# Computational Form-Finding Analysis in the Shape of Bour's

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*Abstract:* - Computational form-finding analysis need to be carried out for tensioned fabric structure in order to determine the initial equilibrium shape under prescribed pre-stress and boundary condition. Tensioned fabric structure is highly suited to be used for realizing surfaces of complex or new forms. Tensioned fabric structures are normally designed to be in the form of equal tensioned surface. However, research study on a new form as a tensioned fabric structure has not attracted much attention. Alternative source of inspiration of minimal surface which could be adopted as form for tensioned fabric structure is very crucial. The aim of this study is to investigate initial equilibrium shape of tensioned fabric structures in the form of Bour's minimal surfaces using nonlinear analysis method. The study proposes an alternative choice for engineer to consider the Bour's minimal surface with radius 0 to 1.4, 0 to 1.2, 0 to 1.0, 0 to 0.8 and 0 to 0.6 applied in tensioned fabric structure. The results on parameter range in Bour's minimal surface can serve as a reference for proper selection of surface parameter for achieving a structurally viable surface. Such in-sight will lead to improvement of rural basic infrastructure, economic gains, sustainability of built environment and green technology initiative.

*Key-Words: - Form-finding, tensioned fabric structure, initial equilibrium shape, minimal surface, Bour's and nonlinear analysis method.*

## 1 Introduction

Tensioned Fabric Structures (TFS) are structures that are composed of the fabric surface as structural members. Fabric in TFS are joined together at seams and are tensioned through mechanical means or cables to rigid supporting system to typically provide a roofing structure. Fig. 1 shows the example of tensioned fabric structure [1]. As shown in Fig. 1, tensioned fabric structure uses fabric roof membrane, cables, clamping systems and a small amount of support structure to create lightweight structures that capable to span to huge area.

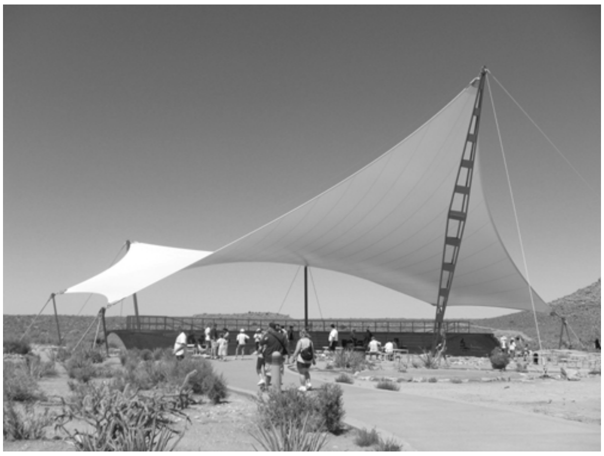


Fig. 1: Tensioned fabric structure

TFS have been employed throughout recorded history as in rope bridges and tents. However, large permanent tension structures were generally a 19th century development in bridges and a 20th century development in buildings. Tensioned fabric structure is the best alternative to cover area with low cost. One of the greatest benefit is their translucent. Woven fabric coated with a polymeric resin allows a light transmission value of around 10%. This provides a very comfortable level of illumination compared to the full brightness of outside the structure. [2] have stated large energy consumption of energy heating for residential and commercial use in the world. [3], [4], [5], [6] and [7] have stated energy saving. The energy saving problem can be overcome by application of TFS.

The selection of membrane material is important to the successful design of the tensioned fabric structure. The material contributes to the structural function of the system, as well as other important properties involving durability, insulation, light transmission and fire protection. Also, the membrane component of the structure determines the long term appearance of the structure for it is the most visible element of the structure. Currently, glass and polyester laminates, composites and fluoroplastic films are most popular. When selecting a membrane, the most importance qualities to consider are the mechanical tensile strength and the elastic properties.

There are several types of primary structures or support structure, one of them is the Arch Supported Structures. The arch form is ideally suited as a primary load bearing element for membrane structure. The appropriate layout of arch member along the support lines facilitates the creation of membrane saddle areas. Another primary structure is the Primary Point Supports which can create hypar surface by connecting four points in different planes via cables. Besides that, by setting out cables with opposite curvature next to each other, a primary structure can be created on which membrane surface with a slight double curvature can be formed and supported forming The Ridge and Valley Principle support structure. Some other famous support structures are Cable Nets, Mast Structures, Arched Roofs and Air supported surface element. Cable net act as a point supported surface were created in the form of cable nets. Mast structures are used in combination with a wide range of membrane system. Arched roof enables is ideal geometrical form for creating double curved membranes while Air supported surface structure using air as the supporting element of the membrane skin to bear the compression loads.

