An Invariant Model of Boltzmann Statistical Mechanics and Some of its Implications to Thermodynamics and Quantum Nature of Space and Time

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Abstract: - Further implications of a scale invariant model of Boltzmann statistical mechanics to generalized thermodynamics and quantum nature of space and time are described. Scale invariant definition of absolute thermodynamic temperature identified as Wien wavelength of thermal oscillations is applied to introduce *internal dependent measures* of space and time called spacetime. The internal spacetime measures $(\lambda_{w\beta-1}, \tau_{w\beta-1})$ allow for definition of *external independent measures* of space and time coordinates (x_β, t_β) for the description of system dynamics. Because of its *hyperbolic geometry*, its discrete or quantum fabric, and its stochastic atomic motions, physical space is called *Lobachevsky-Poincaré-Dirac-Space*.

Key-Words: - Generalized thermodynamics; quantum nature of space and time; spacetime, relativity; TOE.

1 Introduction

The true universality of Boltzmann statistical mechanics is revealed for systems composed of very large numbers of weakly interacting particles considered as ideal gas. The similarities between stochastic quantum fields [1-17] and classical hydrodynamic fields [18-30] resulted in recent introduction of a scale-invariant model of statistical mechanics [31], and its application to thermodynamics [32-34] and fluid mechanics [35, 36] and quantum mechanics [37, 38].

Recently, some of the implications of a invariant model of statistical mechanics to relativistic [33] and classical [34] thermodynamics were investigated. The present study focuses on further implications of the model to the physical foundation and quantum nature of both space and time. Thermodynamic temperature is shown to provide interdependent *internal measures* of a manifold called *Lobachevsky-Poincaré-Dirac-Space*. Compressibility of such a space leads to Poincaré-Lorentz *dynamic* as opposed to Einstein *kinematic* theory of relativity [39].

2 A Scale Invariant Model of Statistical Mechanics

The scale-invariant model of statistical mechanics for equilibrium galactic-, planetary-, hydro-system-, fluid-element-, eddy-, cluster-, molecular-, atomic-, subatomic-, kromo-, and tachyon-dynamics at the scale β = g, p, h, f, e, c, m, a, s, k, and t is schematically shown on the left hand side and the corresponding laminar flows on the right hand side of Fig. 1.

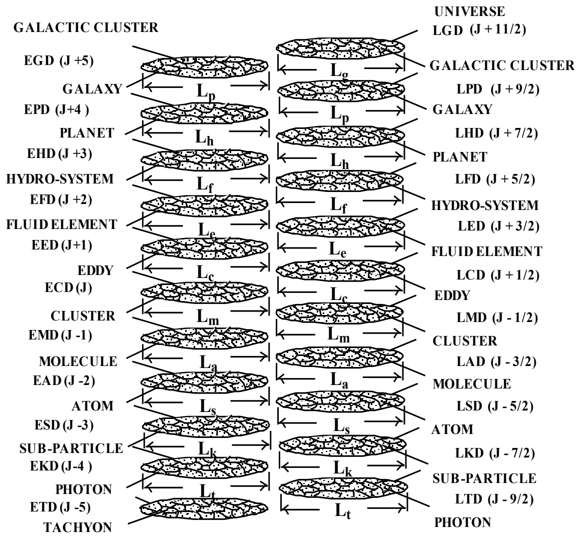

Fig. 1 Scale invariant model of statistical mechanics. Equilibrium–β–Dynamics on the left-hand-side and non-equilibrium Laminar–β–Dynamics on the right-hand-side for scales β = g, p, h, f, e, c, m, a, s, k, and t as defined in Section 2. Characteristic lengths of (system, element, "atom") are $(L_\beta, \lambda_\beta, \ell_\beta)$ and λ_β is the mean-free-path [38].

For each statistical field, one defines particles that form the background fluid and are viewed as point-mass or "*atom*" of the field. Next, the *elements* of the field are defined as finite-sized composite entities composed of an ensemble of "*atoms*" as shown in Fig. 1. Finally, the ensemble of a large number of "*elements*" is defined as the statistical "*system*" at that particular scale.

Following the classical methods [19, 40-44] the invariant definitions of density ρ_β, and velocity of *atom* $\mathbf{u}_\beta$, *element* $\mathbf{v}_\beta$, and *system* $\mathbf{w}_\beta$ at the scale β are given as [38]

$$\rho_\beta = n_\beta m_\beta = m_\beta \int f_\beta du_\beta \quad , \quad \mathbf{u}_\beta = \mathbf{v}_{w\beta-1} \tag{1}$$

$$\mathbf{v}_\beta = \rho_\beta^{-1} m_\beta \int \mathbf{u}_\beta f_\beta d\mathbf{u}_\beta \quad , \quad \mathbf{w}_\beta = \mathbf{v}_{w\beta+1} \tag{2}$$

Similarly, the invariant definition of the peculiar and diffusion velocities are introduced as

$$\mathbf{V}'_\beta = \mathbf{u}_\beta - \mathbf{v}_\beta \quad , \quad \mathbf{V}_\beta = \mathbf{v}_\beta - \mathbf{w}_\beta \tag{3}$$

such that

$$\mathbf{V}_\beta = \mathbf{V}'_{\beta+1} \tag{4}$$

The most probable element of the lower scale β is identified as the "atom" of the next higher scale $\beta + 1$ such that $\mathbf{v}_{mp\beta} = \mathbf{u}_{\beta+1}$, resulting in the hierarchy of embedded statistical fields schematically shown in Fig. 2.

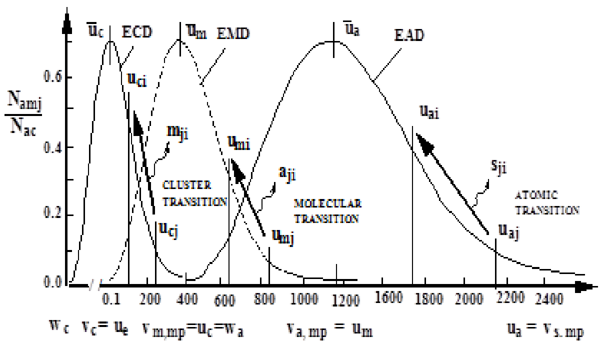

Fig. 2 Maxwell-Boltzmann distributions for ECD, EMD, and EAD scales at 300 K [38].

