Renewable Energy and Demand-Side Management: Micro-Grid Power Market Control Using Stackelberg Discounting Play

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Abstract: - A volume discounting model in the micro-grid power market discussed in this paper. The Generator Companies' (GenCo) discounting model and the special fares for Distribution Companies (DisCo) formulated. The concept of renewable energy intermittent electricity transmissions to certain storage devices in the distribution level presented as a novel contribution in the field. Considering the final unit price which GenCo proposes to Disco for different power consumption levels, GenCo tries to maximize its profit margin by controlling DisCos' order behavior, in term of volumes per order, by using the discounting tool. Respectively DisCo tries to maximize its profit margin by considering the special fares offered by GenCo. In this model, the volume of electricity in the storage devises assumed to continuously drained over the time. The drained power volume assumed to be a fraction of stored power in the storage devices. This model formulated as a Stackelberg game between a micro-grid GenCo and DisCo considering renewable-energy concept as an artwork to reach an optimal discounting and pricing policy. This policy will maximize both parties' profit margins per unit of time. Finally, the model credibility examined by using a numerical example to show benefits of the proposed formulation.

Key-Words: Power Market, Demand-side Management, Discount Policy, Micro-Grid, Renewable Energy Resources, Profit Margin, Stackelberg Game

1 Introduction

The Quantity Discount Strategy is one of the mostly used methods used by sellers to reduce fixed ordering, shipment and material handling associated costs. Monahan [1] formulated the seller and buyer economical relationship ([2],[3]) and proposed an optimal all-unit quantity discount strategy while demand has a fixed and predictable trend. Lee and Rosenblatt [4] modified Monahan's model to reach an exact discounting rate that seller used to offer. Parlar and Wang [5] offered a game theory model to analysis different discount strategies. Developing the former model, Yang [6] offered an optimal vendor pricing policy for the deteriorating products. He used the Taylor series expansion to forecast the total expected profit from vendor-buyer discount policy. As an expansion of this model, Parlar and Sarmah et al. [7] assumed that buyer and seller inventory strategies can be formulated by the classical Economic Order Quantity (EOQ) models. Classical EOQ models are well known as the most successful ones respect to the simplicity and the easiness.

In inventory management, economic order quantity (EOQ) is the order amount which minimizes the holding and ordering costs. The optimal order quantity under the EOQ model is one of the oldest models in the industrial engineering and production planning literature. This model initially extended by Ford W. Harris in 1913 and further applied by R.H. Wilson [8]. An important application of the EOQ is in the quantity discounting models. Two major quantity discounting models are “all-units discounting model” and “incremental or by-threshold discounting model” [9]. The all-units discounting models applies when a seller considers discount for the order amount above certain threshold while in the all-units discounting model, in case order amounts will be higher than an specific threshold then the entire order quantity would be entitled for an specific discount. Considering more recent studies, You [10] developed an advanced sales model where a typical firm sell the perishable inventory using a reservation system during the sales seasons and over a limited number of time intervals. You and Wu [11] investigated the problem of ordering and pricing over a finite time planning horizon for an inventory system with advance sales and spot sales and developed a solution procedure to determine an optimal advance sales price, spot sales price, order size and replenishment frequency. Goyal et al. [12] introduced a new concept where the supplier charges the retailer progressive interest rates if the retailer exceeds the period of permissible delay, and established necessary and sufficient conditions for the unique optimal replenishment interval. Ho et al. [13] proposed an integrated inventory model with retail price sensitive demand and trade credit financing.

Once a loyal strategic customer exists which actively uses the proposed discount framework, then designing an optimal discounting model is a complex and sensitive issue [14]. In such situations, there is a potential that a so called “reverse bull whip” effect happens which means an increase in demand-side uncertainty will reduce the supplier order quantity uncertainty at the same time. Tsao [15] considered retailer’s promotion and replenishment policies with an advance sales discount under the supplier and retailer trade credits and presented an algorithm which simultaneously determines the optimal promotion effort and the replenishment cycle time. Chang et al. [16] formulated an integrated vendor–buyer inventory model with retail price sensitive demand, where the credit terms are linked to the order quantity. Chen and Kang [17] developed an integrated model with permissible delay in payments to determine the optimal replenishment time intervals and the replenishment frequency and in the same context Xin Chen et al. [18] proposed a coordinated inventory control model by considering a pricing strategy for the perishable products. In the same context, Malakooti [19] has introduced a multi–measure EOQ model which minimizes the total cost, order amounts and order backlogs.