Form-finding using nonlinear analysis method or soap film models in the form of Catenoid, Helicoid, Scherk, Enneper, Costa, Moebius Strip, Egg, Oval, Monkey Saddle, Chen-Gackstatter and cable reinforced Bour’s TFS models have been carried out by [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18], [19], [20], [21], [22] and [23].

The design of tensioned fabric structure is different from other types of structure, tensioned fabric structure could be design in various forms enabling it to be more unique than conventional structure. Since the last 50 years, many huge and fascinating tensioned fabric structures have been made. Tensioned fabric structure is highly suitable to be used for realizing new forms of surfaces.

Tensioned fabric structure with different surface form could be realized. The problem related to the determination of new form minimal surface in TFS is needed as a reference for structural engineer to be considered as an alternative shape for TFS. One of the most exciting shapes that captured the focus of the mathematics is Bour’s minimal surface. However, none of the previous example of tensioned fabric structure presents any results on the Bour’s form of minimal surface as load carrying elements. Understanding the potential initial equilibrium shapes for Bour’s minimal surface will provide alternative shapes for engineer to consider. In this study, the shape of Bour’s minimal surface has been developed for TFS. Bour’s surface is a form of minimal surface and the surface is very minimal. The form of Bour’s surface has not been studied by other researchers. Besides, no other work on Bour’s surface as idea in TFS has been found.

In this study, basic formulation for nonlinear analysis and form-finding based on [8] is presented. The strategy for form-finding using nonlinear analysis is proposed by [8] is presented. The results of five Bour’s TFS models by using computational form-finding are described. Conclusions of in this study are summarized. Recommendations for further research are also given.

## 2 Bour’s Minimal Surface

Mathematically, Bour's minimal surface is a two-dimensional minimal surface, embedded with self-crossings into three-dimensional Euclidean space. It has negative Gaussian curvature. The Bour’s as shown in Fig. 2 is a full shape of Bour’s surface, which is intersection with each other are occurred. These intersections have to be eliminated by reducing the radius of the Bour’s minimal surface in its equation. Bour’s with intersecting surface is not suitable apply as TFS. Fig. 3 shows the Bour’s minimal surface shape just before the intersection had occur.

A tensioned fabric structure in Bour’s surface must curve equally in opposite directions to give the resulting surface a three dimensional stability. In an anticlastic doubly curved surface, the sum of all positive and all negative curvatures is zero. The tensioned surface itself is the main structural element in a fabric structure. The surface is an anticlastic form with positive (Gaussian) curvature in one principal direction and negative (Gaussian) curvature in the other.

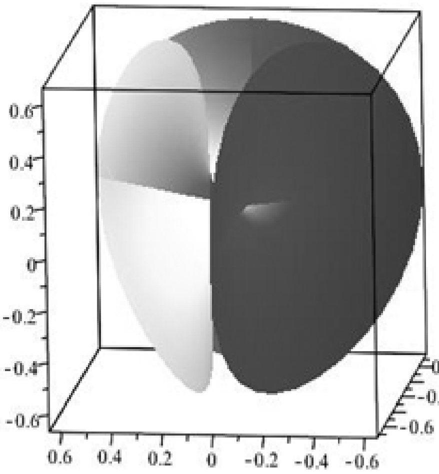


Fig. 2: Model of Bour’s surface intersection with each other

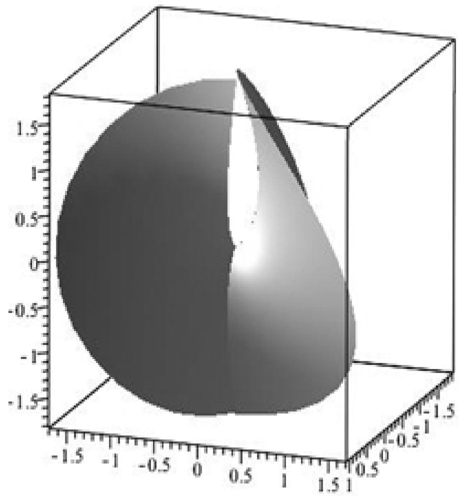


Fig. 3: Bour’s minimal surface

[24] mentioned that the explicit equations of the Bour’s surface in polar coordinates are as following in Equation (1):