Following the classical methods [19, 40-44], the scale-invariant forms of mass, thermal energy, linear and angular momentum conservation equations at scale β are given as [38]

$$\frac{\partial \rho_{i\beta}}{\partial t_\beta} + \nabla.\left(\rho_{i\beta}\mathbf{v}_{i\beta}\right) = \Omega_{i\beta} \tag{5}$$

$$\frac{\partial \varepsilon_{i\beta}}{\partial t_\beta} + \nabla.\left(\varepsilon_{i\beta}\mathbf{v}_{i\beta}\right) = 0 \tag{6}$$

$$\frac{\partial \mathbf{p}_{i\beta}}{\partial t_\beta} + \nabla.\left(\mathbf{p}_{i\beta}\mathbf{v}_{j\beta}\right) = -\nabla.\mathbf{P}_{ij\beta} \tag{7}$$

$$\frac{\partial \boldsymbol{\pi}_{i\beta}}{\partial t_\beta} + \nabla.\left(\boldsymbol{\pi}_{i\beta}\mathbf{v}_{j\beta}\right) = \rho_{i\beta}\boldsymbol{\omega}_\beta .\nabla\mathbf{v}_{i\beta} \tag{8}$$

that involve the *volumetric density* of thermal energy $\varepsilon_{i\beta} = \rho_{i\beta}\tilde{h}_{i\beta}$, linear momentum $\mathbf{p}_{i\beta} = \rho_{i\beta}\mathbf{v}_{i\beta}$, and angular momentum $\boldsymbol{\pi}_{i\beta} = \rho_{i\beta}\boldsymbol{\omega}_{i\beta}$, $\boldsymbol{\omega}_{i\beta} = \nabla \times \mathbf{v}_{i\beta}$. Also, $\Omega_{i\beta}$ is the chemical reaction rate and $\tilde{h}_{i\beta}$ is the absolute enthalpy [38].

At thermodynamic equilibrium the velocity, energy, and speed of "particles" or Heisenberg-Kramers [45] virtual oscillators are expected to be governed by Gaussian (Boltzmann), Planck, and Maxwell-Boltzmann distribution functions [38]

$$f_\beta(v_\beta) = \left(\frac{m_\beta}{2\pi k T_\beta}\right)^{3/2} e^{-m_\beta u_\beta^2/2kT_\beta} \tag{9}$$

$$\frac{\varepsilon_\beta dN_\beta}{V_\beta} = \frac{8\pi h}{u_\beta^3} \frac{\nu_\beta^3}{e^{h\nu_\beta/kT} - 1} d\nu_\beta \tag{10}$$

$$\frac{dN_{u\beta}}{N} = 4\pi \left(\frac{m_\beta}{2\pi k T_\beta}\right)^{3/2} u_\beta^2 e^{-m_\beta u_\beta^2/2kT_\beta} du_\beta \tag{11}$$

The distributions (9)-(11) at thermodynamic equilibrium naturally imply a generalized quantum thermodynamics [38] corresponding to the hierarchies of statistical fields shown in Fig. 1.

For the description of invariant model of statistical mechanics (1)-(4) let us start with the field of laminar molecular dynamics LMD when molecules, clusters of molecules (cluster), and cluster of clusters of molecules (eddy) form the "atom", the "element", and the "system" with the velocities $(\mathbf{u}_m, \mathbf{v}_m, \mathbf{w}_m)$. Similarly, the fields of laminar cluster-dynamics LCD and eddy-dynamics LED will have the velocities $(\mathbf{u}_c, \mathbf{v}_c, \mathbf{w}_c)$, and $(\mathbf{u}_e, \mathbf{v}_e, \mathbf{w}_e)$ in accordance with Eqs. (1-2). For the fields of LED, LCD, and LMD, typical characteristic "atom", element, and system lengths are

$$\text{EED} \quad (\ell_e,\ \lambda_e, L_e) = (10^{-5}, 10^{-3},\ 10^{-1})\ \text{m} \tag{12a}$$
$$\text{ECD} \quad (\ell_c,\ \lambda_c, L_c) = (10^{-7}, 10^{-5},\ 10^{-3})\ \text{m} \tag{12b}$$
$$\text{EMD} \quad (\ell_m,\ \lambda_m, L_m) = (10^{-9}, 10^{-7},\ 10^{-5})\ \text{m} \tag{12c}$$

where $\ell_\beta = 0_\beta = \lambda_{\beta\text{-}1}$ is atomic size, $\lambda_\beta = L_{\beta\text{-}1}$ is the mean free path of atoms or element size, and $L_\beta = \lambda_{\beta+1}$ is the system size. The definitions of lengths $(L_\beta, \lambda_\beta, \ell_\beta)$ in Eq. (12) and velocities $(\mathbf{w}_\beta, \mathbf{v}_\beta, \mathbf{u}_\beta)$ in Eqs. (1-2) result in the following definitions of atomic, element, and system "times" $(t_\beta, \tau_\beta, \Theta_\beta)$ for the statistical field at scale β [37]

$$t_\beta = \ell_\beta / u_\beta = \tau_{\beta\text{-}1} \tag{13a}$$
$$\tau_\beta = \lambda_\beta / v_\beta = t_{\beta+1} \tag{13b}$$
$$\Theta_\beta = L_\beta / w_\beta = \tau_{\beta+1} \tag{13c}$$

If one applies the same (atom, element, system) = $(\ell_\beta,\ \lambda_\beta, L_\beta)$ relative sizes in Eq. (12) to the entire spatial scale of Fig. 1, the resulting cascades or hierarchy of overlapping statistical fields will appear as schematically shown in Fig. 3.

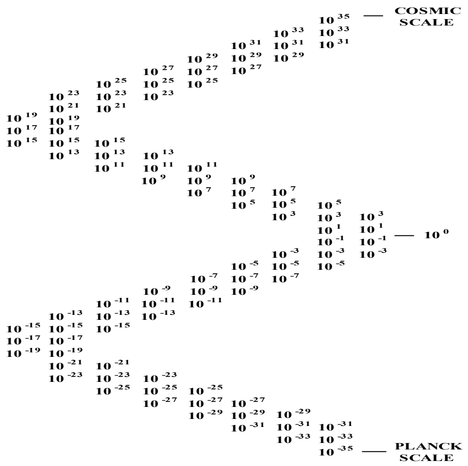

Fig. 3 Hierarchy of statistical fields with $(\ell_\beta,\ \lambda_\beta, L_\beta)$ from cosmic to Planck scales [38].

