class of systems which involve the same uncertain term, $\bar{\varepsilon}(t)$, as (4). In the sequel, one shall use a strategy of observer design for asynchronous machine similar to that adopted in [5], [6] and [7].

One shall firstly introduce an appropriate state transformation allowing to easily design the proposed observers. Then, the equations of these observers will be derived in the new coordinates before being given in the original ones.

3.1 State Transformation

Consider the following change of states:

$\Phi : \mathbb{R}^3 \longmapsto \mathbb{R}^3$;

$$z = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} \longmapsto x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \Phi(z) = \begin{bmatrix} \Phi(z)_1 \\ \Phi(z)_2 \\ \Phi(z)_3 \end{bmatrix}$$

where the $\Phi_j(z)$,$j = \overline{1,3}$ are defined as follows:

$$\left\{ \begin{array}{l} x_1 = \Phi(z)_1 = z_1 \\ x_2 = \Phi(z)_2 = K z_2 \\ x_3 = \Phi(z)_3 = K \begin{bmatrix} -p\mathcal{J}_2\left(\frac{M}{T_r} z_1 - z_2\right) \\ \frac{p}{J}\mathcal{J}_2\psi_r \end{bmatrix}^T z_3 \end{array} \right.$$

$$\Rightarrow \quad x = \Phi(z) = \Lambda z \quad (7)$$

where Λ block diagonal matrix and Λ^{-1} is his left inverse:

$$\Lambda = diag\left(\mathcal{I}_2, \ K\mathcal{I}_2, \ -Kp\mathcal{J}_2 \begin{bmatrix} \frac{M}{T_r} z_1 - z_2 & -\frac{1}{J}\psi_r \end{bmatrix}\right)$$

Using this transformation and a time derivative of these states, we can rewrite from model (1), a following model :

$$\left\{\begin{array}{lcl} \dot{x}_1 & = & x_2 + \varphi_1(u, x_1) \\ \dot{x}_2 & = & x_3 + \varphi_2(u, x_1, x_2) \\ \dot{x}_3 & = & \varphi_3(u, x) \\ y & = & Cx = x_1 \end{array}\right. \tag{8}$$

Where

$$\left\{\begin{array}{lcl} \varphi_1(u, x_1) & = & -\gamma z_1 + \frac{1}{\sigma L_s} u_s \\ \varphi_2(u, x_1, x_2) & = & -pK\dot{z}_3 \mathcal{J}_2 z_2 + \frac{K}{T_r}\left(\frac{M}{T_r} z_1 - z_2\right) \\ \varphi_3(u, x) & = & \frac{\partial \Phi_3(u,z)}{\partial z_1}\dot{z}_1 + \frac{\partial \Phi_3(u,z)}{\partial z_2}\dot{z}_2 \end{array}\right. \tag{9}$$

Proceeding as in [5, 6], one can show that the transformation Φ puts system (1) under the following form:

$$\left\{\begin{array}{l} \dot{x} = Ax + \varphi(u, x) + \frac{\partial \Phi(z)}{\partial z}\bar{\varepsilon}(t) \\ \\ y = Cx = x_1 \end{array}\right. \tag{10}$$

where $\varphi(u; x)$ has a triangular structure i.e.

$$A = \begin{bmatrix} 0 & \mathcal{I}_2 & 0 \\ 0 & 0 & \mathcal{I}_2 \\ 0 & 0 & 0 \end{bmatrix} \text{ and } \varphi(u, x) = \begin{bmatrix} \varphi_1(u, x_1) \\ \varphi_2(u, x_1, x_2) \\ \varphi_3(u, x) \end{bmatrix}$$

3.2 Observer synthesis

As in the works related to the high gain observers synthesis [6, 7, 8, 9], one pose the hypothesis :

$\mathcal{H}3$ **:** The functions $\Phi(z)$ and $\varphi(u, x)$ are globally LIPSCHITZ with respect to x uniformly in u.

Before giving our candidate observers, one introduces the following notations.

1) Let Δ_θ is a block diagonal matrix defined by:

$$\Delta_\theta = diag\left(\mathcal{I}_2, \frac{1}{\theta}\mathcal{I}_2, \frac{1}{\theta^2}\mathcal{I}_2\right);$$

$\theta > 0$ is a real number.

2) Let $S = S_{\theta=1}$ is a definite positive solution of the ALGEBRAIC LYAPUNOV EQUATION:

$$S + A^T S + SA - C^T C = 0 \tag{11}$$

Note that (11) is independent of the system and the solution can be expressed analytically. For a straightforward computation, its stationary solution is given by: $S_{(n,p)} = (-1)^{n+p} C_{n+p-2}^{n-1}$ where $C_n^p = \frac{n!}{p!(n-p)!}$ for $n \geq 1$ and $p \leq 3$; and then we can explicitly determinate the correction gain of (3) as follows:

$$\theta \Lambda^{-1} \Delta_\theta^{-1} S^{-1} C^T = \begin{bmatrix} 3\theta \mathcal{I}_2 \\ \dfrac{3\theta^2}{K}\mathcal{I}_2 \\ -\frac{\theta^3}{Kp}\left(\begin{bmatrix} \mathcal{J}_2\left(\frac{M}{T_r} z_1 - z_2\right) \\ -\frac{1}{J}\mathcal{J}_2 \psi_r \end{bmatrix}^{-1}\right)^T \end{bmatrix} \tag{12}$$