Considering the recent years studies, Jiamin Wang et al. [20] proposed a robust price-control model for management of the perishable products by considering the conventional revenue management concept and when the model parameters are available. This article presents a dynamic price control problem on a continuous-time situation. This model proposed that the formulated min-max price-control model is equal to the conventional revenue management models if demand side parameters are known. Mehmet et al. [21] introduced and EOQ (Economic Order Quantity) respect to a group pf perishable products at the same time. The model objective defined as the maximizing the retailers profit margin under some storage capacity constraints. Similar to the proposed model in this paper, Mehmet assumed that the demand rate is linked to the sales prices and storage capacities and to reach the optimum amount of each order, a Tabu search-based heuristic method proposed. In the same context Maryam Akbari et el. [22] proposed a two calibrated meta-heuristic algorithms to optimize the vendor managed inventory (VMI) in a perishable product supply chain. Geoggrey A. Chuaa et al. [23] considered a short considered a short shelf-life and uncertain demand and discussed the discount and replenishment policy via four alternative models. In the context of the discounting strategies, T. Maiti et al. [24] presented a supply chain model under two periods of time horizons where demand rate for each period is a function of the sales price and applicable discounts. This model assumed the manufacturer as the Stackelberg player which sets the wholesale price for the distributor chain. In this paper several decision scenarios developed through applying various numerical analyses. In the same context, Lina Feng et al. [25] proposed a demand determination model in a multivariable function of different prices and the expected supply for each product.

The Stackelberg model came from the economics and management science [26]. This model initially applied in the industrial engineering practices and recently widely used in the power market literature. The term “Stackelberg” originally came from the leadership science and economics researches. The Stackelberg model stands for a strategic game theory in the economic science. Based on this theory, a leader first acts and then a follower adjusts his subsequent actions by considering the leader prior action. The term itself named after a German economist Heinrich Freiherr von Stackelberg who initially described the Stackelberg model in 1934 in his book, the Market Structure and Equilibrium (Marktform und Gleichgewicht) [26]. The Stackelberg model can be solved by using the Nash Equilibrium. In this case, there will be a strategy or decision policy that can suits the best to all the players in the game. This decision making model assumes that all players are deciding according to the Nash Equilibrium [26]. The Stackelberg models extended to the dynamic Stackelberg games [27]. A survey in the application of Stackelberg differential games in supply, marketing and sales channels held by He et al. [28].

In a technical review work, the electricity generation and possible market models studied by Mariano et al. [29]. The aim of this research was to identify, classify and characterize different approaches in the field of electrical power market modeling. This study reviewed most recent articles and technical publications respect to the electrical power market models. According to the study results, there are three major trends in the field of power market modeling: optimization models, equilibrium problems and simulation methods. In a similar work, the examined approaches so far in the field of power market modeling using the Agent-Based Computational Economics (ACE) studied by Anke et al. [30]. This paper showed the state-of-the-art of the researches in the field. In the same context, Daniel J. Veit et al. [31] developed an agent based model to analyze the German electricity power market. This study analyzed the implications of transmission constraints on the power market of Germany. The implications of price and social welfares analyzed by considering the market behavior of major network players as well as the high wind power generators. In another research Simona et al. [32] formulated a supply chain marginal cost function for the Italian electricity market using the indirect measures. This study suggested a market model by considering the residual demand function and transmission line congestion.

Considering the renewable and sustainable energy areas, Allan et al. [33] provided an in depth review of the literature in the field. This study claimed that decentralization of the electricity resources is a mean for achieving an efficient renewable energy provision. Since decentralization of the electricity system recognized as one of the means to achieve an efficient and renewable energy provision hence this paper focused on a literature review of the economics of individual or groups of distributed energy generators. By considering the economic aspects of the distributed generation, this study provided a detailed review of the conducted researches in the field and offered most likely next steps for the interested researchers to the UK energy market. In a study in the context of integration of national electricity markets into a single European one, Olga [34] studied the electrical power market as a Stackelberg problem. According to this study, a market model with two decision layers presented. The first layer maximizes the profit of the generators companies while they play Cournot game against each other. The last layer determines the electrical flow in the network and maximizes the social welfare subject to a set of physical constraints. Since all major generator companies expected to have an impact on the equilibrium prices and network congestion, then an electric power market optimization model proposed by using the Stackelberg theory. This study suggested that although the line capacity increment is an applicable tool to inspire the competition, nevertheless it is not enough to reshape the large players in the electricity power market. The power market complexity and dynamic environment asks for novel and flexible modeling techniques and methods. These methods should enable decision makers to better understand the inter connections and system dynamics of a typical power market to be able to accurately decide the critical power market decisions when and where needed.

