

3.6 Trajectory Planning

It may be assumed that the angular velocity $\dot{\varphi} = \omega$ of the rotor is constant, then $\ddot{\varphi} = 0$.

Since the desired trajectory is limited to movement in the vertical direction, the following trajectory equations are obtained:

Vertical movement:

$$y^{ref} = R \left[\cos\left(\frac{\varphi_l}{2} - \varphi\right) - \cos\left(\frac{\varphi_l}{2}\right) \right]$$

$$\dot{y}^{ref} = \omega R \cdot \sin\left(\frac{\varphi_l}{2} - \varphi\right)$$

$$\ddot{y}^{ref} = -\omega^2 R \cdot \cos\left(\frac{\varphi_l}{2} - \varphi\right).$$

Movement in the axial direction:

$$x^{ref} = \dot{x}^{ref} = \ddot{x}^{ref} = 0.$$

Movement in the horizontal direction:

$$z^{ref} = \dot{z}^{ref} = \ddot{z}^{ref} = 0.$$

Tilting of the spindle:

$$\psi^{ref} = \dot{\psi}^{ref} = \ddot{\psi}^{ref} = 0,$$

$$\theta^{ref} = \dot{\theta}^{ref} = \ddot{\theta}^{ref} = 0.$$

3.7 Calculating the Control Current

The required currents in each horseshoe magnet must be calculated. This may be done by using (9) to first calculate the net resultant desired force in each bearing. Due to the geometry and design of the radial bearings, there is no unique combination of currents that may be used to obtain the desired resultant force.

Hence, it is necessary to choose of force arbitrarily. Choosing

$$F_{1,j} = \begin{cases} F_{0,j} & \text{if } F_{y,j} \leq -\frac{|F_{y,j}|}{\sqrt{3}} \\ F_{0,j} + F_{y,j} + \frac{|F_{z,j}|}{\sqrt{3}} & \text{otherwise,} \end{cases}$$

where $F_{0,j}$ is some arbitrarily chosen non-negative value and $j \in \{f, r\}$. The other forces can be calculated as follows:

$$F_{2,j} = F_{1,j} - F_{y,j} - \frac{F_{z,j}}{\sqrt{3}},$$

$$F_{3,j} = F_{1,j} - F_{y,j} + \frac{F_{z,j}}{\sqrt{3}}.$$

4 Experimental Results

As noted earlier, the rotor completes one vertical oscillation for every 7.2° change in the rotation angle. Therefore, for a rotational frequency of 100 rpm, the vertical oscillation frequency is 5000 rpm or about 523.6 rad/s or 83.3 Hz.

If the cutter-head has only two cutting edges, then the cutting edge comes into contact with the work piece only for $0 \leq \varphi \leq \varphi_1$ and $180^\circ \leq \varphi \leq 180^\circ + \varphi_1$, where $\varphi_1 = 3.6^\circ$ and the cutting starts at $\varphi = 0$. This was taken into consideration when the controller parameters were fine-tuned.

Fig. 9 shows the desired vertical displacement required along with the actual vertical displacement achieved. An over-shoot is clearly observed along with a delay/phase-angle.

The desired trajectory of the cutting edge can be seen in Fig. 10. Had there been no vertical movement of the spindle the cutting edge would follow the circle, leading to the creation of cutter-marks. If the cutting edge follows the desired trajectory perfectly then a flat surface is produced, as shown by the horizontal dashed line. The actual path is also shown in Fig. 10. It can be seen that the delay and overshoot cause the cutting edge to deviate from the desired path; nevertheless, significant improvement is achieved, that is, the size of the cutter-marks is reduced.

It is obvious from Fig. 9 that there is a lag in the system. The actual trajectory seems to follow the desired trajectory with a lag which appears to be rather constant. One way to eliminate lag is to shift the reference trajectory so that it coincides with actual trajectory.

