Vibration reduction of spindle-bearing system by design optimization

KHAIRUL JAUHARI
Center for Machine Tools, Production & Automation (BT MEPPO),
Badan Pengkajian & Penerapan Teknologi (BPPT)
2nd Floor, Technology Building 2 No 251, PUSPIPTEK, Tangerang Selatan,
INDONESIA 15314
khairul.j4uhari@gmail.com; khairul.jauhari@bppt.go.id

Abstract: - This paper presents a vibration reduction model of the radial vibration in a high precision spindle caused by unbalance force. The spindle-bearing system is considered as a flexible rotor supported by two sets of angular contact ball bearings. The finite element method (FEM) has been adopted for obtaining the spindle-bearing system equation of motion. In this study, natural frequencies, critical frequencies and amplitude of the unbalance response caused by residual unbalance are determined in order to investigate the spindle-bearing system behavior. In this paper, we proposed a new combination stochastic algorithm model such as hybrid genetic algorithm (HGA) for minimizing radial vibration of the spindle-bearing system by raising the critical frequencies and reducing the amplitude of unbalance response, which considers shaft diameter, dynamic characteristic of the bearing, critical frequencies, and amplitude of the unbalance response, and computes optimum spindle diameter and the values of damping and stiffness of the bearing. In numerical simulation results show that by optimizing shaft diameter, and the values of damping and stiffness of the bearing, the spindle vibration amplitude at operating speed can be minimized. A spindle-bearing system about 4.25 µm radial vibration amplitude can be reduced with 2.33 µm accuracy.

Key-Words: - Flexible rotor, high precision spindle, optimization model, radial vibration amplitude, spindle-bearing system

1 Introduction

Many high precision spindles are widely used in most of high precision grinding machine tools. Therefore, the higher accuracy of machining process can be achieved by using these spindles [1]. Moreover, the benefits of using high precision bearing encouraged machine tool engineers to contribute for development in technology of this spindle-bearing system. The fundamental methods for designing machine tool spindle-bearing system can be seen in reference [2]. An important function when employing spindle equipped with angular contact ball bearing, arises from error correction capability.

In this research, the vibration reduction model of radial vibration in spindle-bearing system was studied due to unbalance mass of the grinding wheel, which is influenced by the parameters such as the spindle shaft diameter, and coefficient value of damping and stiffness of the angular contact ball bearing. Few papers have reported that parameters of the system such as shaft diameter, shaft stiffness and the dynamic coefficients of the bearing, the radial vibration amplitude of the shaft can be minimized [3 - 5].

As An illustration, for spindle-bearing system with an initial radial vibration amplitude of 20 µm. has the optimum radial vibration amplitude about 2 µm [6] when mounted on the optimized bearing. A higher stiffness coefficient in the bearing can be raised by increasing the initial preload [7], enabling an optimum design to be achieved as in reference [8]. However, further investigations show that for an optimal performance not only stiffness parameter of the bearing must be increased, but the bearing damping also shall be adjusted [9]. Natural frequencies of spindle-bearing system can be maximized by the optimal locations of bearings installed on a spindle shaft [10, 11].

In this article, a spindle shaft is modeled as flexible rotor supported by two sets of high precision angular contact ball bearing. Finite element model (FEM) is employed to build the spindle-bearing system equation of motion in order to describe its dynamic behavior. In this study, natural frequencies, critical frequencies, and amplitude of the unbalance response caused by residual unbalance are determined in order to investigate the behavior of spindle-bearing system.

An optimization design technique is developed in order to minimize radial vibration amplitude of the spindle and computes the optimum values of spindle shaft diameter, and damping and stiffness of the bearings which considers shaft diameters, dynamic characteristic of the bearing, critical frequencies and amplitude of the unbalance response. The optimum values are obtained by raising critical frequencies and reducing maximum amplitude of the residual unbalance response. Due to complexity equation of the constraint and objective function, describing critical frequencies and unbalance response, an enhanced stochastic algorithm searching method such as hybrid genetic algorithm (HGA) [12] is employed for computing optimum shaft diameter, and damping-stiffness bearing coefficient values.

In numerical simulation results show that by optimizing shaft diameter, and damping-stiffness coefficient, the radial vibration of the spindle can be achieved in certain operating speed. As the simulation example result, an initial design of spindle radial vibration has run-out about 4.25 µm can be minimized with 2.2 µm at operating speed (8000 rpm).