In the context of using Stackelberg game approach in the smart grid power management and demand response, Yu et al. [35] proposed a price-based demand response model to balance both supply and demand of a smart grid. The power market decisions modeled as a Stackelberg game. Same as the former study, this study suggested an iterative algorithm to define the optimal generation and demands quantities. In the other side, Katarzyna Maciejowskaa et al. [36] used factoring model and regression tool to define the forecast intervals in a typical power market, the studied British power market. This model extended a Quantile Regression

Averaging (QRA) approach by using principal component analysis. Guodong Liu et al. [37] proposed an optimal bidding strategy for the distributed generations (DGs) by considering the load of the storage devices and the responsive price. To minimize the model total cost, a hybrid stochastic-robust optimization algorithm proposed where a mixed-integer linear programming approach used to solve this problem. As a practical model, Quang Duy et al. [38] discussed the interactions between several smart houses in the context of the energy market. They proposed a dynamic pricing model using differential game theory to analyze the interactions between these players and to maximize their profit margins. Similar to the proposed model in this paper, Hongming Yang et al. [39] discussed a pricing scheme in which power consumers and distributors urged to response to the power market demand by considering the provided price plans. This model categorized customers in different groups and then proposed a nonlinear pricing model at the second stage. To reach the optimum order amounts, a bayesian discrete probability distribution function (PDF) proposed in this model.

Considering more advanced researches in the field of discounting models in the power market, Habib Allah Alami et al. [40] studied the impact of demand responses and the demand biddings on the power market price. To find an optimum price, a nonlinear model proposed in this model which considers both demand responses and demand bidding programs. Finally in a most recent study in the field of power market modeling using Stackelberg theory, Wei et el. [41] proposed a Multiple Energies Trading problem (MET). This study proposed a Multi-Leader Multi-Follower (MLMF) model based on the Stackelberg game theory and suggested a best response algorithm to obtain the Stackelberg Equilibrium in an iterative way.

Following the former studies which presented in the literature review, this paper discussed an optimal volume discount strategy between GenCo and DisCo, where the energy storage levels are continuously drained due to the regular market demand, the promo offers to the special group of customers and the technical loss of the electricity charges in the storage devices. Since the number of energy storage devices in the distribution layer is by far greater than its number in the generation layer, then the energy deterioration rate at the storage devices in DisCo is greater than the respected rate at GenCo. In this model, GenCo has a kind of purchase agreement with a number of renewable-energy resources as a set of micro-grids. To relax the total volume constraint at the GenCo layer, we assumed that GenCo has access to an unlimited volume of electricity with a contracted price through a number of limited intermittent energy storage recharges. GenCo recharges DisCo storage devices or the power network against a price proposal which offered to the chain of subsidiary DisCos. GenCo tries to maximize its profit margin by controlling DisCo order volumes by a discounted approach. At the same time DisCo tries to maximize its profit margin by putting its orders in-line with the proposed discounting frame which offered by GenCo. This model expressed as a Stackelberg game between the micro-grid GenCo and DisCo considering the renewable-energy concept as an artwork to reach an optimal discounting and pricing policy which maximizes both parties' profit margins per unit of time.

In this model we assumed that a dedicated GenCo exists for each market region. This GenCo uses certain energy storage devices (such as pomp storage) to reserve intermittent electricity loads from different renewable energy resources. Consequently, GenCo plays the role of a wholesaler which deals with several DisCos in that region. Depending on the electrical energy demand patterns, DisCo can either distribute the currency to its contracted end-users or can recharge its storage devices to be used in different time laps. Fig.1 illustrated the schematic concept of the proposed model. According to this model, DisCo enjoys from a discounted fares offered by GenCo. We assumed that DisCo can either accept or reject a discounting frame which proposed by GenCo. This frame can potentially encourage DisCo to align its power consumption behavior in terms of order volumes in-line with the GenCo preferences. Using such discounting frame will strengthens GenCos' control over the demand-side and is offered by considering the flow restriction of the intermittent renewable-energy resources round the clock. Although we considered DisCo as part of the demand-side that is in contact with the end-users directly, nevertheless the relationship between DisCo and end-users and the possible sales tools and tactics ignored in this paper. The next session orderly present the notation used, model formulation, numerical analysis. Finally the conclusion highlights potential further studies following this work.

Fig. 1 – Schematic concept of the proposed model

2 Notation and Assumptions

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Mathematical Equations must be numbered as follows: (1), (2), …, (99) and not (1.1), (1.2),…, (2.1), (2.2),… depending on your various Sections.