The system

$$\begin{array}{rcl} \dot{\hat{x}} & = & A\hat{x} + \varphi(u, \hat{x}) - \theta \Delta_\theta^{-1} S^{-1} C^T \tilde{x_1} \\ & & -\dfrac{\partial \Phi(z)}{\partial z}\left(\Lambda^{-1} - \left(\dfrac{\partial \Phi(z)}{\partial z}\right)^{-1}\right) \\ & & \theta \Delta_\theta^{-1} S^{-1} C^T \tilde{x_1} \end{array} \tag{13}$$

is an observer for (10); Where $\tilde{x} = \hat{x} - x$ error in estimation; u is the input of system (10) and $\theta > 0$ is a real number.

Finally one gives the following lemma (see[7]):

Lemma:

Assume that system (10) satisfies hypothesis $\mathcal{H}1$ to $\mathcal{H}3$. Then,

$$\exists \theta_0 > 0;\ \forall \theta > \theta_0;\ \exists \lambda > 0;\ \exists \mu_\theta > 0;\ \exists M_\theta > 0;$$

$$\forall u \in \mathbb{U};\ \forall \hat{x}(0) \in \mathbb{R};\ \text{ one has :}$$

$$\|\tilde{x}\| \leq \lambda \theta^{-2} \exp^{-\mu_\theta t} \|\tilde{x}(0)\| + M_\theta \delta$$

where x is the unknown trajectory of (10) associated to the input u, $\hat{x}$ is any trajectory of system (13) associated to (u, y) and δ is the upper bound of $\varepsilon_3(t)$. Moreover, one has $\lim_{\theta \longrightarrow \infty} \{\mu_\theta\} = +\infty$ and $\lim_{\theta \longrightarrow \infty} \{M_\theta\} = 0$.

3.2.1 Stability Analysis

: One has:

$$\begin{array}{rcl} \dot{\tilde{x}} & = & A\tilde{x} - \theta \Delta_\theta^{-1} S^{-1} C^T \tilde{x_1} + \varphi(u, \hat{x}) - \varphi(u, x) \\ & & -\dfrac{\partial \Phi(\Phi^{-1}(x))}{\partial z}\bar{\varepsilon}(t) - \Gamma(\hat{x})\theta \Delta_\theta^{-1} S^{-1} C^T \tilde{x_1} \end{array}$$

where

$$\Gamma(\hat{x}) = \frac{\partial \Phi(\Phi^{-1}(\hat{x}))}{\partial z}\left(\Lambda^{-1} - \left(\frac{\partial \Phi(\Phi^{-1}(\hat{x}))}{\partial z}\right)^{-1}\right)$$

Notice that $\Gamma(\hat{x})$ is a lower triangular matrix with zeros on its main diagonal. Moreover, using hypothesis $\mathcal{H}1$ and $\mathcal{H}3$, one can easily deduce that $\Gamma(\hat{x})$ is bounded.

Now, one can easily check the following identities: $\theta \Delta_\theta^{-1} A \Delta_\theta = A$ and $C\Delta_\theta = C$. Set $\bar{x} = \Delta_\theta \tilde{x}$. One obtains :

$$\begin{array}{rcl} \dot{\bar{x}} & = & \theta A\bar{x} - \theta S^{-1} C^T \tilde{x_1} + \Delta_\theta(\varphi(\hat{x}) - \varphi(u, x)) \\ & & -\Delta_\theta \dfrac{\partial \Phi(\Phi^{-1}(x))}{\partial z}\bar{\varepsilon}(t) - \theta \Delta_\theta \Gamma(\hat{x}) \Delta_\theta^{-1} S^{-1} C^T \tilde{x_1} \end{array} \tag{14}$$

To prove convergence, let us consider the following equation of LYAPUNOV $V(\bar{x}) = \bar{x}^T S \bar{x}$. By calculating the derivative of V along the $\tilde{x}$ trajectories, we obtains:

$$\begin{array}{rcl} \dot{V} & = & 2\bar{x}^T S \dot{\bar{x}} \\ & = & 2\theta \bar{x}^T SA\bar{x} - 2\theta \bar{x}^T C^T \tilde{x_1} \end{array}$$

$$\begin{aligned}
&+2\bar{x}^T S\Delta_\theta\left(\varphi(u,\hat{x})-\varphi(u,x)\right)\\
&-2\bar{x}^T S\Delta_\theta\frac{\partial\Phi(\Phi^{-1}(x))}{\partial z}\bar{\varepsilon}(t)\\
&-2\theta\bar{x}^T S\Delta_\theta\Gamma(\hat{x})\Delta_\theta^{-1}S^{-1}C^T\tilde{x_1}\\
=\;&\theta\bar{x}^T(-S+C^TC)\bar{x}\\
&-2\theta\bar{x}^T C^T\tilde{x_1}+2\bar{x}^T S\Delta_\theta\left(\varphi(u,\hat{x})-\varphi(u,x)\right)\\
&-2\bar{x}^T S\Delta_\theta\frac{\partial\Phi(\Phi^{-1}(x))}{\partial z}\bar{\varepsilon}(t)\\
&-2\theta\bar{x}^T S\Delta_\theta\Gamma(\hat{x})\Delta_\theta^{-1}S^{-1}C^T\tilde{x_1}\\
=\;&-\theta V+\theta\bar{x}^T C^TC\bar{x}-2\theta\bar{x}^T C^T\tilde{x_1}\\
&+2\bar{x}^T S\Delta_\theta\left(\varphi(u,\hat{x})-\varphi(u,x)\right)\\
&-2\bar{x}^T S\Delta_\theta\frac{\partial\Phi(\Phi^{-1}(x))}{\partial z}\bar{\varepsilon}(t)\\
&-2\theta\bar{x}^T S\Delta_\theta\Gamma(\hat{x})\Delta_\theta^{-1}S^{-1}C^T\tilde{x_1}
\end{aligned}$$