According to Fig. 3, exactly seven generations of statistical fields and a factor of 10^{-17} separates statistical fields of galactic-dynamics (cosmology), astrophysics, hydrodynamics, electrodynamics, and finally chromodynamics at Planck length scale $(\hbar G / c^3)^{1/2} \simeq 10^{-35}\,\text{m}$, where G is the gravitational constant [38].

3 Stochastic Definitions of Planck and Boltzmann Constants Related to Spatio-Temporal Aspects of Casimir Vacuum Fluctuations

Because at thermodynamic equilibrium the mean particle velocity must vanish $\langle \mathbf{u}_\beta \rangle = 0$, particle energy $\varepsilon_\beta = m_\beta \langle u_\beta^2 \rangle = \bar{p}_\beta\ \langle \lambda_\beta^2 \rangle^{1/2} \langle \nu_\beta^2 \rangle^{1/2}$ can be expressed in terms of either frequency $\varepsilon_\beta = h_\beta \langle \nu_\beta^2 \rangle^{1/2}$ or wavelength $\varepsilon_\beta = k_\beta \langle \lambda_\beta^2 \rangle^{1/2}$ when stochastic (Planck, Boltzmann) factors are defined as ($h_\beta = \bar{p}_\beta \langle \lambda_\beta^2 \rangle^{1/2}$, $k_\beta = \bar{p}_\beta \langle \nu_\beta^2 \rangle^{1/2}$) and $\bar{p}_\beta = m_\beta \langle u_\beta^2 \rangle^{1/2}$ is the *root-mean-square momentum* of particle [34]. Particles are considered to have harmonic translational motions in two spatial directions (x+, x-) such that $\varepsilon_\beta = m_\beta \langle u_{\beta x+}^2 \rangle / 2 + m_\beta \langle u_{\beta x+}^2 \rangle / 2 = m_\beta \langle u_{\beta x+}^2 \rangle$.

The important scale of equilibrium chromodynamics EKD β = k (Fig. 1) is identified as *physical space* or, Aristotle *fifth element*, Casimir *vacuum* [46], Dirac *stochastic ether* [47] or de Broglie *hidden thermostat* [3]. Naturally, Planck and Boltzmann factors are identified as the stochastic definitions of Planck and Boltzmann universal constants associated with spatial $\langle \lambda_\beta^2 \rangle^{1/2}$ and temporal $\langle \nu_\beta^2 \rangle^{1/2}$ aspects of Casimir [47] vacuum fluctuations [34].

$$\begin{aligned} h &= h_k = m_k c \lambda_{rk} = 6.626 \times 10^{-34}\ [\text{J}-\text{s}] \\ k &= k_k = m_k c \nu_{rk} = 1.381 \times 10^{-23}\ [\text{J}/\text{m}] \end{aligned} \tag{14a}$$

that lead to *spatial* and *temporal* uncertainty principles [34]

$$\begin{aligned} \Delta\lambda_{r\beta} \Delta\bar{p}_{r\beta} &\geq h \\ \Delta\nu_{r\beta} \Delta\bar{p}_{r\beta} &\geq k \end{aligned} \tag{14b}$$

Because of definition $\bar{p}_k = \bar{p}_{rmsk} = m_k v_{rk} = m_k c$, stochastic Planck and Boltzmann constants (h, k) are respectively defined in terms of *root-mean-square* wavelength and frequency $(\lambda_{rmsk}, \nu_{rmsk}) = (\lambda_{rk}, \nu_{rk})$ and $\lambda_{rk}\nu_{rk} = c$ where c is the speed of light in vacuum [34] such that

$$\begin{aligned} \varepsilon_{rk} &= h\nu_{rk} = m_k v_{rk}^2 = m_k c^2 \\ \varepsilon_{rk} &= k\lambda_{rk} = m_k v_{rk}^2 = m_k c^2 \end{aligned} \tag{15}$$

Hence, the *propagation speed of light wave* is identified as the *root-mean-square speed of photons* in Casimir vacuum [46] at EKD scale β = k. Although the velocity of light appears to be nearly constant it actually decreases extremely slowly at cosmological time scale (i.e. eons) with the temperature of vacuum due to expansion of the universe [38].

In view of recent [38] closure of the gap between radiation and gas theory Planck [48] law of equilibrium radiation could be viewed as energy spectrum *of clusters of photons*, Sackur's "clusters", or Planck's "quantum sphere of action" as described by Darrigol [49,50] in harmony with the perceptions of de Broglie [38,51,52]

"Existence of conglomerations of atoms of light whose movements are not independent but coherent"

with cluster sizes given by Maxwell-Boltzmann law of speed distribution [38]. Direct derivation of Maxwell-Boltzmann distribution function from Planck distribution was recently described [53].

Maxwell-Boltzmann distribution relates the most-probable and root-mean-square speeds as [34]

$$v_{mpk}^2 = v_{wk}^2 = v_{rk}^2 / 3 = c^2 / 3 \tag{16a}$$

such that

$$k\lambda_{rk} = m_k c^2 = 3m_k v_{mpk}^2 = 3k\lambda_{mpk} = 3kT \tag{16b}$$

In view of Eq. (14a) and (16b), Kelvin absolute temperature is identified as the Wien wavelength associated with the *most-probable speed* of particles at thermodynamic equilibrium [34]

$$T_\beta = \langle \lambda_{mp\beta}^2 \rangle^{1/2} = \lambda_{mp\beta} = \lambda_{w\beta} = \lambda_{r\beta} / 3 \tag{17}$$

The root-mean-square and most-probable wavelength and frequency relationships

$$\lambda_{r\beta} = 3\lambda_{w\beta} \quad , \quad \nu_{r\beta} = \lambda_{w\beta} / \sqrt{3} \tag{18}$$

satisfy the corresponding speed relation

$$v_{rk} = \sqrt{3} v_{wk} \tag{19}$$

The definitions of (h, k) in Eq. (14a) lead to the gravitational mass of photon m_k, Avogadro-Loschmidt number N^o, atomic-mass-unit *amu*, and universal gas constant R^o that are given as [34]

$$m_k = (hk / c^3)^{1/2} \approx 1.84278 \times 10^{-41} \ \text{[kg]} \tag{20a}$$

$$N^o = 1 / (hkc)^{1/2} \approx 6.0376 \times 10^{26} \ [\text{kmol}^{-1}] \tag{20b}$$

$$amu = m_k c^2 = (hkc)^{1/2} \approx 1.6563 \times 10^{-27} \ \text{[kg]} \tag{20c}$$

$$R^o = N^o k = (k / hc)^{1/2} \approx 8.338 \ \text{[kJ/kmol-m]} \tag{20d}$$

4 Invariant Definitions of External Space and Time Versus Internal Spacetime

According to the model in Fig. 1, physical space, Aristotle fifth element, Casimir vacuum [46], Dirac stochastic ether [47], or de Broglie hidden thermostat [3] is identified a tachyon fluid [38, 54]. It is important to emphasize that *space is the tachyonic fluid itself and not the container that is occupied by this fluid*, as in the classical theories of ether [55]. Using a glass of water as an example, the physical space is analogous to the water itself, and not to the glass. Such a model is harmonious with perceptions of Euler [56] concerning reality of space and time.