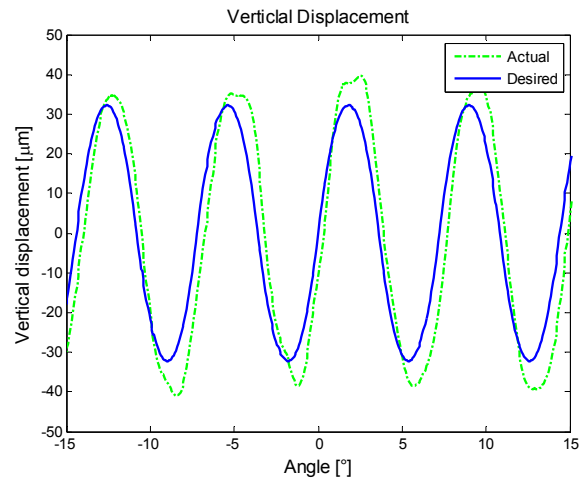


Figure 9: Plot of the desired and actual vertical displacement of the rotor

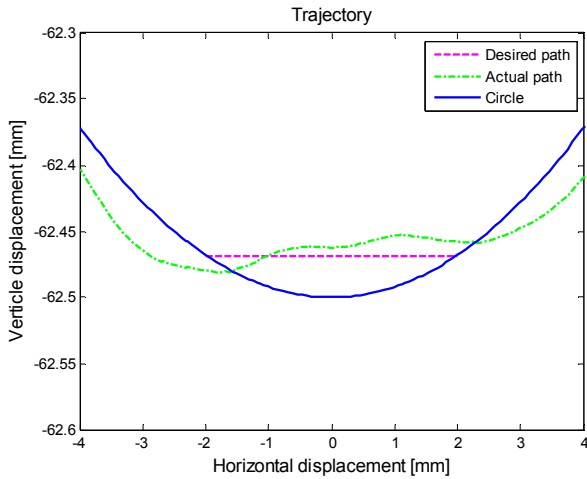


Figure 10: Trajectory of the cutting-edge (scaled)

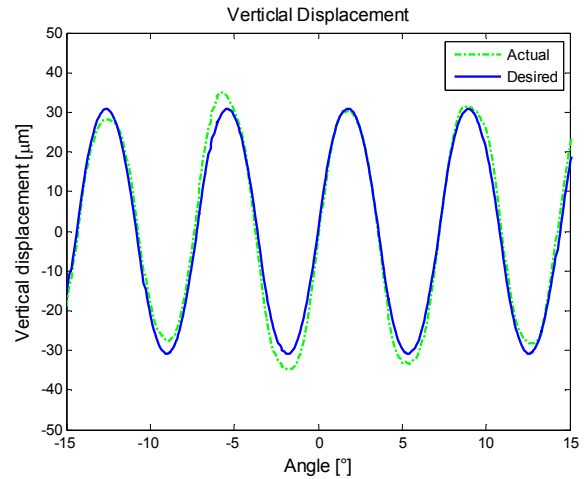


Figure 11: Plot of the desired and actual vertical displacement of the rotor

Define

$$\varphi_f = \varphi + k_1 T_a \omega$$

where k_1 is an integer, T_a is the sampling time and φ_f is the angle of rotation after k_1 number of time steps.

The lag can be eliminated by choosing an appropriate value for k_1 . To achieve this goal, define

$$y_f^{ref} = R \left[\cos\left(\frac{\varphi_l}{2} - \varphi_f\right) - \cos\left(\frac{\varphi_l}{2}\right) \right],$$

where y_f^{ref} is the desired reference trajectory with respect to φ_f . This yields the reference acceleration:

$$\ddot{y}_f^{ref} = -\omega^2 R \cdot \cos\left(\frac{\varphi_l}{2} - \varphi_f\right).$$

Then a new reference trajectory is defined as follows:

$$y_{new}^{ref} = y^{ref} + k_2 (y_f^{ref} - y^{ref}).$$

The new reference trajectory, y_{new}^{ref} , is a linear combination of the current reference trajectory and the reference trajectory k_1 number of time steps in the future. The use of this new reference trajectory helps eliminate the time lag.