By taking account of the (11) the derivative of V becomes:

$$\begin{aligned}
\dot{V}\;=\;&-\theta V+2\theta\left(\frac{1}{2}\bar{x}^T C^TC\bar{x}-\bar{x}^T\Upsilon(x_1)\right)\\
&+2\bar{x}^T S\Delta_\theta\left(\varphi(u,\hat{x})-\varphi(u,x)\right)\\
&-2\bar{x}^T S\Delta_\theta\frac{\partial\Phi(\Phi^{-1}(x))}{\partial z}\bar{\varepsilon}(t)\\
&-2\theta\bar{x}^T S\Delta_\theta\Gamma(\hat{x})\Delta_\theta^{-1}S^{-1}C^T\tilde{x_1}\\
\le\;&-\theta V+2\bar{x}^T S\Delta_\theta\left(\varphi(u,\hat{x})-\varphi(u,x)\right)\\
&-2\bar{x}^T S\Delta_\theta\frac{\partial\Phi(\Phi^{-1}(x))}{\partial z}\bar{\varepsilon}(t)\\
&-2\theta\bar{x}^T S\Delta_\theta\Gamma(\hat{x})\Delta_\theta^{-1}S^{-1}C^T\tilde{x_1}
\end{aligned}\tag{15}$$

Now, assume that $\theta\ge 1$, then, because of the triangular structure and the LIPSCHITZ assumption on φ, one can show that :

$$\left\|\Delta_\theta\left(\varphi(u,\hat{x})-\varphi(u,x)\right)\right\|\le\zeta\left\|\bar{x}\right\|\tag{16}$$

where ζ is a constant of LIPSCHITZ. Similarly, according to hypothesis $\mathcal{H}1$ and to the LIPSCHITZ assumption on Φ (hypothesis $\mathcal{H}3$) , $\Gamma(\hat{x})$ is bounded. Moreover, and since $\Gamma(\hat{x})$ is lower triangular with zeros on the main diagonal, one has:

$$\left\|\theta\Delta_\theta\Gamma(\hat{x})\Delta_\theta^{-1}\right\|\le\varrho\;\text{ for }\;\theta\ge 1\tag{17}$$

where $\varrho>0$ is a constant that does not depend on θ. Finally, according to the structure of $\bar{\varepsilon}(t)$ and since $\frac{\partial\Phi(\Phi^{-1}(x))}{\partial z}$ is triangular, one can show that:

$$\left\|\Delta_\theta\frac{\partial\Phi(\Phi^{-1}(x))}{\partial z}\bar{\varepsilon}(t)\right\|=\frac{1}{\theta^2}\left\|\|\right\|\le\frac{\beta^2}{\theta^2}\delta\tag{18}$$

where $\delta=\sup\left\{\bar{\varepsilon}(t)\right\}$ given in hypothesis $\mathcal{H}2$; β is given in hypothesis $\mathcal{H}1$. Using inequalities (16), (17) and (18) inequality (15) becomes:

$$\begin{aligned}
\dot{V}\;\le\;&-\theta V+2\lambda_{\max}(S)\left\|\bar{x}\right\|\left(\zeta\left\|\bar{x}\right\|+\varrho\eta(S)\left\|\tilde{x}_1\right\|\right)\\
&+2\lambda_{\max}(S)\frac{\beta^2}{\theta^2}\delta\left\|\bar{x}\right\|\\
\le\;&-(\theta-c_1)V+\frac{c_2}{\theta^2}\delta\sqrt{V}
\end{aligned}$$

where $c_1=2\eta^2(S)(\zeta+\varrho\eta(S))$ and $c_2=2\beta^2\eta(S)\sqrt{\lambda_{\max}(S)}$ with $\lambda_{\min}(S)$ and $\lambda_{\max}(S)$ being respectively the smallest and the largest eigenvalues of S and $\eta(S)=\sqrt{\frac{\lambda_{\max}(S)}{\lambda_{\min}(S)}}$.

Now taking $\theta_0=\max\{1,c_1\}$ and using the fact that for $\theta\ge 1$, $\|\bar{x}\|\le\|\tilde{x}\|\le\theta^2\|\bar{x}\|$, one can show that for $\theta>\theta_0$, one has :

$$\|\tilde{x}\|\le\theta\eta(S)\exp\left[-\left(\frac{\theta-c_1}{2}\right)t\right]\|\tilde{x}(0)\|+2\beta\frac{\eta^2(S)}{(\theta-c_1)}\delta$$

It is easy to see that λ,μ_θ and M_θ needed by the result 1 are: $\lambda=\eta(S)$, $\mu_\theta=\frac{\theta-c_1}{2}$ and $M_\theta=2\beta^2\frac{\eta^2(S)}{(\theta-c_1)}$. This completes the proof.