In this model GenCo assumed to use a volume discount strategy to manage demand-side consumption behavior. GenCo proposes electrical energy volumes together with a possible discounting proposal to encourage DisCo to align its power consumption behavior in terms of order volumes in-line with the GenCo preferences. Below are two major scenarios that studied in this paper.

2.1 Scenario A – Demand-side is not controllable; DisCo orders based on EOQ

Under this scenario, for what so ever reason, DisCo cannot make its orders in-line with the proposed discounting frame. DisCo mainly is not capable to steer business based on GenCos' proposal and therefore prefers to order electricity against market price and according to a volume which is defined by the EOQ model. Since DisCo preference is to have adhoc orders against market price, then it cannot theoretically reach any kind of service level agreement and promotional offers from GenCo to push an special power volume in the market for sales.

2.2 Scenario B – Demand-side is controllable; DisCo orders based on GenCos' offer

In this scenario DisCo uses the proposed discount offer which offered by GenCo. Respectively DisCo has certain service level agreement with a nominated GenCo which enables him to sell special prices to the power market by using a discount grid based on GenCo discount proposal.

2.3 Notation

Following notation used to formulate the model:

q_i Volume of DisCo power order each time from a GenCo under scenario A and B

O_i Volume of GenCo power order each time from a micro-grid under scenario A and B

T_i Length of DisCo order cycle under strategy A and B

ts_c GenCo storage associated cost per unit of time

ts_a DisCo storage associated cost per unit of time

sc_c GenCo order cost per power load request from the micro-grid

sc_a DisCo order cost per power load request from the GenCo

p_c GenCo unit acquisition price from the micro-grid

p_a GenCo unit acquisition sales price to DisCo without discount, e.g. DisCo unit acquisition price from GenCo

ξ_c Discount rate based on GenCo proposal; GenCo offers discounted fare (1- ξ_c) p_a while ($0 \le \xi_c < 1$)

p_p DisCo selling price

θ_c Power deterioration rate at the GenCos storage devices

θ_a Power deterioration rate at the DisCo storage devices

μ Power market natural demand rate

$S^{(a)}(t)$ DisCo power storage devices' electricity recharge level

$S^{(c)}(t)$ GenCo power storage devices' electricity recharge level

$\alpha(T_1)$ Cumulative stored electricity power in DisCo storage devises

$\beta(T_1)$ Cumulative stored electricity power in GenCo storage devises

N_i Number of electricity load transmissions from GenCo to DisCo

2.4 Model Assumptions

The following assumptions considered in this model formulation,

1) GenCo and DisCo power reserves in the storage devices drains continuously over the time due to the continuous power market demand
2) Power recharge capacity from micro-grid to GenCo considered infinite without any time delay to relax respective load constraint here
3) Power volumes assumed to be continuous
4) Power shortage (black-out) is not permitted
5) Both GenCo and DisCo deciding purely based on a rational model
6) GenCo order cycle is given by $N_i\, T_i; N_i$ is a positive number.

3 DisCo Total Profit Margin

In this section DisCos total profit margin under two different scenarios formulated.

3.1 DisCo Profit Margin under Scenario A

If DisCo selects scenario A, its power load order volume each time and respected acquisition cost are given by $q_1 = q\ (T_1)$ and p_a, where p_a is per load acquisition cost without existence of GenCo discount. In this situation DisCo determines the optimal volume $q_1 = q_1^*$ which maximizes its profit margin per unit of time by using an EOQ model. Because of the market demand, the special offers from the DisCo to its loyal customers and the technical power loss in the storage devices, the power storage electricity level $S^{(a)}(t)$ at the time t in $[0, T_1\ )$ is equal to the following equation,

$$dS^{(a)}(t)/dt = -\theta_a S^{(a)}(t) - \mu \qquad (1)$$

Equation (1) respect to $S^{(a)}(T_1) = 0$ gives power storage level at time t as below,

$$S^{(a)}(t) = \lambda[e^{\theta_a(T_1 - t)} - 1] \qquad (2)$$

Where $\lambda = \mu / \theta_a$

Therefore, $S^{(a)}(0)(= q_1 = q(T_1))$ in order cycle is

$$q(T_1) = \lambda(e^{\theta_a T_1} - 1) \qquad (3)$$

And the cumulative stored electricity power in DisCo storage devises, $\alpha(T_1)$, for period $[0, T_1)$ is

$$\alpha(T_1) = \int_0^{T_1} S^{(a)}(t)dt = \chi\ [\frac{(e^{\theta_a T_1} - 1)}{\theta_a} - 1] \qquad (4)$$

DisCo profit margin per unit of time under scenario A is

$$R_1(T_1) = \frac{p_p \int_0^{T_1} \mu dt - p_a q(T_1) - ts_a \alpha(T_1) - sc_a}{T_1} =$$

$$\lambda(p_p\ \theta_a + ts_a) - \frac{(p_a + \frac{ts_a}{\theta_a})q(T_1) + sc_a}{T_1} \qquad (5)$$

Fig. 2 – GenCo (A) and DisCo (B) Electricity Power Storage Behavior while $N_i = 3$.