This paper is organized into few sections as follow. In Section 2, the modeling of the problem including the derivation of motion equation model is explained. Furthermore, the general analyses for eigen-value and unbalance response are derived. The parameter variables for our design optimization, critical frequencies, unbalance responses, are determined, and a search strategy method for our optimization process is presented. Finally, numerical results for the spindle-bearing system which reducing the total spindle mass are shown in section 3.

2 Methods

2.1 Model of rotating spindle-bearing system

Generally, the spindle-bearing system is considered as an assembling of discrete disks, bearings and the spindle segments with distributed mass. In order to obtain an analysis of the complicated spindle-bearing system, the vibrations are calculated based on the procedure of the finite element discrete in many literatures [13 – 15], detail of those equations will not be derived here and only the general motion equations are shown below. The system equations that describe behavior of entire spindle-bearing system are formulated by taking into account the contributions from all elements in the model. The assembled equation of motion with N_e elements in the global coordinates is of the form [3].

$$M_G \ddot{q} - C_G \dot{q} + K_G q = F \tag{1}$$

where $M_G = (M_{re} + M_{te})$ is the global mass matrix, M_{re}, M_{te} are the rotational and translational mass element matrices, $C_G = (-\Omega G + C_b)$, $K_G = (K_b + K_s)$ are the global matrices of damping and stiffness, G is a gyroscopic matrix, C_b, K_b are the damping and stiffness element matrices of the bearing, and F is a force vector, respectively.

2.1.1 Analysis of eigen-value

In order to obtain the natural frequency of system, then eigen-value must be solved and expressed by Eq. (1), the equation of global system can be set as state variable vector.

$$A_G \dot{x} + B_G x = 0 \tag{2}$$

where the matrices of A_G, B_G, and displacement x consist of element matrices given as:

$$A_G = \begin{bmatrix} M_G & C_G \\ 0 & I \end{bmatrix}, \ B_G = \begin{bmatrix} 0 & K_G \\ -I & 0 \end{bmatrix}, \quad x = \begin{bmatrix} \dot{q} \\ q \end{bmatrix}$$

For assuming harmonic solution $x = x_0 e^{\lambda t}$ of Eq. (2), the solution of an eigen-value problem is

$$(A_G \lambda + B_G) x_0 = 0 \tag{3}$$

where, λ is the eigen-value. The eigen-values are usually complex number and conjugate roots.

$$\lambda_k = \alpha_k \pm i\omega_k \tag{4}$$

where, α_k and ω_k are the stability factor of growth and the k mode of damped frequencies, respectively.

2.1.2 Analysis of the unbalance response

The force of unbalance mass (F) which is shown in Eq. (1) can be expressed as:

$$F = F_u \Omega^2 e^{i\Omega t} \tag{5}$$

where F_u is force which independent of time and rotating speed. The steady-state response due to unbalance mass is considered to be as the form.

$$A = A_u e^{i\Omega t} \tag{6}$$

by substituting Eqs. (5) and (6) into (1), the equation can be expressed as

$$(K_G - \Omega^2 M_G + i\Omega C_G) A_u = F_u \Omega^2 \tag{7}$$

by solving Eq. (7) for A_u, the steady-state response can be obtained.

2.2 Optimization model

In this article, we proposed a new optimization formulation model for the radial vibration reduction problem that can be called as vibration level optimization problem. Optimum values of the spindle, diameter and the damping-stiffness of the bearings could be obtained by raising the critical frequencies and reducing amplitude of the unbalance response. For the formulation model, in this study the objective function is to minimize the total spindle mass M (Q) and the inequality constraints are subject to the non-linear function of the critical frequencies and the amplitude of the unbalance responses. In this case, the spindle diameter, and the damping and stiffness of the bearings were selected as the design variables. As we have described above, the optimization formulation model can be expressed as follows:

Minimize mass M (Q)

$$\Omega_m (Q) \geq \Omega_m^*,$$
$$A_m (\Omega_m) \leq A_m^*,$$
$$Q_L \leq Q \leq Q_U. \quad (8)$$

where, Ω_m and A_m (m = number of mode) are the new values of critical frequencies and unbalance responses for the optimum model, and Ω_m^* and A_m^* are the target constraint values of critical frequencies and unbalance response for the initial model. Therefore, it means that the critical frequencies, Ω_m, should be increased above given initial values Ω_m^*, and decreasing the unbalance response, A_m, below the given values A_m^*. Moreover, the upper Q_U and lower Q_L bounds on the design variables are set due to manufacturing constraint and to prevent critical stress.