3.3 Observer equation in the original coordinates

Proceeding as in [6], one can show that observer (13) can be written in the original coordinates z as follows: (see e.g. [9, 5, 6]):

$$\dot{\hat{z}}=f(u,\hat{z})-\theta\Lambda^{-1}\Delta_\theta^{-1}S^{-1}C^T(\hat{z}_1-z_1)\tag{19}$$

Or

$$\left\{\begin{aligned}
\dot{\hat{z}}_1\;=\;&-\gamma\hat{z}_1+K\hat{z}_2+\tfrac{1}{\sigma L_s}u_s-3\theta(\hat{z}_1-z_1)\\
\dot{\hat{z}}_2\;=\;&\tfrac{1}{T_r}\left(\tfrac{M}{T_r}\hat{z}_1-\hat{z}_2\right)-p\hat{z}_{31}\mathcal{J}_2\left(\tfrac{M}{T_r}\hat{z}_1-\hat{z}_2\right)\\
&-p\hat{d\omega}\mathcal{J}_2\hat{\psi}_r+\tfrac{p}{J}T_l\mathcal{J}_2\psi_r-\tfrac{3\theta^2}{K}(\hat{z}_1-z_1)\\
\dot{\hat{z}}_3\;=\;&\begin{bmatrix}\frac{p}{J}\frac{M}{L_r}\hat{i}_s^T\mathcal{J}_2\hat{\psi}_r\\ 0\end{bmatrix}\\
&+\tfrac{\theta^3}{Kp}\begin{bmatrix}\mathcal{J}_2\left(\frac{M}{T_r}\hat{z}_1-\hat{z}_2\right) & -\frac{1}{J}\mathcal{J}_2\psi_r\end{bmatrix}^{-1}\\
&(\hat{z}_1-z_1)
\end{aligned}\right.\tag{20}$$

Referring to (7), the rotor flux is governed by the following equations:

$$\hat{\psi}_r=\left(\frac{1}{T_r}\mathcal{I}_2-p\hat{\Omega}\mathcal{J}_2\right)^{-1}\hat{z}_2\tag{21}$$

4 Synthesis of Field-Oriented Control

The field-oriented control is based on the transformation of the vector of the stator currents and rotor fluxes $\begin{bmatrix}\hat{i}_{s\alpha} & \hat{i}_{s\beta} & \hat{\psi}_{r\alpha} & \hat{\psi}_{r\beta}\end{bmatrix}$ represented in the reference

(α,β) who turns with the vector of flux $\left[\begin{array}{cc}\hat{\psi}_{r\alpha} & \hat{\psi}_{r\beta}\end{array}\right]$, to see [12], [13] and [14]. This transformation is given by

$$\left[\begin{array}{c}\hat{i}_{sd}\\ \hat{i}_{sq}\end{array}\right]=\left[\begin{array}{cc}\cos(\hat{\vartheta}) & \sin(\hat{\vartheta})\\ -\sin(\hat{\vartheta}) & \cos(\hat{\vartheta})\end{array}\right]\left[\begin{array}{c}\hat{i}_{s\alpha}\\ \hat{i}_{s\beta}\end{array}\right] \quad (22)$$

$$\left[\begin{array}{c}\hat{\psi}_{rd}\\ \hat{\psi}_{rq}\end{array}\right]=\left[\begin{array}{cc}\cos(\hat{\vartheta}) & \sin(\hat{\vartheta})\\ -\sin(\hat{\vartheta}) & \cos(\hat{\vartheta})\end{array}\right]\left[\begin{array}{c}\hat{\psi}_{r\alpha}\\ \hat{\psi}_{r\beta}\end{array}\right] \quad (23)$$

with

$$\hat{\vartheta}=\arctan\left(\frac{\hat{\psi}_{r\beta}}{\hat{\psi}_{r\alpha}}\right) \quad (24)$$

Knowing that $\cos(\hat{\vartheta})=\frac{\hat{\psi}_{r\alpha}}{\hat{\Psi}}$, $\sin(\hat{\vartheta})=\frac{\hat{\psi}_{r\beta}}{\hat{\Psi}}$ and $\hat{\Psi}=\sqrt{\hat{\psi}_{r\alpha}^2+\hat{\psi}_{r\beta}^2}$, then of the two equations (22) and (23), we obtains:

$$\left\{\begin{array}{lcl}\hat{i}_{sd} & = & \frac{\hat{\psi}_{r\alpha}\hat{i}_{s\alpha}+\hat{\psi}_{r\beta}\hat{i}_{s\beta}}{\hat{\Psi}}\\ \hat{i}_{sq} & = & \frac{\hat{\psi}_{r\alpha}\hat{i}_{s\beta}-\hat{\psi}_{r\beta}\hat{i}_{s\alpha}}{\hat{\Psi}}\\ \hat{\psi}_{rd} & = & \hat{\Psi}\\ \hat{\psi}_{rq} & = & 0\end{array}\right. \quad (25)$$