According to Nash equilibriums [8] [9], a unique $T_1 = T_1^*$ (>0, Finite) exists that maximizes $R_1(T_1)$ in equation (5) therefore optimal order quantity for DisCo is

$$q_1^* = \lambda(e^{\theta_a T_1^*} - 1) \qquad (6)$$

And respectively, the total profit margin per unit of time is

$$R_1(T_1^*) = \lambda[(p_p \theta_a + ts_a) - \theta_a(p_p + \frac{ts_a}{\theta_a})e^{\theta_a T_1^*}] \qquad (7)$$

3.2 DisCo Profit Margin under Scenario B

If DisCo selects scenario B, the power recharge volume per order and the discounted electricity price are $q_2 = q_2(T_2) = \lambda(e^{\theta_a T_2} - 1)$ and $(1-\xi)p_a$ respectively. Hence the profit function will be as the following,

$$R_2(T_2, \xi) = \lambda(p_p \theta_a + ts_a) - ([(1-\xi)p_a + \frac{ts_a}{\theta_a}]$$
$$q_2(T_2) + sc_a)/T_2 \qquad (8)$$

4 GenCo Total Profit Margin

In this section GenCo total profit margin per unit of time formulated. The presented formulation is based on DisCo decision respect to the introduced scenarios A and B. Fig. 2 shows the electricity storage behavior in the storage devices over the time while $N_i = 3$.

4.1 GenCo Profit Margin under Scenario A

If GenCo uses strategy A, its storage load volume per time and the unit acquisition cost are respectively q_1 and p_a. The length of GenCo order cycle from the respected micro-grid can be divided to N_i recharge cycles (N_1=1,2,…) as earlier explained in the assumption number 6. Based on this assumption, Ni is a decision variable for GenCo. The GenCo storage volume is drained due to deterioration effect during $[(j-1)T_1, jT_1)$ in the j^{th} electricity recharge cycle (j=1,2,…,N_1). Therefore GenCo power storage level at time t is

$$dS^{(c)}(t)/dt = -\theta_c S^{(c)}(t) \qquad (9)$$

Where $S^{(c)}(jT_1) = \varepsilon_j(T_1)$ and $\varepsilon_j(T_1)$ shows the remaining electricity volume in GenCo storage devices at the end of the j^{th} recharge cycle. Based on equation 9, GenCo storage level at time t in the j^{th} recharge is

$$S_j^{(c)}(t) = \varepsilon_j(T_1)e^{\theta_c(jT_1 - t)} \qquad (10)$$

The currency load level at the end of the $(N_1-1)^{th}$ recharge cycle is equal to q_1, i.e. $\varepsilon_{N_1-1}(T_1)=q_1$ as it is shown in Fig. 1.

$$\varepsilon_j(T_1)=\frac{q(T_1)[e^{\theta_c(N_1-j)T_1}-1]}{[e^{\theta_c T_1}-1]} \tag{11}$$

GenCo order volume from the available micro-grid, $O_1=O(N_1,T_1)\ (=\varepsilon_0(T_1))$, per order is equal to

$$O(N_1,T_1)=\frac{q(T_1)[e^{\theta_c N_1 T_1}-1]}{[e^{\theta_c T_1}-1]} \tag{12}$$

And the cumulative stored electricity power in GenCo storage devises which is held during the j^{th} power recharge cycle, $\beta_j(T_1)$, is

$$\beta_j(T_1)=\int_{(j-1)T_1}^{jT_1} S^{(c)}(t)dt=\frac{q(T_1)}{\theta_c}[e^{\theta_c(Ni-j)T_1}-1] \tag{13}$$

Therefore GenCo cumulative power storage on $[0,\ N_1T_1)$ is

$$\beta(N_1,T_1)=\sum_{j=1}^{N_1-1}\beta_j(T_1)= \\ \frac{q_1(T_1)}{\theta_c}\left(\frac{e^{\theta_c N_1 T_1}-1}{e^{\theta_c T_1}-1}-N_1\right) \tag{14}$$