Table 1 Search strategy and parameters for global search (genetic algorithm)

GA strategy	Description of values
Population size	40
Scaling function	Rank
Selection function	Stochastic uniform
Elite count	2
Crossover fraction	85%
Mutation probability	Constraint dependent
Constraint tolerance	1×10^{-8}
Max number of generation	150

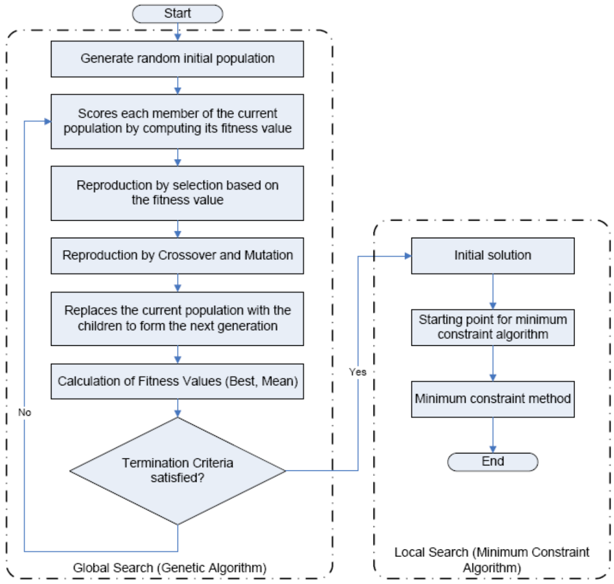

Fig. 1 Flowchart of hybrid genetic algorithm (HGA)

Due to the non-linearity and complexity functions of critical frequencies and unbalance responses, the derivatives of these functions are difficult to obtain. Therefore, an enhanced stochastic search optimization approach without derivatives such as hybrid genetic algorithm (HGA) [4] was employed to solve the model of optimization, which performed in MATLAB programming language (M-file). There are two main processes in hybrid genetic algorithm (HGA), the global search (genetic algorithm) and the local search (minimum constraint algorithm) processes. A hybrid function is an optimization function that runs after the genetic algorithm terminates in order to improve the value of the fitness function. The hybrid function uses the final point from the genetic algorithm as its initial point. In this study, we used optimization function minimum constraint, such as a constrained minimization function. The first process is running the genetic algorithm to find a point close to the optimal point and then uses that point as the initial point for minimum constraint algorithm process. The flowchart process of hybrid genetic algorithm (HGA) for searching the optimum values of single-objective function and design variables are described in Fig. 1. Table 1 shows the strategy of input parameter for performing process of genetic algorithm (GA).

3 Problem Solution and Example Case Results

In order to illustrate how the vibration level optimization design technique can be used to minimize the radial displacement of the spindle system, a numerical simulation of example was done. A schematic of the finite element model of the spindle system is shown in Fig. 2. In this case, the spindle shaft is modeled into 17 beam elements with a node at both ends of the shaft element. The mass of grinding wheel and pulley system can be considered as four elements of the rigid disk which are located at node 1, 14, 15 and 16. In addition, the two sets of bearing are located at node 5 and 12, and the residual unbalance is assumed to occur at node 1.

In the case of vibration level optimization, the diameters of shaft element, d_n, (n = element number), and the stiffness and damping coefficient of the bearing, K_m, C_m, (m = 1, 2) are chosen as design variables. Thus, the design variables Q for the spindle-bearing system model should be written as follows:

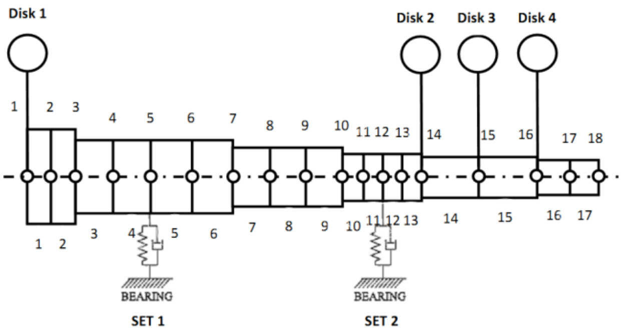

Fig. 2 Discretization (finite element) model of spindle system.

$$Q = [d_1, d_2, \ldots, d_{17}, K_1, K_2, C_1, C_2] \tag{9}$$

Due to the bearing dimension constraint, avoiding critical stress and the stability of optimization process is ensured then the upper and lower values on the diameters of shaft need to be set. The lower and upper bounds on the diameter of the shaft elements are given by Q_L = 0.017 m and Q_U = 0.106 m except in the vicinity of the bearings there is no change of the shaft diameter due to limitations of the bearing size.