$$x = \frac{r^2}{2}\cos(2\theta) - \frac{r^4}{4}\cos(4\theta)$$

$$y = -\frac{r^2}{2}\sin(2\theta) - \frac{r^4}{4}\sin(4\theta)$$

$$z = \frac{2r^3}{3}\cos(3\theta) \quad (1)$$

Where $r \in [-1, 1]$, $\theta \in [0, \pi]$. This polar coordinates equation had been applied in maple software to view the shape and the respective x, y and z coordinate. The changing value of *r* parameter will affect the generated form.

From the mathematical finding, the radius that does not have the intersection is from 0 to 1.41. In this study, form-finding using nonlinear analysis method in the shape of Bour’s with radius 0 to 1.4, 0 to 1.2, 0 to 1.0, 0 to 0.8 and 0 to 0.6 have been carried out. From this study, the software ADINA [25] has been used for the purpose of model generation. Aspect of modeling of surface of Bour’s minimal surface and form as well pre-stress pattern of the resulting TFS through form-finding using nonlinear analysis method are studied.

## 3 Numerical Method using Nonlinear Analysis Method

YEE [8] stated the principle of nonlinear analysis method is based on the large displacement finite element formulation used for analysis of structural behaviour under external loads. Since the method can be used for both the initial equilibrium problem and load analysis, the approach using nonlinear analysis is quite common. The basic equation used is expressed as Equation 2:

$$({}_0^t\boldsymbol{K}_L + {}_0^t\boldsymbol{K}_G)\boldsymbol{u} = {}^{t+\Delta t}\boldsymbol{F} - {}_0^t\boldsymbol{f} \quad (2)$$

Where ${}_0^t\boldsymbol{K}_L$ is linear strain incremental stiffness matrix, ${}_0^t\boldsymbol{K}_G$ is nonlinear strain incremental stiffness matrix, ${}_0^t\boldsymbol{f}$ is vector internal forces, ${}^{t+\Delta t}\boldsymbol{F}$ is load vector and $\boldsymbol{u}$ is vector of increment in displacement.

A nonlinear finite element analysis program by [8] for the analysis of TFS has been used in this study. The procedure adopted is based on the work by [8]. 3-node plane stress element has been used as element to model the surface of TFS. All x, y and z translation of nodes lying along the boundary edge of the Bour's minimal surface have been restrained. The member pretension in warp and fill direction, is 2000N/m, respectively. The shear stress is zero.

Two stages of analysis were involved in the procedures of form-finding in one cycle proposed by [8]. First stage (denoted as SF1) is analysis which starts with an initial assumed shape in order to obtain an updated shape for initial equilibrium surface. The initial assumed shape can be obtained from any pre-processing software and reference [8] is chosen for this study. This is then followed by the second stage of analysis (SS1) aiming at checking the convergence of updated shape obtained at the end of stage (SF1). During stage (SF1), artificial tensioned fabric properties, E with very small values are used. Both warp and fill tensioned fabric stresses are kept constant. In the second stage of (SS1), the actual values of tensioned fabric properties are used. Resulting warp and fill tensioned fabric stresses are checked at the end of the analysis against prescribed tensioned fabric stresses. Then, iterative calculation has to be carried out in order to achieve convergence where the criteria adopted is that the average of warp and fill stress deviation should be < 0.01. The resultant shape at the end of iterative step n (SSn) is considered in the state of initial equilibrium under the prescribed warp and fill stresses and boundary condition if difference between the obtained and the prescribed membrane stresses relative to the prescribed stress is negligibly small. Such checking of difference in the obtained and prescribed stresses has been presented in the form of total stress deviation in warp and fill direction versus analysis step.

As a first shape for the start of form-finding procedure adopted in this study, initial assumed shape is needed. For the generation of such initial assumed shape, knowledge of the requirement of anti-clastic nature of TFS is used. The incorporation of anti-clastic feature into the model will help to produce a better initial assumed shape.