And for a specific N_1, the GenCo total profit margin per unit of time under scenario A is

$$\bar{R}_1(N_1,T^*{}_1)=\frac{1}{N_1T^*{}_1}.[p_aN_1q(T^*{}_1)-p_cO(N_1,T^*{}_1) \\ -ts_c\beta(N_1,T^*{}_1)-sc_c] \tag{15}$$

4.2 GenCo Profit Margin under Scenario B

When DisCo uses scenario B, it will purchase $q_2=q(T_2)$ volume of electricity power against the discounted price $(1-\xi)p_a$ from GenCo. The GenCo electrical power volume per order under scenario B is $O_2=O(N_2,T_2)$ and respectively GenCo total profit margin per unit of time under scenario B is

$$\bar{R}_2(N_2,T_2,\xi)=\frac{1}{N_2T_2}.[(1-\xi)p_aN_2q(T_2)- \\ p_cO(N_2,T_2)-ts_c\beta(N_2,T_2)-sc_c] \tag{16}$$

Where

$$q(T_2)=\lambda(e^{\theta_a T_2}-1), \tag{17}$$

$$O(N_2,T_2)=\frac{q(T_2)[e^{\theta_c N_2 T_2}-1]}{[e^{\theta_c T_2}-1]} \tag{18}$$

5 The Optimal Decision for DisCo

This section analyzes DisCo optimal decision. DisCo prefers the scenario A rather than B if $R^*{}_1>\bar{R}_2(T_2,\xi)$. If $R^*{}_1<\bar{R}_2(T_2,\xi)$ one will prefer the strategy B to A. There is no priority for DisCo when $R^*{}_1=\bar{R}_2(T_2,\xi)$ which

$$\xi=\frac{(p_a+\frac{ts_a}{\theta_a})[q\ (T_2)-\lambda\theta_aT_2e^{\theta_aT_1^*}]+sc_a}{p_aq\ (T_2)} \tag{19}$$

We introduce ψ for right hand side of equation 19. Base on this equation we can show that ψ is increasing when $(T_2\geq T^*{}_1)$. (Figure 3)

Fig. 3 – GenCo Optimal Decision

6 The Optimal Decision for GenCo

GenCo optimal quantities for T_2 and ξ can be reached by maximizing total profit margin respect to DisCo decisions under the discussed scenarios. Let define $\pi_i(i=1,2)$ as the following,

$\pi_1=\{(T_2,\xi)|\xi\leq\psi(T_2)\}$,

$\pi_2=\{(T_2,\xi)|\xi\geq\psi(T_2)\}$

Fig. 3 shows the region $\pi_i(i=1,2)$ on (T_2,ξ).

6.1 GenCo Optimal Decision under Scenario A

If $(T_2,\xi)\in\pi_1\setminus\pi_2$ in Fig. 2, then DisCo selects scenario A and can maximize its profit margin per unit of time based on the mentioned condition. GenCo total profit margin in this case becomes

$$\bar{R}_1^*=\max_{N_1\in N}\bar{R}_1(N_1,T^*{}_1),\forall N\rightarrow Int. \tag{20}$$

6.2 GenCo Optimal Decision under Scenario B

If $(T_2,\xi)\in\pi_2\setminus\pi_1$, then DisCo optimal decision is to select scenario B. GenCo total profit margin in this case is

$$\bar{R}_2^{*}=\max_{N_2\in N}\bar{P}_2(N_2) \qquad (21)$$

Where

$$\bar{P}_2(N_2)=\max\bar{R}_2(N_2,T_2,\xi), \quad \forall(T_2,\xi)\in\pi_2\setminus\pi_1 \qquad (22)$$

For a specific N_2 we can show existence of an optimal discounting policy for a typical GenCo which satisfies the equation 22 [8][9]. It can be proven that $\bar{R}_2(N_2,T_2,\xi)$ in the equation 16 is decreasing on ξ and respectively the GenCo can reach $\bar{R}_2(N_2)$ in equation 22 if $\xi\to\psi(T_2)$. If $\xi=\psi(T_2)$ total profit margin per unit of time is as below,

$$\bar{R}_2(N_2,T_2)=\lambda(p_a+\frac{ts_a}{\theta_a})\theta_a e^{\theta_a T^{*}{}_1}-\frac{1}{N_2T_2}.[\gamma.O(N_2,T_2)-TS(N_2)q(T_2)+sc_a(N_2)+sc_c] \qquad (23)$$