For solving the optimization problem, the first is to determine critical frequencies in the main concern of operating speeds range, and then proceed to calculate the magnitude of the unbalance response caused by these critical frequencies. These two things will give an overview about the vibration level of the spindle-bearing system behavior, and the responses with high amplitude chosen as a target value of the optimization process in which the amplitude needs to be reduced.

Initial simulation results show that, the spindle system has two forward modes of the two-first critical frequencies, which are first forward mode Ω_{1F} = 11910 rpm and second forward mode Ω_{2F} = 21120 rpm, respectively. Due to the first forward mode has a small modal damping ratio (ζ_{1F} = 0.05), it may lead to a very high response peak as illustrated in Fig. 3. The initial values of critical frequency and maximum amplitude of vibration at the first forward mode $(1F)^{(0)}$ are

$$\Omega_{1F}^{(0)} = 11910 \text{ rpm}, \quad A_{1F}^{(0)} = 5.032\times10^{-5} \text{ m}$$

For the optimization procedure, by substituting the original model values into Eq. (8), re-arranged should be written as

$$\begin{aligned} &\text{Minimize mass } M(Q) \\ &\Omega_m(Q) \geq \Omega_m^* = \Omega_{1F}^{(0)}, \\ &A_m(\Omega_m) \leq A_m^* = A_{1F}^{(0)}, \\ &0.017 \leq Q \leq 0.106. \end{aligned} \tag{10}$$

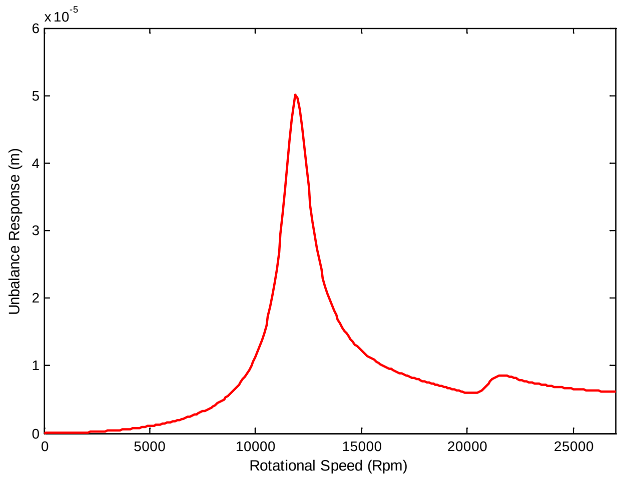

Fig. 3 Unbalance response of the spindle system.

The numerical values are initial mass M = 14.4 kg, operating speed Ω = 8000 rpm and initial values of the design variables are $d_1 \sim d_2$ = 88 mm, $d_3 \sim d_6$ = 70 mm, $d_7 \sim d_9$ = 64.5 mm, $d_{10} \sim d_{13}$ = 60 mm, $d_{14} \sim d_{15}$ = 54.5, $d_{16} \sim d_{17}$ = 50.4. Furthermore, in this case characteristics of the spindle-bearings are considered as an isotropic bearing [17], which initial values are such as $K_{yy1} = K_{zz1} = 1.911\times10^8$ N/m, $K_{yy2} = K_{zz2} = 2.476\times10^8$ N/m, $C_{yy1} = C_{zz1} = 191.1\times10^2$ N.s/m and $C_{yy2} = C_{zz2} = 247.6\times10^2$ N.s/m. The initial radial displacement in Eq. (7) is A = 4.25 μm when the allowance residual unbalance (ISO 1940 G1) [16] was applied to the grinding wheel. In Figs. 4 and 5 the time response of the displacement in the y and z axis direction and the absolute displacement of the spindle system respectively before optimization are shown.