The difficulty to control final shape has overcome by starting the form-finding form on properly selected initially assumed shape. Initially equilibrium shapes of membrane structures were solved under a condition of equal tension in the warp and fill directions corresponds to minimal surfaces.

ADINA [25] has chosen as a tool for mesh generation in this study. The checking of the orientation of mesh in terms of element node sequence is essential because the orientation of element nodes directly affects the direction of material axes. Fig. 4 shows the direction of element nodes for an element. The first edge connecting i and j nodes corresponds to the warp direction which has used in the Finite Element (FE) formulation. The direction that was perpendicular to the warp direction (edge i-j) corresponds to the fill direction. The edge i-j which has oriented to the warp direction was defined first. Then the third element node k was numbered in such a way that the sequence i-j-k was in a counterclockwise direction. The proper sequence of node i-j-k for all elements was important for the generation of a good mesh representing the shape of fabric model.

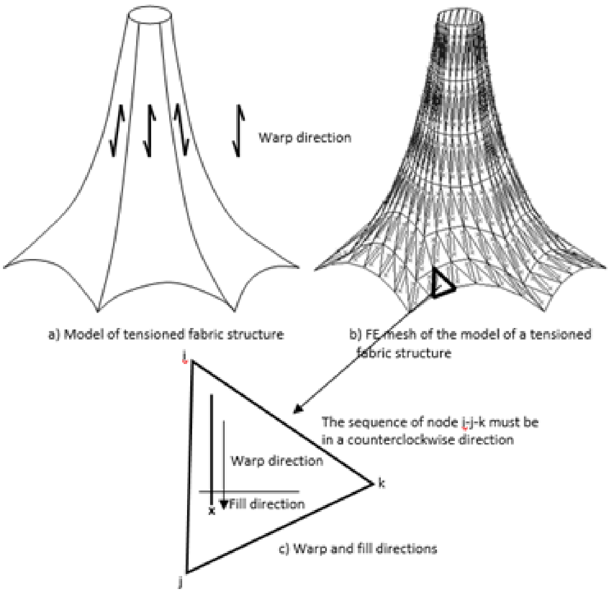
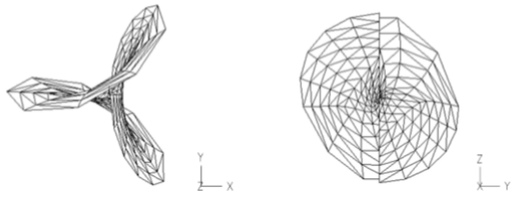


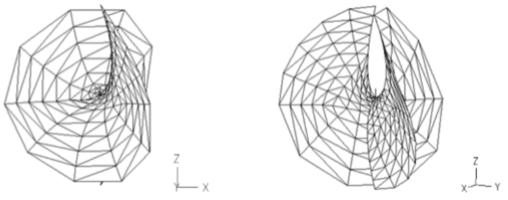
Fig. 4: Meshing and element node sequence

Form-finding on TFS models in the form of Bour's minimal surface have been carried out. The radius used in this study is 0 to 1.4, 0 to 1.2 and 0 to 1.0, 0 to 0.8 and 0 to 0.6. For computational form-finding analysis, initial equilibrium shape is determined.

Fig. 5 shows the initial equilibrium shape of Bour's minimal surface with radius 0 to 1.4. Variation of total stress deviation in warp and fill direction versus stress analysis of Bour's minimal surface with radius 0 to 1.4 as shown in Fig. 6. The Bour's minimal surface with radius 0 to 1.4 has been found to converge with least square error of total warp and fill stress deviation less than 0.01.