with the associated control

$$\left[\begin{array}{c}u_{s\alpha}\\ u_{s\beta}\end{array}\right]=\hat{\Psi}\left[\begin{array}{cc}\hat{\psi}_{r\alpha} & \hat{\psi}_{r\beta}\\ -\hat{\psi}_{r\beta} & \hat{\psi}_{r\alpha}\end{array}\right]^{-1}\left[\begin{array}{c}u_{sd}\\ u_{sq}\end{array}\right] \quad (26)$$

then, the asynchronous machine model given by the equation (1), becomes

$$\left\{\begin{array}{lcl}\frac{d\hat{i}_{sd}}{dt} & = & -\gamma\hat{i}_{sd}+\frac{K}{T_r}\hat{\psi}_{rd}+p\hat{\Omega}\hat{i}_{sq}+\frac{M\hat{i}_{sq}^2}{T_r\hat{\psi}_{rd}}+\frac{u_{sd}}{\sigma L_s}\\ \frac{d\hat{i}_{sq}}{dt} & = & -\gamma\hat{i}_{sq}-pK\hat{\Omega}\hat{\psi}_{rd}-p\hat{\Omega}\hat{i}_{sd}-\frac{M\hat{i}_{sd}\hat{i}_{sq}}{T_r\hat{\psi}_{rd}}\\ & & +\frac{u_{sq}}{\sigma L_s}\\ \frac{d\hat{\psi}_{rd}}{dt} & = & -\frac{\hat{\psi}_{rd}}{T_r}+\frac{M\hat{i}_{sd}}{T_r}\\ \frac{d\hat{\Omega}}{dt} & = & \frac{pM}{JL_r}\hat{\psi}_{rd}\hat{i}_{sq}-\frac{T_l}{J}\\ \frac{d\hat{\theta}}{dt} & = & p\hat{\Omega}+\frac{M\hat{i}_{sq}}{T_r\hat{\psi}_{rd}}\end{array}\right.$$

Now, Let us consider the following nonlinear feedback state

$$\left[\begin{array}{c}u_{sd}\\ u_{sq}\end{array}\right]=\sigma L_s\left[\begin{array}{c}-\frac{K}{T_r}\hat{\psi}_{rd}-p\hat{\Omega}\hat{i}_{sq}-\frac{M\hat{i}_{sq}^2}{T_r\hat{\psi}_{rd}}+v_d\\ pK\hat{\Omega}\hat{\psi}_{rd}+p\hat{\Omega}\hat{i}_{sd}+\frac{M\hat{i}_{sd}\hat{i}_{sq}}{T_r\hat{\psi}_{rd}}+v_q\end{array}\right] \quad (27)$$

the equation (26) is written then:

$$\left[\begin{array}{c}u_{s\alpha}\\ u_{s\beta}\end{array}\right] = \sigma L_s\hat{\Psi}\left[\begin{array}{cc}\hat{\psi}_{r\alpha} & \hat{\psi}_{r\beta}\\ -\hat{\psi}_{r\beta} & \hat{\psi}_{r\alpha}\end{array}\right]^{-1}\left[\begin{array}{c}-\frac{K}{T_r}\hat{\psi}_{rd}-p\hat{\Omega}\hat{i}_{sq}-\frac{M\hat{i}_{sq}^2}{T_r\hat{\psi}_{rd}}+v_d\\ pK\hat{\Omega}\hat{\psi}_{rd}+p\hat{\Omega}\hat{i}_{sd}+\frac{M\hat{i}_{sd}\hat{i}_{sq}}{T_r\hat{\psi}_{rd}}+v_q\end{array}\right] \quad (28)$$

Consequently, we leads to a simple system, with a dynamics of the linear norm flux

$$\left\{\begin{array}{lcl}\frac{d\hat{i}_{sd}}{dt} & = & -\gamma\hat{i}_{sd}+v_d\\ \frac{d\hat{i}_{sq}}{dt} & = & -\gamma\hat{i}_{sq}-v_q\\ \frac{d\hat{\psi}_{rd}}{dt} & = & -\frac{\hat{\psi}_{rd}}{T_r}+\frac{M\hat{i}_{sd}}{T_r}\\ \frac{d\hat{\Omega}}{dt} & = & \frac{pM}{JL_r}\hat{\psi}_{rd}\hat{i}_{sq}-\frac{T_l}{J}\\ \frac{d\hat{\theta}}{dt} & = & p\hat{\Omega}+\frac{M\hat{i}_{sq}}{T_r\hat{\psi}_{rd}}\end{array}\right.$$

When the amplitude of rotor flux $\hat{\psi}_{rd}=\hat{\Psi}$ reached its reference ψ_{ref} who is constant, the dynamics rotor speed becomes also linear. If we considers the following subsystem:

$$\left\{\begin{array}{lcl}\frac{d\hat{\psi}_{rd}}{dt} & = & -\frac{\hat{\psi}_{rd}}{T_r}+\frac{M\hat{i}_{sd}}{T_r}\\ \frac{d\hat{i}_{sd}}{dt} & = & -\gamma\hat{i}_{sd}+v_d\end{array}\right. \quad (29)$$

and like it was shown in [15] and [16], the dynamics of the amplitude of flux can be controlled by v_d by simple PI as follows

$$v_d=k_{d1}(\psi_{ref}-\hat{\Psi})+k_{d2}\int_0^t\left(\psi_{ref}-\hat{\Psi}(\tau)\right)d\tau \quad (30)$$