The optimum values of the spindle diameter and the damping and stiffness of the bearings that minimize the radial displacement of the spindle system are tabulated in Tables 2 and 3.

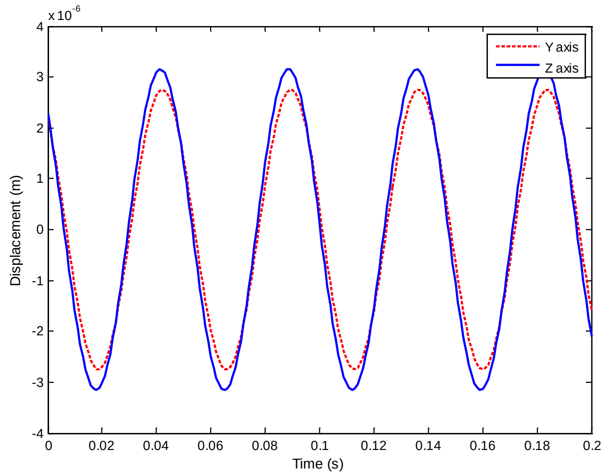

Fig. 4 Displacement amplitude of spindle-bearing system in the y and z axis (before optimization).

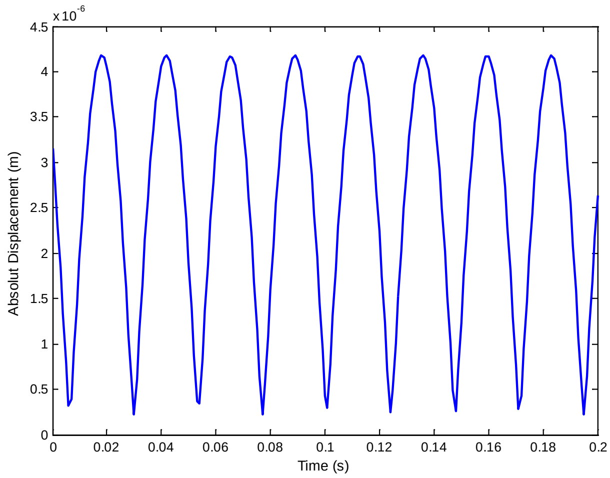

Fig. 5 Absolute displacement amplitude of the spindle-bearing system (before optimization)

Table 2. Shaft diameter of the spindle

Diameter	Initial values	Optimum values
$d_1 \sim d_2$	88	88
$d_3 \sim d_6$	70	70
$d_7 \sim d_9$	64.5	60
$d_{10} \sim d_{13}$	60	60
$d_{14} \sim d_{15}$	54.5	54.5
$d_{16} \sim d_{17}$	50.4	17

Table 3. Dynamic characteristics of the bearing

Bearing	Initial values	Optimum values
Stiffness (N/m)		
$K_{yy1} = K_{zz1}$	1.911×10^8	3.797×10^8
$K_{yy2} = K_{zz2}$	2.476×10^8	3.240×10^8
Damping (N.s/m)		
$C_{yy1} = C_{zz1}$	191.1×10^2	192.8×10^2
$C_{yy2} = C_{zz2}$	247.6×10^2	249.3×10^2

A graphic comparison of the unbalance response at the node 1 due to the residual unbalance before and after optimization is shown in Fig. 6. It can be seen that the spindle diameter and the damping and stiffness of the bearings are the design variables which effective to increase the critical frequency and to decrease the amplitude of the unbalance response for the first mode (1F). The total shaft mass, the 1st critical frequency and the unbalance response for the initial and optimum model which was optimized by hybrid genetic algorithm (HGA) are presented in Table 4.

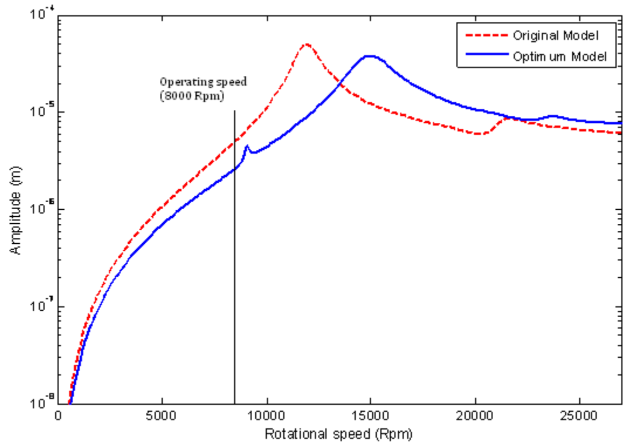

Fig. 6 Comparison of unbalance respon amplitude

Table 4. Optimum values computed by HGA

	Initial values	Optimum values
Total mass of shaft (kg)	14.40	13.44
1st critical speed Ω_{1F} (rpm)	11910	14838
Amplitude of unbalance response A_{1F} (m) at first mode	5.032×10^{-5}	3.691×10^{-5}