XY-View YZ-View



XZ-View 3D-View

Fig. 5: Bour’s minimal surface with radius 0 to 1.4 after form-finding analysis

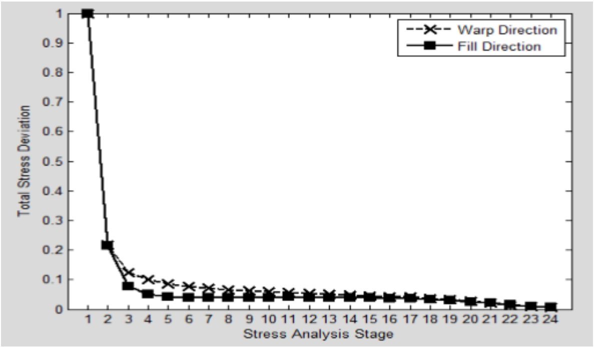
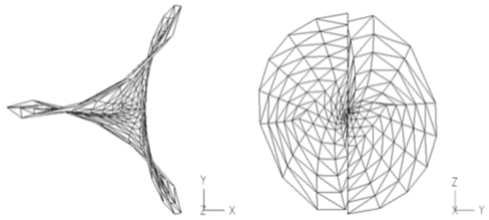


Fig. 6: Variation of total stress deviation in warp and fill direction versus stress analysis stage for Bour’s minimal surface with radius 0 to 1.4

Similar converged shape of the Bour’s TFS model in the radius 0 to 1.2, 0 to 1.0, 0 to 0.8 and 0 to 0.6. Fig. 7 and Fig. 8 show initial equilibrium shape and variation of total stress deviation in warp and fill direction versus stress analysis stage for Bour’s minimal surface with radius 0 to 1.2, respectively.



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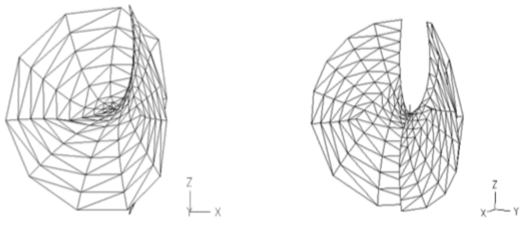

XZ-View 3D-View

Fig. 7: Bour’s minimal surface with radius 0 to 1.2 after form-finding analysis

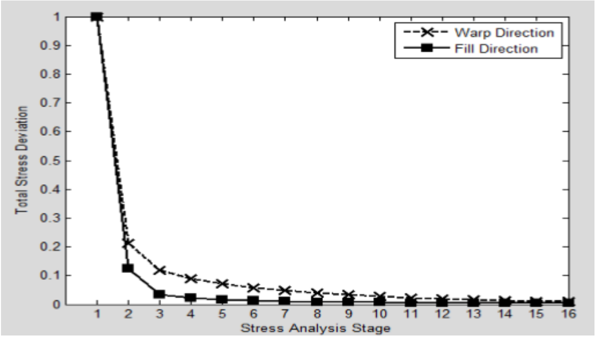

Fig. 8: Variation of total stress deviation in warp and fill direction versus stress analysis stage for Bour’s minimal surface with radius 0 to 1.2

Fig. 9 and Fig. 10 show initial equilibrium shape and variation of total stress deviation in warp and fill direction versus stress analysis stage for Bour’s minimal surface with radius 0 to 1.0, respectively.

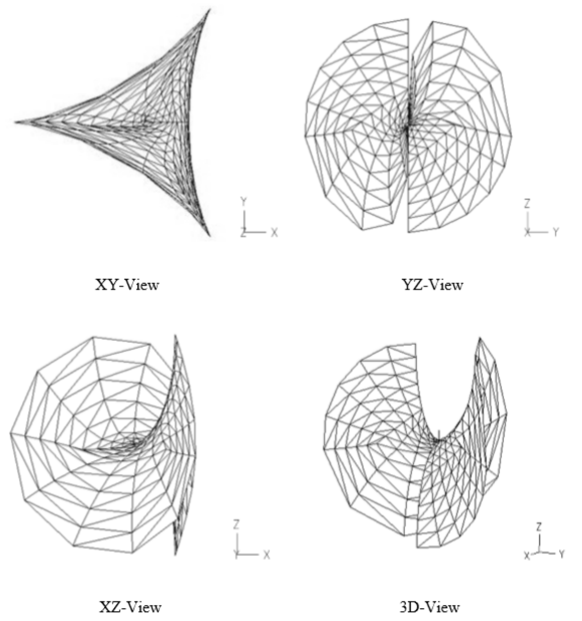

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Fig. 9: Bour’s minimal surface with radius 0 to 1.0 after form-finding analysis