To describe the nature of physical space and time, we consider the large scale of cosmology β = g with (atom, element, system) velocities $(\mathbf{u}_g, \mathbf{v}_g, \mathbf{w}_g)$ and the adjacent lower scale of astrophysics β = s with the velocities $(\mathbf{u}_s, \mathbf{v}_s, \mathbf{w}_s)$. According to Eq. (1) the "atom" of cosmic statistical field is defined as the most probable element of astrophysical field $\mathbf{u}_g \equiv \mathbf{v}_{ws}$. Also, by Eq. (2) the most-probable element of cosmic field is identified as "system" of astrophysical statistical field $\mathbf{v}_{wg} \equiv \mathbf{w}_s$. Planck and Maxwell-Boltzmann distribution functions in Eqs. (10-11) in non-dimensional coordinates are shown in Figs. 4 and 5 [54, 57]

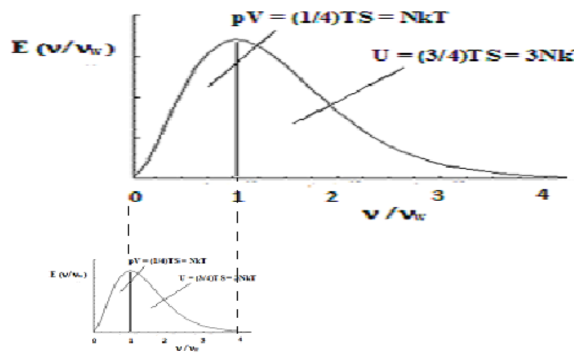

Fig. 4 Normalized Planck energy distribution as a function of oscillator frequency (ν/νw) [57].

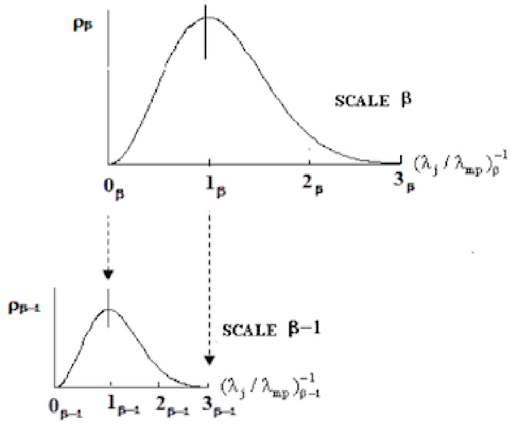

Fig. 5 Normalized Maxwell-Boltzmann speed distribution as a function of oscillator wavelengths $(\lambda/\lambda_w)^{-1}$ **[54].**

According to normalized Planck distribution in Fig. 4, maximum frequency and most probable frequency are related as

$$\nu_{\infty k} = 4\nu_{wk} \tag{21}$$

that by Planck quanta of energy in Eq. (14a) leads to energy relation

$$\varepsilon_{\infty k} = h\nu_{\infty k} = 4h\nu_{wk} = 4\varepsilon_{wk} \tag{22}$$

One can express Eq. (22) as [34]

$$\varepsilon_{\infty k} = 4m_{wk}v_{wk}^2 = \frac{4}{3}m_{wk}v_{rk}^2 = \frac{4}{3}m_{wk}c^2 \tag{23}$$

that is in accordance with Hasenöhrl [58] factor of 4/3 in his pioneering prediction of total energy of equilibrium radiation discussed in [59]

$$\varepsilon_{\infty k} = \hat{h}_k = \frac{4}{3}m_{wk}c^2 = \frac{4}{3}\hat{u}_k = 4kT \tag{24}$$

Similarly, according to normalized Maxwell-Boltzmann speed distribution in Fig. 5, by

$$v_{\infty k}/v_{wk} = \sqrt{h\nu_\infty / h\nu_w} = \sqrt{v_\infty\lambda_w / v_w\lambda_\infty} \tag{25}$$

the maximum and most-probable speeds are related as

$$v_{\infty k} = 3v_{wk} \tag{26}$$

that in view of Eq. (19) can be expressed as

$$v_{\infty k} = \sqrt{3}v_{rk} = \sqrt{3}c \tag{27}$$

Finally, one further notes that by $v_{\infty k} = u_k = v_{wt} = v_{rt}/\sqrt{3}$, the root-mean-square speeds of adjacent EKD and ETD scales are related as $v_{rt} = 3v_{rk} = 3c$, or in general

$$v_{r\beta} = 3v_{r\beta+1} \tag{28}$$

According to Figs. 1, 4, and 5, every "point" or "atom" constituting the "space" of cosmic field will be composed of either [54]

$$\begin{matrix} g & \text{Galaxy} \\ \varnothing & \text{Casimir Vacuum} \end{matrix} \tag{29}$$

since in the present discussion we do not address the issue of Dirac *anti-matter* $\overline{g}$. By Eq. (3) the most-probable element of cosmic scale decompactifies into the system of astrophysical scale $\mathbf{v}_{wg} \equiv \mathbf{w}_s$ as shown in Fig. 2. Also, because of the definition $\mathbf{u}_g \equiv \mathbf{v}_{ws}$ the atomic energy at cosmic scale $\beta = g$ is identical to the most-probable energy of astrophysical field

$$m_g u_g^2 = m_{ws}v_{ws}^2 = kT_{\beta=s} = k\lambda_{ws} \tag{30}$$

Hence, under local thermodynamic equilibrium cosmic field will have a homogenous constant temperature given by

$$kT_{\beta=s} = k\lambda_{ws} = m_{ws}v_{ws}^2 \tag{31}$$

By definition of most-probable speed $v_{ws} = \lambda_{ws}\nu_{ws} = \lambda_{ws}/\tau_{ws}$ and Eqs. (30-31), one can associate constant measures of (extension, duration)