Numerical simulation result shows that, after optimizing the diameter of spindle shaft, and adjusting the bearings to an optimal stiffness and damping, which the allowance residual unbalance (1 gr.mm/kg) according to ISO 1940-1 G1 was applied to the grinding wheel, therefore the maximum radial displacement of the spindle for operating speed Ω at 8000 rpm would be A = 2.33 μm as illustrated in Fig. 6. In Figs. 7 and 8 the time response of the displacement in the y and z axis direction and the absolute displacement of the spindle system after optimization are shown respectively. In Fig. 8 the absolute displacement of the spindle system shows a great reducing, about 45.1% in the amplitude when compared with Fig. 5. This certainly can contribute to improve accuracy of the product of the machining.

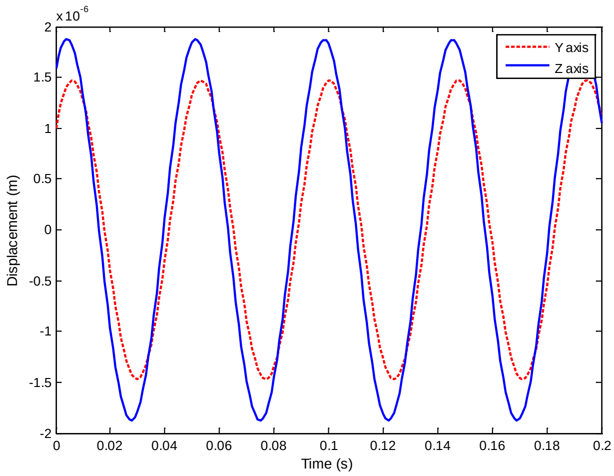

Fig. 7 Displacement amplitude of the spindle-bearing system in the y and z axis (after optimization)

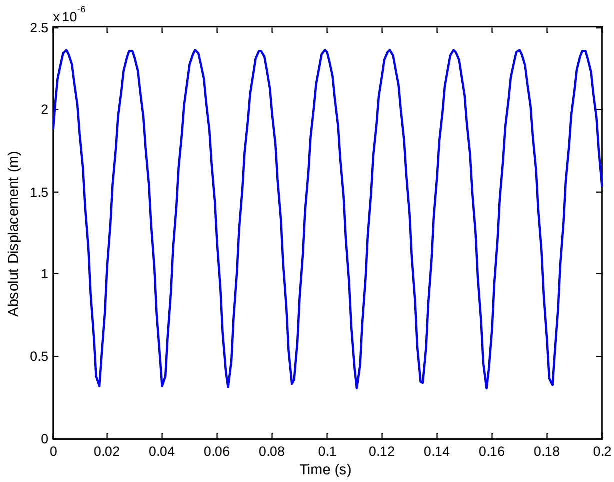

Fig. 8 Absolute displacement amplitude of the spindle-bearing system (after optimization)

4 Conclusion

An optimization design technique such as vibration level optimization has been implemented successfully in order to minimize radial vibration amplitude of the spindle-bearing system. In this study, vibration level optimization model was developed to find spindle diameter and damping-stiffness coefficient of the bearings optimum values by raising critical frequencies and reducing the amplitude of unbalance response. The objective function of this optimization problem is categorized as a single-objective problem which only to minimize the spindle mass under of the critical frequencies and the unbalance response constraints. Simulation results show that the radial vibration amplitude of the spindle for operating speed Ω at 8000 rpm is reduced satisfactory, about 45.1% when optimizing the diameter of spindle shaft, and adjusting the dynamic characteristics of the bearing to an optimum damping and stiffness coefficient. This certainly can improve the accuracy of the machining process. For future work, this vibration level optimization model need to be considered as a multi-objective problem, which including the spindle mass, critical frequencies, and unbalance responses as objective function.

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