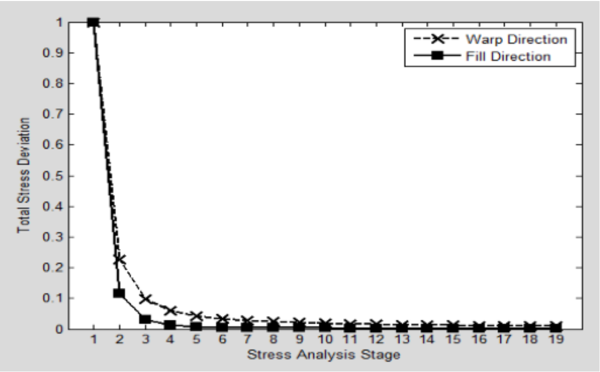

Fig. 10: Variation of total stress deviation in warp and fill direction versus stress analysis stage for Bour’s minimal surface with radius 0 to 1.0

Fig. 11 and Fig. 12 show initial equilibrium shape and variation of total stress deviation in warp and fill direction versus stress analysis stage for Bour’s minimal surface with radius 0 to 0.8, respectively.

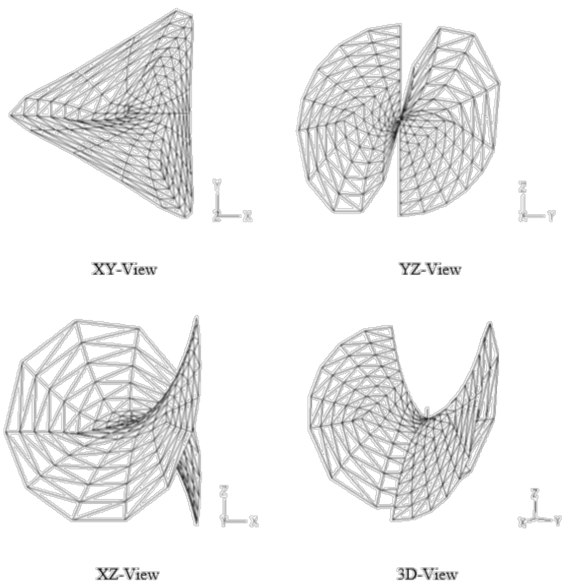

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Fig. 11: Bour’s minimal surface with radius 0 to 0.8 after form-finding analysis

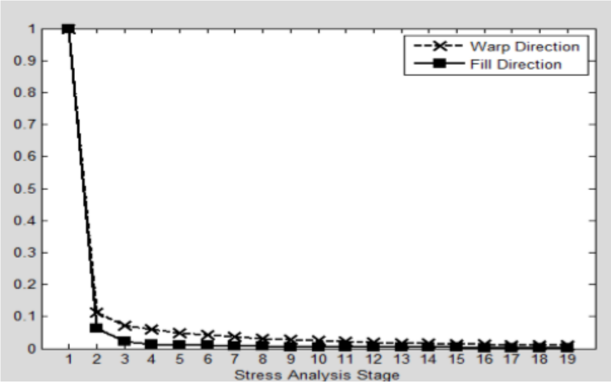

Fig. 12: Variation of total stress deviation in warp and fill direction versus stress analysis stage for Bour’s minimal surface with radius 0 to 0.8

Fig. 13 and Fig. 14 show initial equilibrium shape and variation of total stress deviation in warp and fill direction versus stress analysis stage for Bour’s minimal surface with radius 0 to 0.6, respectively.