$$\begin{matrix} \lambda_{ws} & \text{Internal measure of extension} \\ \tau_{ws} & \text{Internal measure of duration} \end{matrix} \tag{32}$$

to every "point" or "atom" of space in a universe at constant temperature $T_g = T_s = \lambda_{ws}$. Thus, cosmological temperature of physical space or Casimir vacuum in effect fixes the local spatio-temporal measures of *spacetime* because the measures of *extension* λ_{ws} and *duration* τ_{ws} are no longer *independent* since $v_{ws} = \lambda_{ws}\nu_{ws}$ must satisfy the constant vacuum temperature $T_g = T_s$ in Eqs. (30). The internal coordinates in Eq. (32) provide for physical foundation of universal measures of spacetime that change at cosmic time scale (eons) as the universe cools due to its expansion in harmony with observed cosmic background radiation of Penzias and Wilson [60].

When expressed as $(\lambda_{wxs}, \lambda_{wys}, \lambda_{wzs}, \tau_{ws})$ the internal coordinates are in harmony with the four-dimensional spacetime of Poincaré [61] and Minkowski [62]. Because of its direct relevance to thermodynamic temperature $T_\beta = \lambda_{mp\beta}$ viewed as a "*measure*" of spatial extensions, besides the sheer beauty of its eloquence and intuitive clarity, we present the following long quotation from Poincaré [63]

"Suppose, for example, a world enclosed in a large sphere and subject to the following laws: --The temperature is not uniform; it is greatest at the center, and gradually decreases as we move towards the circumference of the sphere, where it is absolute zero. The law of this temperature is as follows:--If R be the radius of the sphere, and r the distance of the point considered from the center, the absolute temperature will be proportional to $R^2 - r^2$. Further, I shall suppose that in this world all bodies have the same coefficient of dilatation, so that the linear dilatation of any body is proportional to its absolute temperature. Finally, I shall assume that a body transported form one point to another of different temperature is instantaneously in thermal equilibrium with its new environment. There is nothing in these hypotheses either contradictory or unimaginable. A moving object will become smaller and smaller at it approaches the circumference of the sphere. Let us observe, in the first place, that although from the point of view of our ordinary geometry this world is finite, to the inhabitants it will appear infinite. As they approach the surface of the sphere they become colder, and at the same time smaller and smaller. The steps they take are therefore smaller and smaller, so that they can never reach the boundary of the sphere. If to us geometry is only the study of the laws according to which invariable solids move, to these imaginary beings it will be the study of the laws of motion of solids deformed by the differences of temperature alluded to."

A pictorial representation of hyperbolic geometry of physical space described in the above quotation from Poincaré [63] is the wonderful painting by Escher [64] called *Circle Limit IV* shown in Fig. 6.

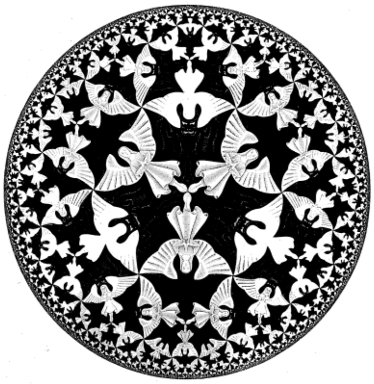

Fig. 6 Circle Limit IV by M.C. Escher [64] as model of Poincaré description of hyperbolic geometry of Lobachevski.

One notes that the approach to circumference of the sphere in Fig. 6 is similar to that of event horizon of black hole when internal measures of spacetime $(\lambda_{w\beta}, \tau_{w\beta})$ asymptotically approach zero as $T_\beta \to 0_\beta$. Hence, Poincaré [63] anticipated the importance of hyperbolic geometry of physical space that parallel to his anticipation of an external stress (Poincaré stress) being responsible for particle stability [37, 61], is another testimony to the true genius of this great mathematician, physicist, and philosopher. Because of its *hyperbolic geometry* (Fig. 6), its discrete or quantum fabric, and its stochastic atomic motions, physical space is called *Lobachevsky-Poincaré- Dirac-Space*.

The problem of time emphasized by Aristotle [65] concerns the nature of past, present, and future and was most eloquently described by *St.* Augustine [66]. The problem of how could a finite time duration be constructed from multitudes of instantons [67], "Nows", that do not exist is resolved by the invariant definition of internal atomic time in Eq. (32) [37]

$$t_\beta = \sum\nolimits' \tau_{\beta\text{-}1} \tag{33}$$

This is because the "atomic" instant $t_\beta = 0$ of scale β has a finite duration $\tau_{\beta-1}$ at the lower scale $\beta-1$. Hence, one employs clock of $\beta-1$ scale to measure time of β scale, clock of $\beta-2$ scale to measure time of $\beta-1$ scale, and so on ad infinitum.

In view of Figs. 1-3 and Eqs. (30-32), at thermodynamic equilibrium there exist *internal* (clocks, rulers) associated with random thermal motion of atoms ($t_\beta = \tau_{w\beta-1}$, $x_\beta = \lambda_{w\beta-1}$) of each statistical field shown on the left-hand-side of Fig. 1 from cosmic to tachyon scales [37, 68]

$$... > \tau_{we} > \tau_{wc} > \tau_{wm} > \tau_{wa} > \tau_{ws} ... \quad (34a)$$

$$... > \lambda_{we} > \lambda_{wc} > \lambda_{wm} > \lambda_{wa} > \lambda_{ws} ... \quad (34b)$$

Such hierarchy of embedded clocks each associated with its own stochastically stationary *periodic motions* is schematically shown in Fig. 7.

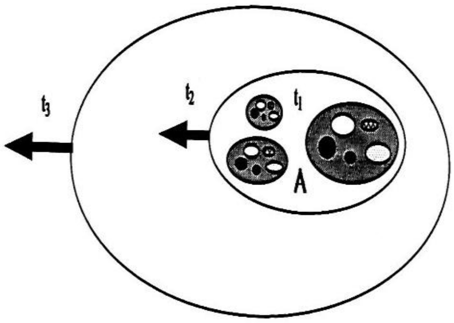

Fig. 7 Hierarchies of embedded clocks [68].

Clearly, the problem of time reversal at any scale β is now much more complex since it requires reversal of the entire hierarchy of "times" at lower scales shown in Eq. (34a).