If one is interested now in the second subsystem given by:

$$\left\{\begin{array}{lcl}\frac{d\hat{\Omega}}{dt} & = & \frac{pM}{JL_r}\hat{\psi}_{rd}\hat{i}_{sq}-\frac{T_l}{J}\\ \frac{d\hat{i}_{sq}}{dt} & = & -\gamma\hat{i}_{sq}+v_q\end{array}\right. \quad (31)$$

one can notice that the dynamics speed is linear if $\hat{\psi}_{rd}$ is constant, and can also be also controled independently by v_q by using a regulator of the type PI as follows:

$$v_q=k_{q1}(\Omega_{ref}-\hat{\Omega})+k_{q2}\int_0^t\left(\Omega_{ref}-\hat{\Omega}(\tau)\right)d\tau \quad (32)$$

The adjustment of the gains is rather easy when the model is perfect, knowing that it is sufficient to calculate the gains of PI for a first order system. When the parameters vary, the gains become difficult to regulate.

5 Results and Simulations

We conceived simulation by carrying out the diagram general in blocks as the figure shows it (Fig.4).

5.1 Simulation Block Diagrams, Machine Data and a Benchmark

We show a detailed scheme SIMULINK of the control with the observer in Fig.5.

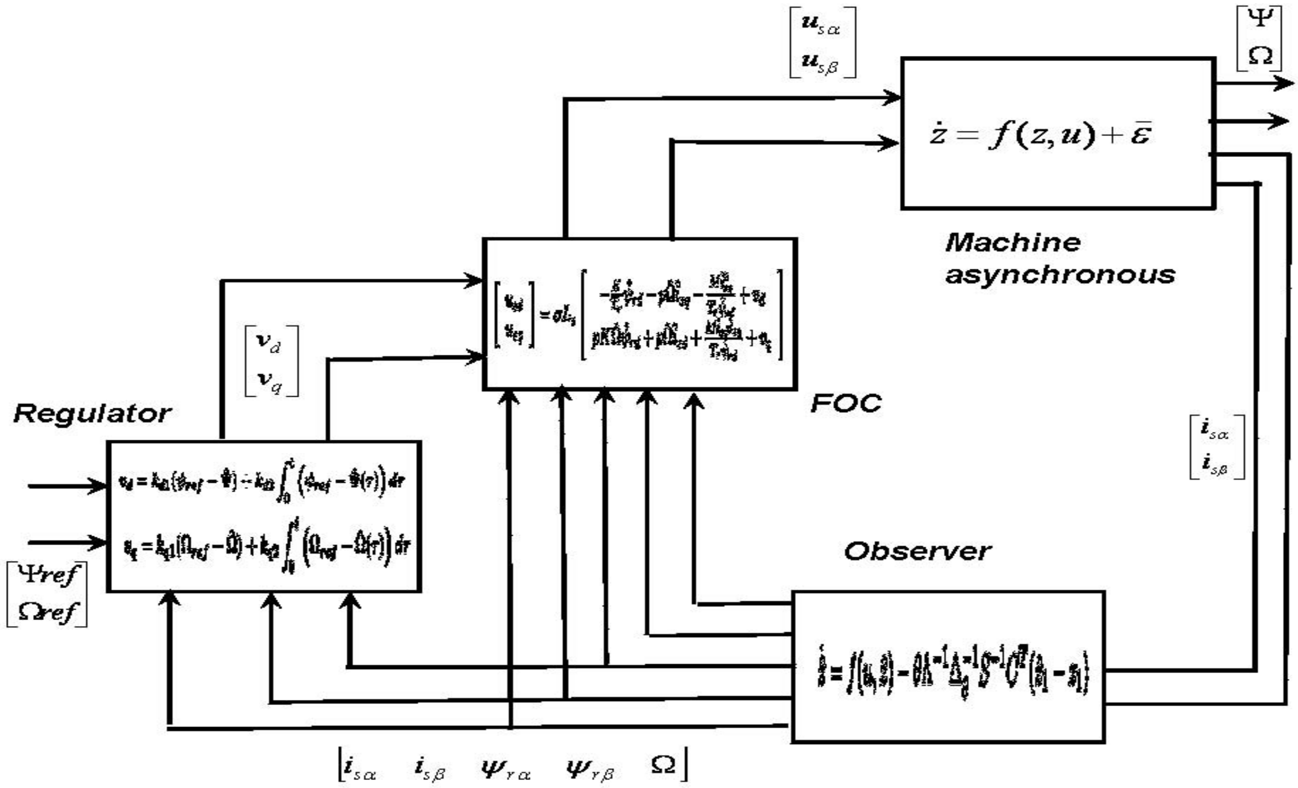

Figure 4: Diagram general of the field-oriented control with high gain observer.