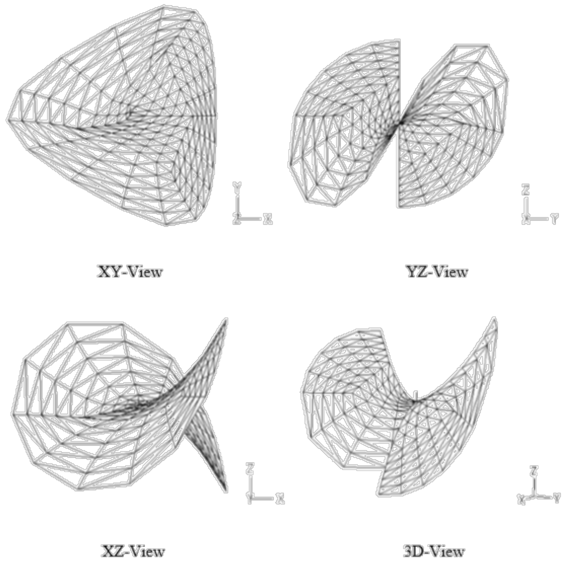

Fig. 13: Bour’s minimal surface with radius 0 to 0.6 after form-finding analysis

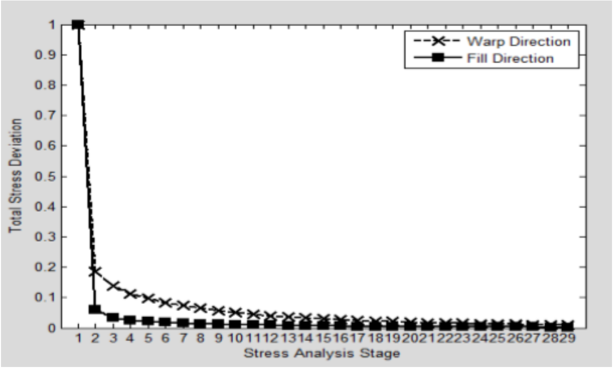

Fig. 14: Variation of total stress deviation in warp and fill direction versus stress analysis stage for Bour’s minimal surface with radius 0 to 0.6

The five models with radius 0 to 1.4, 0 to 1.2, 0 to 1.0, 0 to 0.8 and 0 to 0.6 have shown the changes in its forms. Fig. 15 shows initial assumed shape and initial equilibrium shape with radius 0 to 1.4.

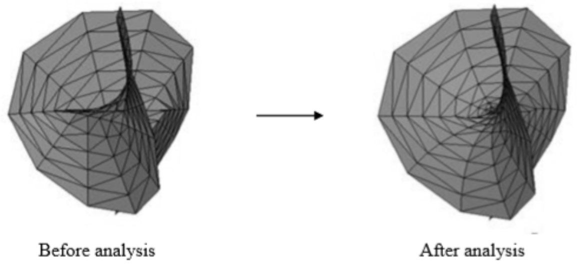

Fig. 15: Initial assumed shape before the form-finding to the initial equilibrium shape after form-finding for Bour’s minimal surface with radius 0 to 1.4

Fig. 16 shows initial assumed shape and initial equilibrium shape with radius 0 to 1.2.

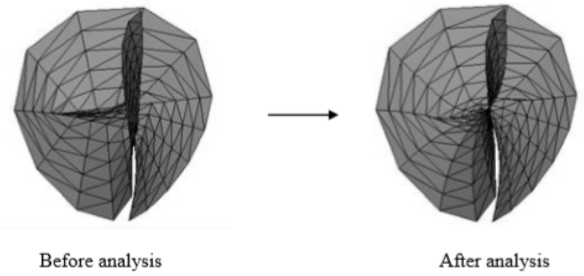

Fig. 16: Initial assumed shape before the form-finding to the initial equilibrium shape after form-finding for Bour’s minimal surface with radius 0 to 1.2

Fig. 17 shows initial assumed shape and initial equilibrium shape with radius 0 to 1.0.

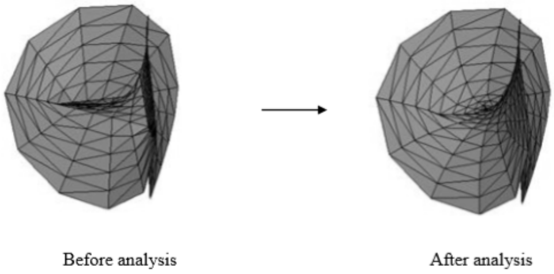

Fig. 17: Initial assumed shape before the form-finding to the initial equilibrium shape after form-finding for Bour’s minimal surface with radius 0 to 1.0

Fig. 18 shows initial assumed shape and initial equilibrium shape with radius 0 to 0.8.