Where

$$\gamma=(p_c+\frac{ts_c}{\theta_c}) \qquad (24)$$

And

$$TS(N_2)=(\frac{ts_c}{\theta_c}-\frac{ts_a}{\theta_a})N_2 \qquad (25)$$

Let to define $\sigma(T_2)$ as below, (26)

$$\sigma(T_2)\equiv[\gamma.\theta_c.T_2.q(T_2).((N_2e^{N_2\theta_cT_2}(e^{\theta_cT_2}-1)) -e^{\theta_cT_2}(e^{N_2\theta_cT_2}-1))/(e^{\theta_cT_2}-1)^2] +[\lambda\theta_a e^{\theta_aT_2}T_2-q(T_2)].[\gamma.\frac{e^{N_2\theta_cT_2}-1}{e^{\theta_2T_2}-1}-TS(N_2)]$$

Here we summarize our discussed analyzes respect to the optimal number of N_2 which reach equation (22) optimal amount.

If $N_2=1$ and $(p_c+\frac{ts_a}{\theta_a})>0$, then equation (23) will be maximized by a unique $(\bar{T}_2>T_1^{*})$ therefore,

$$(T_2^{*},\xi^{*})\to(\bar{T}_2,\bar{\xi}) \qquad (27)$$

Where

$$\bar{\xi}=\psi(\bar{T}_2)$$

Therefore GenCo total profit margin is

$$\bar{R}_2(N_2)=\lambda\theta_a[(p_a+\frac{ts_a}{\theta_a})e^{\theta_aT^{*}{}_1}-(p_c+\frac{ts_a}{\theta_a})e^{\theta_aT^{*}{}_2}] \qquad (28)$$

From the other side, if $N_2=1$ and $(p_c+\frac{ts_a}{\theta_a})\le 0$, then the optimal strategy is as below,

$$(T_2^{*},\xi^{*})\to(\bar{T}_2,1) \qquad (29)$$

Where $(\bar{T}_2>T_1^{*})$ is a unique finite and positive solution for $\psi(T_2)=1$ and respectively GenCo total profit margin is

$$\bar{R}_2(N_2)=-\frac{p_2q(\bar{T}_2)}{\bar{T}_2}-sc_c \qquad (30)$$

And finally if $N_2\ge 2$, then we define $\bar{T}_2=T_2$ (which $\bar{T}_2>T_1^{*}$) as a unique optimal solution and then

$$\sigma(T_2)=sc_aN_2+sc_c$$

While the optimal solution for this case is based on equation (27).

6.3 GenCo Optimal Decision under the Scenarios A and B together

If $(T_2,\xi)\in\pi_1\cap\pi_2$, there is no priority between two scenarios. Based on this reason, discussed study is limited to the situations in which GenCo does not propose any discounting frame in $(T_2,\xi)\in\pi_1\cap\pi_2$.

6.4 GenCo Optimal Number of N_i

This section discusses lower and upper limits for the optimal numbrt of N_i^* which satisfies $R_i^*(i=1,2)$ in equations 20 and 21. Consider $\rho(T_i^*)$ as

$$\rho(T_i^*) \equiv \frac{(p_c + \frac{ts_c}{\theta_c})q(T_i^*)}{(e^{\theta_c T^*_i} - 1)} \qquad (32)$$

Respected analysis summarized here. For the Lower Limit of the number of N_i, replace N_i with $N_i^{(L)}(T_i^*)$ where $N_i^{(L)}$ assumed as the lower limit for N_i and is smaller or equal than the optimal N_i^* amount. Therefore two situations are expected,

1) $\{(e^{\theta_c T^*_i} - 1)^2 \geq \frac{sc_c}{\rho(T_i^*)}\} \Rightarrow$ In this situation $N_i^{(L)}(T_i^*) = 1$
2) $\{(e^{\theta_c T^*_i} - 1)^2 < \frac{sc_c}{\rho(T_i^*)}\} \Rightarrow$ In this situation a unique finite $N_i^{(L)}$ exists that is equal or greater than 1 and therefore

$$N_i e^{N_i \theta_c T^*_i}(e^{\theta_c T^*_i} - 1) - (e^{N_i \theta_c T^*_i} - 1) = \frac{sc_c}{\rho(T_i^*)} \qquad (33)$$

Respectively, for the Upper Limit of the number of N_i, replace N_i with $N_i^{(U)}(T_i^*)$ where $N_i^{(U)}$ assumed as the upper limit for N_i and is greater or equal than the optimal N_i^* amount. Therefore a final unique $N_i^{(U)}$ exists that is the solution for equation (34) as below,

$$N_i e^{(N_i - 1)\theta_c T^*_i}(e^{\theta_c T^*_i} - 1) - (e^{N_i \theta_c T^*_i} - 1) = \frac{sc_c}{\rho(T_i^*)} \qquad (34)$$