Possible spatial anisotropy of time is explained by the fact that by Eqs. (30-32) internal "atomic" time $t_\beta = \tau_{w\beta-1}$ depends on the thermodynamic temperature [68]. Therefore, temperature anisotropy will lead to anisotropy of *internal measures* of both space and time. Also, as suggested by Fig. 7, both space and time have vector property that is *intrinsic* in the same sense as Gaussian curvature. Also, by Eq. (34a), the reason for coincidence of directions of arrows of times at all scale shown in Fig. 1 becomes apparent.

The most fundamental and *universal physical time* is the time associated with the tachyon fluctuations $\tau_{wt} = t_k$ [68] of *Casimir* vacuum [46] at Planck scale. One may associate the absolute mathematical time of Newton to the equilibrium state of tachyon-dynamics (t_t) that in the absence of any non-homogeneity (light) will be a timeless (eternal) world of darkness irrespective of its stochastic dynamics because, in accordance with the perceptions of Aristotle [65], the concept of time without any change is meaningless.

To describe dynamics of billions of galaxies in the universe one requires *independent* space and time coordinates that are not provided by *inter-dependent* internal spacetime coordinates. However, according to Eq. (34), at cosmic scale β = g one employs internal (ruler, clock) of the lower scale of astrophysics β = s to define *external* space and time coordinates (x_β, t_β) as

$$x_\beta = N_x \lambda_{w\beta-1}$$
$$t_\beta = N_t \tau_{w\beta-1} \quad (35)$$

Because of *independence* of the numbers N_x and N_t in Eq. (35), coordinates (x_β, t_β) could be employed in the conservation equations (5-8) to determine the dynamics of the universe.

Recent closure of the gap between kinetic theory of ideal gas and photon gas in equilibrium radiation resulted in number of photons in volume V of Casimir vacuum given by [38]

$$N = \frac{8\pi^5 V}{45}\left(\frac{kT}{hc}\right)^3 \quad (36)$$

Substituting from Eq. (36) in Stefan-Boltzmann law

$$\tilde{u} = \frac{\pi^2 k_B^4}{45\hbar^3 c^3} T^4 = \sigma T^4 \quad (37)$$

for volumetric internal energy density $\tilde{u}$ of photon gas results in [34, 38]

$$U = \tilde{u}V = 3NkT \quad (38)$$

in harmony with generalized Stefan-Boltzmann law recently discussed by Montambaux [69]. Equation (38) when combined with the ideal gas law

$$pV = NkT \quad (39)$$

results in enthalpy of Casimir vacuum [34]

$$H = U + pV = 4NkT \quad (40)$$

The thermodynamic arrow of time of Eddington is in the direction of increasing entropy or disorder. According to the modified form of the second law of thermodynamics [34], entropy of ideal gas relates to the number of Heisenberg-Kramers [45] virtual oscillators

$$S_\beta = 4N_\beta k \quad (41)$$

From equations (40-41) the total thermal energy, Sommerfeld "*total heat*" [70], or enthalpy of Casimir vacuum [46] is given by [34]

$$H = Q = TS = U + pV = 4NkT \quad (42)$$

Finally, Helmholtz decomposition of total heat H into *free* and *latent* heats results in definition of *dark energy DE* and *dark matter DM* [33, 34]

$$H_\beta = T_\beta S_\beta = \frac{3}{4} T_\beta S_\beta + \frac{1}{4} T_\beta S_\beta = DE + DM \quad (43)$$

It is known that exactly ¾ and ¼ of the total energy of *Planck* black body equilibrium radiation falls on $\lambda > \lambda_w$ and $\lambda < \lambda_w$ sides of λ_w given by the Wien displacement law as shown in Fig. 4. Therefore, of the total energy of the field T*S*, ¾ is associated with *free heat* and identified as kinetic energy, electromagnetic mass, or *dark energy* and ¼ is associated with *latent heat* and identified as potential energy, gravitational mass, or *dark matter* [33]. The predicted [33] fractions ¾ and ¼ are found to be in close agreement with the recent observations by Riess et al., [71, 72], Schmidt et al., [73], and Perlmutter et al., [74].

The result in Eq. (43) is also in accordance with general theory of relativity [75-76] as described by Pauli [76]

"*The energy of a spatially finite universe is three-quarters electromagnetic and one-quarter gravitational in origin*"

Scale invariance of the model suggests that dark matter in Eq. (43) may be expressed as [33]

$$DM_\beta = E_{\beta-1} = DE_{\beta-1} + DM_{\beta-1} \quad (44)$$

Therefore, according to equations (20c) and (43-44) all matter in the universe is composed of dark energy hence electromagnetic mass as anticipated by both Lorentz [77] and Poincaré [61, 78-80].

It is known that general theory of relativity confronts what is called "time problem" [81] since the theory aims to describe *global* dynamics of the universe in terms of *local* geometry of spacetime governed by Einstein field equation. As a result, the very existence of time is being debated [81-87]. As discussed above, when physical space or Casimir vacuum [46] is identified as a compressible tachyon fluid a new paradigm for the physical foundation of quantum gravity [54, 68] become possible. Because of the definition of *internal atomic time* in Eq. (32), quantum theories of gravity [88-91] may have wave functions Ψ_g that instead of Wheeler-DeWitt equation [88-90]

$$H\Psi_g = 0 \quad (45)$$

satisfy the *modified Wheeler-DeWitt* equation

$$i\hbar \frac{\partial \Psi_g}{\partial t_{\beta-1}} = H\Psi_g \quad (46)$$

that is invariant Schrödinger equation [37,39,54]. The resurrection of time in (46) is made possible because the new "*atomic*" *time* arises from internal degrees of freedom, permitting Ψ_g (x_1, x_2, x_3, t_β, $\tau_{\beta-1}$) and the associated $g_{ij}(x_1, x_2, x_3, t_\beta, \tau_{\beta-1})$, that by thermodynamic considerations is related to the temperature $T_g = T_s$ of the field [54,68].

5 Compressibility of Physical Space and its Impact on Special Theory of Relativity

Huygens [92] recognized the analogy between propagation of light in ether and that of sound in air. Also, parallel to ideas of Lorentz [93-94],

"I cannot but regard the ether, which is the seat of an electromagnetic field with its energy and its vibrations, as endowed with certain degree of substantiality, however different it may be from all ordinary matter"

the concept of ether always played a crucial role in Poincaré's perceptions of relativity [95, 96] as he explicitly stated in his Principle of Relativity [97]

"We might imagine for example, that it is the ether which is modified when it is in relative motion in reference to the material medium which it penetrates, that when it is thus modified, it no longer transmits perturbations with the same velocity in every direction."