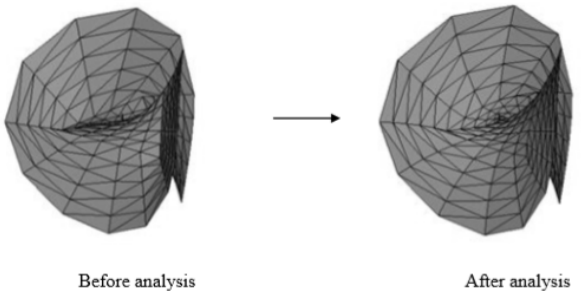

Fig. 18: Initial assumed shape before the form-finding to the initial equilibrium shape after form-finding for Bour’s minimal surface with radius 0 to 0.8

Fig. 19 shows initial assumed shape and initial equilibrium shape with radius 0 to 0.6.

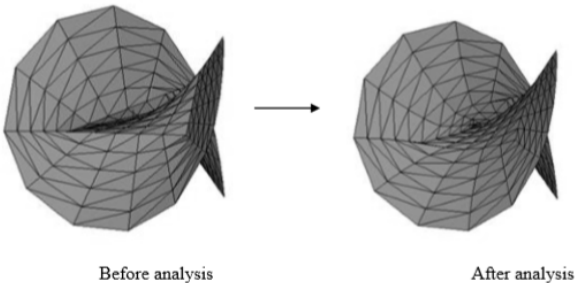

Fig. 19: Initial assumed shape before the form-finding to the initial equilibrium shape after form-finding for Bour’s minimal surface with radius 0 to 0.6

The form-finding computational analysis using Nonlinear Analysis Method of Bour’s minimal surface with radius 0 to 1.4, 0 to 1.2, 0 to 1.0, 0 to 0.8 and 0 to 0.6 have studied. The Bour’s minimal surface pattern has intersections between its surfaces, thus the parameter chosen is between 0 to 0.6 so that there are no surface intersections occurs on the shape.

At first, for starting the computational of the proposed form-finding using nonlinear analysis, initial assumed shape with anticlastic feature is needed. High distortion of elements will be encountered if it is started with a flat shape for the form-finding, it is difficult to control the final shape generated as the mesh formed will have high distortion.

The results of form-finding using computational strategies proposed have been able to determine initial equilibrium shape for Bour’s minimal surface with radius 0 to 1.4, 0 to 1.2, 0 to 1.0, 0 to 0.8 and 0 to 0.6 with good agreement with the mathematically defined shape. The computational analysis shows that Bour’s minimal surface with radius 0 to 1.4, 0 to 1.2, 0 to 1.0, 0 to 0.8 and 0 to 0.6 have found to converge with least square error of total warp and fill stress deviation less than 0.01.

## 4 Conclusion

This study has been carried out with the main objective of proposing initial equilibrium shapes of TFS in the form of Bour’s minimal surface by using the proposed computational strategies by [8]. Form-finding on TFS models in the form of Bour’s minimal surface with radius 0 to 1.4, 0 to 1.2, 0 to 1.0, 0 to 0.8 and 0 to 0.6 have provided an alternative architectural surface for architect and structural engineer to consider the tensioned fabric green structure in the form of Bour’s minimal surface. The Bour’s minimal surface with radius 0 to 1.4, 0 to 1.2, 0 to 1.0, 0 to 0.8 and 0 to 0.6 would enhance the understanding on the suitable choice of Bour’s minimal surface for TFS among structural designer.

TFS in the form of Bour’s minimal surface have been studied in detailed for the first time in the area of tensioned structure. The following studies should be further carried out in order to take advantage of the applicability of Bour’s minimal surface to form-finding and analysis under loading conditions:

i. TFS require a sound supporting system in order that they can develop the necessary tension to resist load. Supporting systems can be generally classified into systems which are relatively rigid such as concrete frames and relatively flexible such as grid of compression post and tensioned cables. In this study, form-finding has been carried out assuming that the Bour’s minimal surface supporting system is rigid. Further work should be carried out to develop computational strategies for form-finding Bour’s minimal surface in TFS considering non-rigid supporting structures.

ii. Extension of the nonlinear analysis to simulate inflation process of air supported Bour’s minimal surface should be carried out. Suitable computational strategy to be used in combination with nonlinear analysis should be proposed in Bour’s minimal surface in TFS.

iii. Analysis of TFS in the form of Bour's minimal surface including behaviour under loading and cutting pattern analysis should be carried out.

iv. Physical model by conducting experimental form-finding of Bour's minimal surface using soap film model should be proposed.

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