Table 1 – Fixed and variable parameters

7 Numerical Analysis

The presented model credibility examined by using a numerical simulation and analysis. For this reason Lingo software used and in absence of the real world data, we replaced model fixed parameters according to the virtual figures which presented in Tabel 1. The fixed parameters' values obtained by using expert knowledges in the field and do not necessarily matches with the real world power market business cases. After the model simulation, a sensitivity analysis conducted respect to the stated possible scenarios. Sensitivity analysis results under both scenarios A and B respect to the available model parameters to reach the $q_i^*, p_a, O_1,^*$ $N_1^*, \overline{R}_1^*, q_2^*, p_2^*, O_2^*, N_2^*$ and $\overline{R}_2^*$ shown in Table 2 in the following.

Table 2 – Sensitivity Analysis Results

As it is shown in Table 2, O_1^* and N_1^* are non-decreasing in sc_c. As discussed earlier in scenario A, DisCo is not changing the order quantities according to the discounting proposal which offered by GenCo. In this case, GenCo cannot hold any control over the DisCo order behavior. Therefore GenCo should have a greater electricity storage volume to be used in the market rate. From the other side, GenCo should extend the power acquisition cycles lengths from the renewable energy micro-grids. This decision will help GenCo to maintain the storage volume acquisition costs as much as possible in a low level. According to Table 2, in scenario B, O_2^* increases in sc_c against a fixed value for N_2^*. In this scenario, DisCo will sell the electricity power according to the available discounting frame offered by GenCo. If GenCo increases N_2, it's the electricity recharge volume per order will increase as well. Therefor in this scenario, GenCo should increase the electricity recharge volume from the renewable energy micro-grid rather than raising the number of power acquisition rounds N_2, specially once sc_c takes a greater amount. Finally, the simulation results verified model credibility to be used as a supportive tool for power market decision making problems respect to the economic measures.

8 Conclusion

The Quantity Discount Strategy is one of the mostly used methods by the sellers to reduce fixed ordering, shipment and material handling associated costs. In inventory management, the Economic Order Quantity (EOQ) is the order amount which minimizes the holding and ordering costs. The optimal order quantity under EOQ is one of the oldest models in the field of industrial engineering and production planning. From the other side, the Stackelberg model brought from the economics and management science to the power market literature and in the resent years, it is widely used in electricity market studies. In this paper a volume discounting model in the micro-grid power market presented. The Generator Company (GenCo) discounting model and the special fares for Distribution Companies (DisCo) formulated respectively. This model assumed that a GenCo has specific deal with its partner DisCo for certain power consumption levels. The concept of intermittent electricity load transmissions generated by a set of renewable energy resources, to the certain storage devices in distribution level presented as a novel contribution to the field. In this model we assumed that a dedicated GenCo exists for each market region. This GenCo uses certain energy storage devices (such as pomp storage) to reserve intermittent electricity loads from different renewable energy resources. Consequently, GenCo plays the role of a wholesaler and deals with several DisCos in a region. Depending on the electrical energy demand patterns, DisCo can either distribute the electricity to the contracted end-users or can recharge the storage devices to use the energy in different time laps. According to this model, DisCo enjoys of a discounted fares which is offered by GenCo. We assumed that DisCo can either accept or reject a discount proposal which is offered by GenCo. This proposal can potentially encourage DisCo to align its power consumption behavior in terms of order volumes in-line with GenCo preferences. Using such discounting frame will strengthens GenCo to hold better control over the demand-side. The discounting proposal structured by considering the flow restrictions of the intermittent renewable-energy resources round the clock. Although we considered DisCo as part of the demand-side, nevertheless the relationship between DisCo and power market ignored in this model. This model expressed as a Stackelberg game between GenCo, including the renewable energy micro-grids, and DisCo as a retailer. This model structured by considering the expected behavior of the renewable-energy resources as an artwork to reach an optimal discounting and pricing policy which maximizes GenCo and DisCo profit margins per unit of time and under two different scenarios. The presented model credibility examined by using numerical simulation and showing a sensitivity analysis. For this reason Lingo software used and in absence of the real world data, we replaced model fixed parameters with a set of virtual figures. Finally, simulation results verified the model credibility to be used as a supportive tool for the power market decision making problems respect to the economic measures. Developing more heuristic models to optimize power market reliability, considering network failures, incorporating the emergency loads and developing the power market behavior under different circumstances are potential next steps which identified by the authors.

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