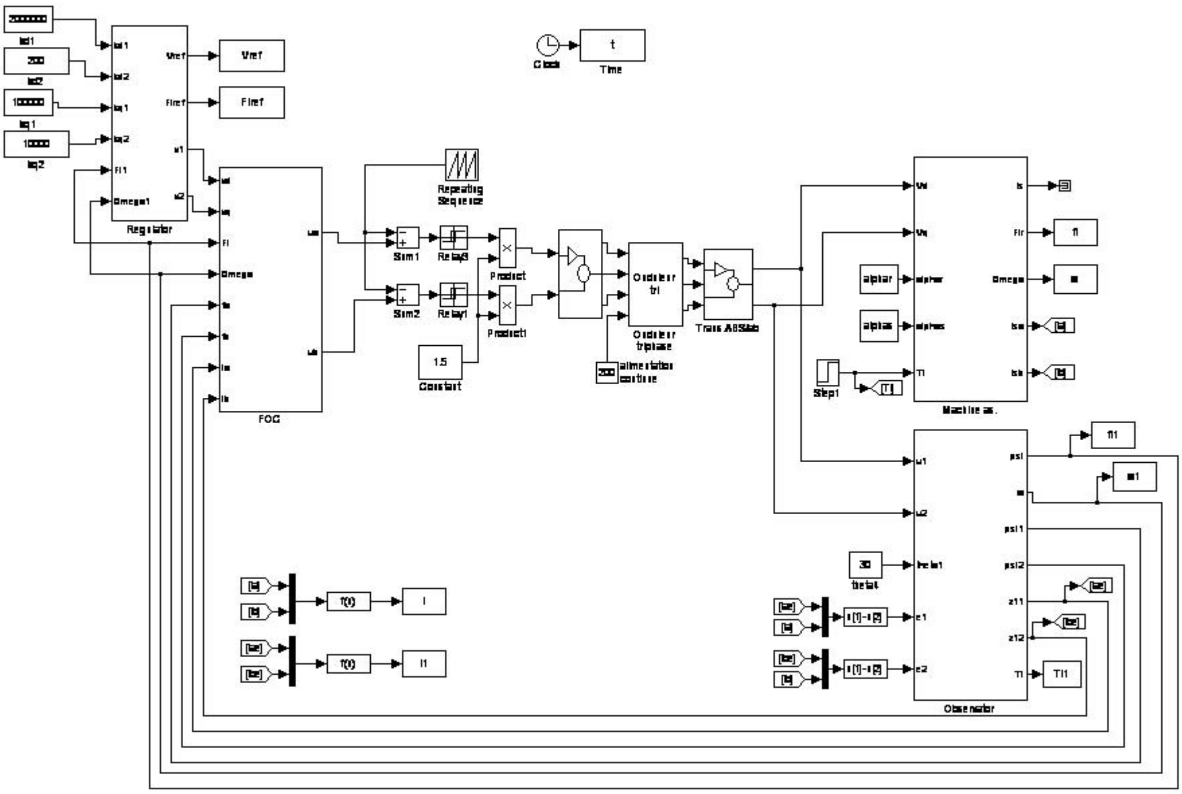

Figure 5: General block scheme in SIMULINK.

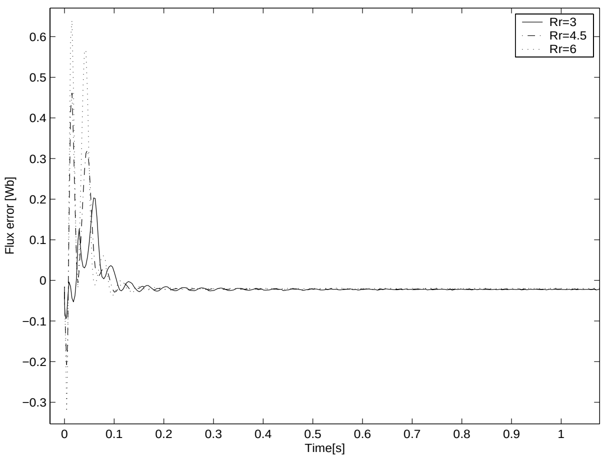

Figure 6: Sensitivity to rotor resistance.

R_r		rotor resistance error
3Ω	Mean	-19×10^{-4}
	Var	8.1507×10^{-5}
4.5Ω	Mean	-9.8577×10^{-4}
	Var	4.9437×10^{-4}
6Ω	Mean	-4.8098×10^{-4}
	Var	10×10^{-4}

Table 3: Means and variances of rotor resistance error.

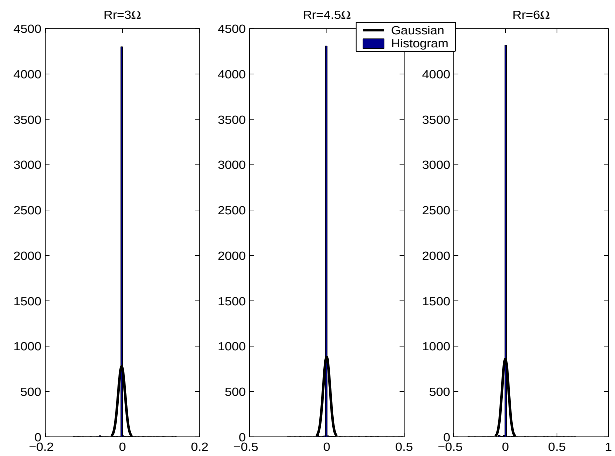

Figure 7: Gaussian and histogram of error rotor resistance.