As opposed to Einstein who at the time found the ether to be superfluous [98], Poincaré anticipated the granular structure of the ether and its possible role in electrodynamics [99]

"We know nothing of the ether, how its molecules are disposed, whether they attract or repel each other; but we know this medium transmits at the same time the optical perturbations and the electrical perturbations;"

"The electrons, therefore, act upon one another, but this action is not direct, it is accomplished through the ether as intermediary."

Also, the true physical significance of Lorentz's local time [93] was first recognized by Poincaré [99].

In his lecture delivered in London in 1912 shortly before he died Poincaré stated [96, 100]

"Today some physicists want to adopt a new convention. It is not that they are constrained to do so; they consider this new convention more convenient; that is all. And those who are not of this opinion can legitimately retain the old one in order not to disturb their old habits. I believe, just between us, that this is what they shall do for a long time to come."

The perceptions of Poincaré concerning relativity theory are known to be also shared by Lorentz who stated in a 1915 lecture at the Royal Academy of Sciences in Amsterdam [101]

I could point out to you [if I had more time] how Poincaré in his study of dynamics of electron, about the same time as Einstein, formulated many ideas that are characteristic for his theory, and also formulated what he calls "le postulat de relativité"

The tachyonic fluid that constitutes the physical space is considered to be *compressible* in accordance with Planck's compressible ether [93]. If the compressible tachyonic fluid is viewed as an ideal gas, its change of density when brought isentropically to rest will be given by the expression involving Michelson number $Mi = v/c$ [37, 39]

$$\rho = \rho_o \left[1 + \frac{\gamma - 1}{2}\frac{v^2}{c^2}\right]^{\frac{1}{\gamma-1}} = \rho_o \left[1 + \frac{\gamma - 1}{2} Mi^2\right]^{\frac{1}{\gamma-1}} \quad (47)$$

With γ = 4/3 for photon gas, Eq. (47) leads to Lorentz-FitzGerlad contraction [33,37,39]

$$\lambda = \lambda_o \sqrt{1-(v/c)^2} \quad , \quad \rho = \rho_o / (1 - v^2/c^2) \quad (48)$$

that accounts for the null result of Michelson–Morley experiment [102].

Therefore, supersonic *Ma* > 1 (superchromatic *Mi* > 1) flow of air (tachyonic fluid) leads to the formation of Mach (Poincaré-Minkowski) cone that separates the zone of sound (light) from the zone of silence (darkness) as schematically shown in Fig. 8. Compressibility of physical space can therefore result in Lorentz-FitzGerald contraction [93,103], thus accounting for relativistic effects [61,77,78-80,93,97-99] and providing a *causal explanation* [76] of such effects in accordance with the perceptions of Poincaré and Lorentz [61,77,78-80].

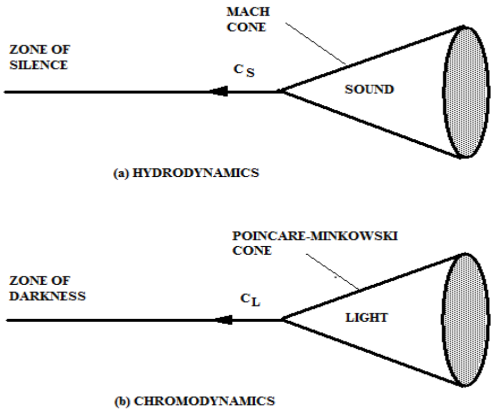

Fig. 8 (a) Mach cone separating sound and silence in supersonic flow (b) Poincaré-Minkowski cone separating light and darkness in super-chromatic flows [53].

In view of the above considerations and in harmony with ideas of Darrigol [95] and Galison [96], one can identify two distinct paradigms of the Special Theory of Relativity [37,39]:

(A) Poincaré-Lorentz
Dynamic Theory of Relativity
Space and time (x, t) are altered due to causal effects of motion on the ether.

(B) Einstein
Kinematic Theory of Relativity
Space and time (x, t) are altered due to the two postulates of relativity:

1- The laws of physics do not change form for all inertial frames of reference.

2- Velocity of light is a universal constant independent of the motion of its source.

The result in Eq. (48) and the almost constant cosmic temperature T_g and hence the speed of light

$$\lambda_{rk} \nu_{rk} = \lambda_{ork} \nu_{ork} = v_{rk}(T_g) \approx c \quad (49)$$

lead to the frequency transformation

$$\nu = \nu_o / \sqrt{1-(v/c)^2} \quad (50)$$

The relativistic transformation of frequency in Eq. (50) may also be expressed as contraction of time duration or transformation of period $\tau = 1/\nu$ as

$$\tau = \tau_o \sqrt{1-(v/c)^2} \tag{51}$$

Hence, time durations and space extensions contract by Eqs. (51) and (48) such that the speed of light in Eq. (49) remains invariant.

It is important to emphasize again that the relation between space and time according to the dynamic theory of relativity of Poincaré-Lorentz is causal as noted by Pauli [76] and is induced by compressibility of the manifold of the physical space itself rather than being a purely kinematic effect as suggested by Einstein [98] according to paradigm **(B)** above. Also, because space compressibility effects are complex and 3-diemsional, it is anticipated that future experiments will distinguish the difference between the dynamic versus kinematic paradigms [39, 54].

In supersonic flows at β = m scale, although sound waves move at velocity of sound $v_{rm} = c_m$ and are confined to Mach cone (Fig. 8), signals due to molecular *emission* can travel ahead of the cone since by Eq. (27)

$$v_{\infty m} = u_m = \sqrt{3} v_{rm} = \sqrt{3} c_m \tag{52}$$

Similarly, in superluminal flows, although light waves (gravitational waves) with velocity $v_{rk} = c_k = c$ are confined to Poincaré-Minkowski cone (Fig. 8), exceedingly weaker tachyon signals associated with gravitational radiation [104] or photon-emission can travel ahead of the cone

$$v_{\infty k} = u_k = \sqrt{3} v_{rk} = \sqrt{3} c_k = \sqrt{3} c \tag{53}$$

The name Poincaré-Minkowki cone is suggested because both Poincaré in 1906 [61] as well as Minkowski in 1908 [62] described a four dimensional manifold with three real space and one complex time dimensions.