5.3 Powerful of Control Associated with the High Gain Observer

The vector of state of the motor is initialized with the stopped state $\begin{bmatrix} i_{s\alpha} & i_{s\beta} & \psi_{r\alpha} & \psi_{r\beta} & \Omega \end{bmatrix}^T = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$; whereas that of the observer in a functional state $\begin{bmatrix} \hat{i}_{s\alpha} & \hat{i}_{s\beta} & \hat{\psi}_{r\alpha} & \hat{\psi}_{r\beta} & \hat{\Omega} \end{bmatrix}^T = \begin{bmatrix} 0.2 & 0.2 & 1 & 1 & 10 \end{bmatrix}^T$. The results of simulation are given by the curves obtained law of field-oriented control which regulator coefficients $k_{d1} = 2000000$, $k_{d2} = 200$, $k_{q1} = 200000$, $k_{q2} = 10000$.

5.3.1 Rotor flux

Before $t = 1.5s$, speed is null (blocked rotor). The rotor flux norm follows the trajectory. When speed starts increase; it creates a peak (see figure 8). At moment $t = 2.5s$ the speed takes the opposite direction; there is a reduction in flux. The gaussian error density and empirical error of observation and regulation show in table 4.

Error	Observation	Regulation
Mean	-57×10^{-4}	-95×10^{-4}
Var	7.1452×10^{-4}	17×10^{-4}

Table 4: Mean and variance flux errors of observation and regulation.

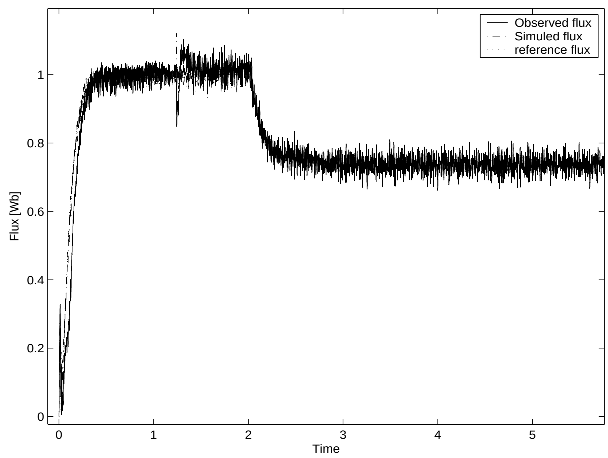

Figure 8: Observed, simulated and reference flux norm.

5.3.2 Rotating speed

After the transient state due to the conditions initial; we note a very good tracking (Figure 9). The gaussian error density and empirical error of observation and regulation show in table 5.

Error	Observation	Regulation
Mean	2267×10^{-4}	1524×10^{-4}
Var	2.3126	5.4344

Table 5: Mean and variance speed errors of observation and regulation.

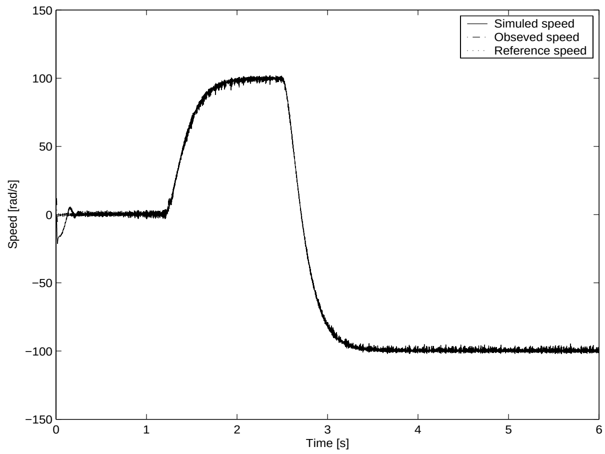

Figure 9: Observed, simulated and reference rotating speed.

5.3.3 Stator current error

We show the plot of error stator current in figure 10(a). There are oscillations at moment (t=0, 0.5, 1, 2, 3); we notice at these time flux and speed changes their values. In graph 10(b) appear gaussian errors density and empirical errors histogram of stator current error where means equal 17×10^{-4} and variance equal 53×10^{-4}.

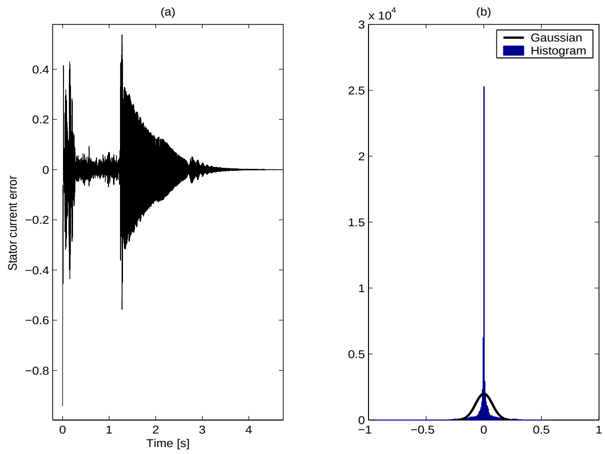

Figure 10: (a) Current error. (b) Gaussian and histogram of current error

6 Conclusion

In this paper, we presented a new robust high gain observer based on a field-oriented control scheme for machine asynchronous. The observer proposed in this study offers the advantage of only one tuning parameter θ. The stability observer is proofed with a carefully built LYAPUNOV function. The results of simulations regarding the observation of rotor flux, the robustness of the observer, the speed tracking and the rotor flux tracking confirm the theory suggested. We wish to validate these results in real time.

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