It is interesting to introduce, in analogy to Mach $Ma = v/c_m$ and Michelson $Mi = v/c_k$ numbers for supersonic and superluminal flows, Lorentz number

$$Lo = v/c_e \tag{54}$$

for super-sonic flow of electrons in electrodynamics where c_e is the speed of "electrodynamic sound" in plasma. The ratio Mi / Lo was recently identified as Sommerfeld *fine structure constant* [105].

$$\frac{Mi}{Lo} = \frac{c_e}{c_k} = \frac{v_{re}}{c} = \alpha \approx 1/137.036 \tag{55}$$

In view of closure of the gap between kinetic theory of ideal gas and equilibrium radiation composed of photon gas [38] and observed Bose-Einstein condensation (BEC) of photon gas to liquid photon, it is reasonable to anticipate that at still lower temperatures, BEC superfluid will undergo a phase transition to *solid photon* that was identified as black hole [106].

Such phase transition of space as BEC superfluid is harmonious with finite pressure of Casimir vacuum [46] that resulted in introduction of modified form of the van der Waals *law of corresponding states* [107]

$$p_{rg} = p_{ra} - p_{rv} = \frac{1}{Z_c}\left[\frac{T_r}{\tilde{v}_r - 1/3} - \frac{9}{8\tilde{v}_r^2} + Z_c - \frac{3}{8}\right] \tag{56}$$

with predictions that are closer to experimental observations as shown in Fig. 9.

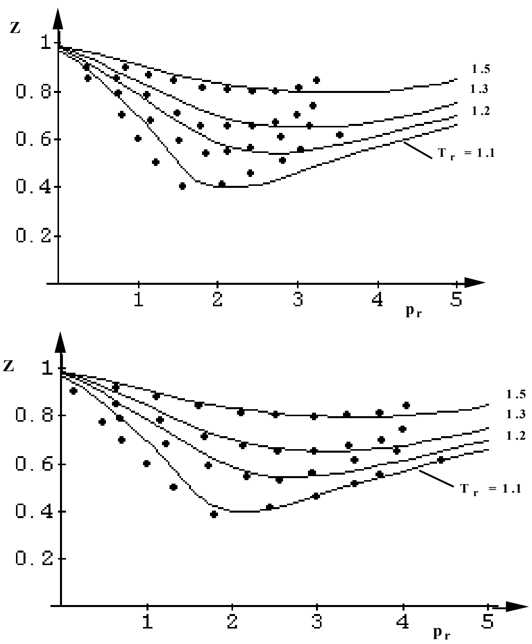

Fig. 9 Comparisons between experimental data (solid lines) and the calculations (solid circles) based on (top) van der Waals (bottom) modified van der Waals equation of state [107].

In the limit $\tilde{v}_r \to \infty$ since absolute pressure vanishes $p_{ra} \to 0$, one obtains from Eq. (56) a finite and positive reduced vacuum pressure [107]

$$p_{rv} = \frac{3/8}{Z_c} - 1 \geq 0 \tag{57}$$

It is then reasonable to suggest that *Dirac Sea* associated with anti-matter should correspond to pressures lower than that of Casimir [46] vacuum. Hence the pressure of matter p_m and anti-matter p_{am} fields will be respectively larger and smaller than vacuum pressure p_v [107]

$$0 = p_{WH} < p_{am} < p_v < p_m < p_{BH} = \infty \tag{58}$$

and ultimately limited by the pressures of *white hole* $p_{WH} = 0$ and *black hole* $p_{BH} = \infty$ that are the two singularities of the field.

Under such a model, the conservation of physical space i.e. Casimir vacuum [46], requires a symmetry between matter particles and their conjugate antimatter particles such that their interactions leads to mutual annihilation and generation of vacuum, thereby resolving the flatness-paradox of cosmology. Also, the conservation of angular momentum requires that the spin of matter versus their conjugate anti-matter particles be reversed, thereby accounting for the *time-reversal paradox*.

Since spacetime $(\lambda_{w\beta-1}, \tau_{w\beta-1})$ is physical attribute of a compressible fluid, the *causal connections* between space and time in relativistic physics become apparent. For example, in the classical problem of *twin paradox* of the special theory of relativity, the different times experienced by the twins is due to the different rates of biological reactions in their body induced by the compressibility of physical space in accordance with Poincaré-Lorentz dynamic theory of relativity [37, 39, 54].

It is interesting to examine the implication of the internal versus external times in Eqs. (32) and (35) to the problem of time travel suggested by closed time-like world lines in Gödel [108] solution of Einstein field equation in rotating universe, in harmony with Kerr [109] rotating black hole, discussed in an excellent book by Yourgrau [110]. Clearly, according to the model described in the previous Section, past is non-existent (except in human memory) and future remains only a potentiality of what may happen. Hence, time reversal in the present model requires the reversal of all chemical reactions as well as dissipations at the entire hierarchy of time scales shown in Eq. (34a). The near impossibility of such time reversal is evidenced by the diverse spectrum of hierarchies of time scales described in an excellent recent book by ‘t Hooft and Vandoren [111].

The internal versus external times defined in previous Section are harmonious with perceptions of Aristotle [65] and St Augustine [66] that the essence of time is in what it is not. Life at each instant appears as a chemically reactive mathematical surface of discontinuity *propagating* through the manifold of spacetime like a flame-front separating two non-existent worlds, a past already lived and a future yet to come. Poincaré [112] gives an eloquent description of the mystery of life through instanton [67] or “now” at the very end of his wonderful book

“And yet--strange contradictions for those who believe in time--geologic history shows us that life is only a short episode between two eternities of death, and that, even in this episode, conscious thought has lasted and will last only a moment. Thought is only a gleam in the midst of a long night.

But it is this gleam which is everything.”

6 Concluding Remarks

A scale-invariant model of statistical mechanics was applied to describe the physical foundation and quantum nature of space and time. It was shown that the temperature of cosmos leads to internal measures of extension and duration thus providing for the physical basis of spacetime. Because of its *hyperbolic geometry*, its discrete or quantum fabric, and its stochastic atomic motions, physical space is called *Lobachevsky-Poincaré-Dirac-Space*. The compressibility of physical space provided the causal basis of Lorentz-FitzGeral contractions thus leading to Poincaré-Lorentz *dynamic* as opposed to Einstein *kinematic* theory of relativity.

Acknowledgments: Some of this research was in part supported by NASA grant No. NAG3-1863.

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