Similarly, the new reference acceleration is defined as

$$\ddot{y}_{new}^{ref} = \ddot{y}^{ref} + k_3 (\ddot{y}_f^{ref} - \ddot{y}^{ref}).$$

As opposed to the new reference trajectory, which helps eliminate lag, the new reference acceleration can be used to eliminate the overshoot that is apparent in Fig. 9, with the appropriate choice of the tuning parameter k_3 .

It can be seen in Fig. 11 and Fig. 12 that significant improvement is achieved. There exists certain systematic noise in the system due to structural inhomogeneity (see Fig. 14). This can be observed by looking at the tracking error over a number of revolutions of the spindle. Therefore, it is important to adjust for k_2 and k_3 according to the angle during which the cutting will take place.

It can be seen from Fig. 13 that the tracking error for the case with adjustment is much lower than the case without adjustment. Specifically, during the cutting phase $0^\circ < \varphi < 3.6^\circ$ the error is extremely small. The large error for the case without adjustment can be attributed to the lag and not the overshoot.

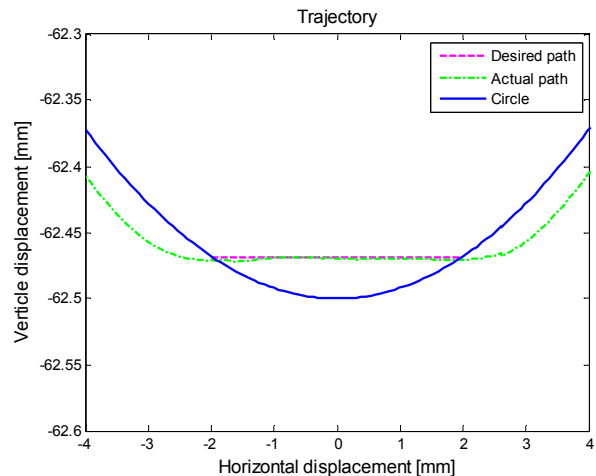


Figure 12: Plot trajectory of the cutting-edge (scaled)

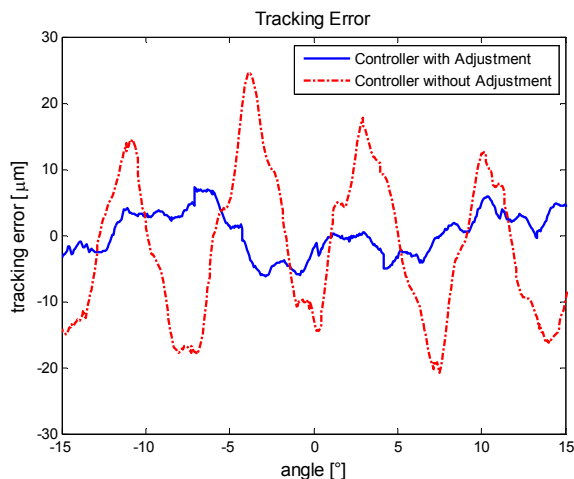


Figure 13: Tracking error

5 Conclusions and Outlook

Milling operation on wood produces specific surface patterns known as cutter-marks. This is due to the kinematic relation between cutting tool and the work piece, and the exact shape of the cutter-marks are dependent on the cutting parameters such as the number of cutting edges, feed speed, etc. The method used to reduce or eliminate cutter-marks is by periodic actuation of the cutting tool in order to produce linear motion during the cutting phase. The vertical motion of the cutting tool is in the micron-range.

The authors had shown in [8] that active magnetic bearings can be used to reduce cutter-mark depth. Improvements were suggested and implemented in this study and the results show significant improvement in the reduction of the grooves formed. In particular, both the overshoot and the delay have been eliminated for all practical purposes.

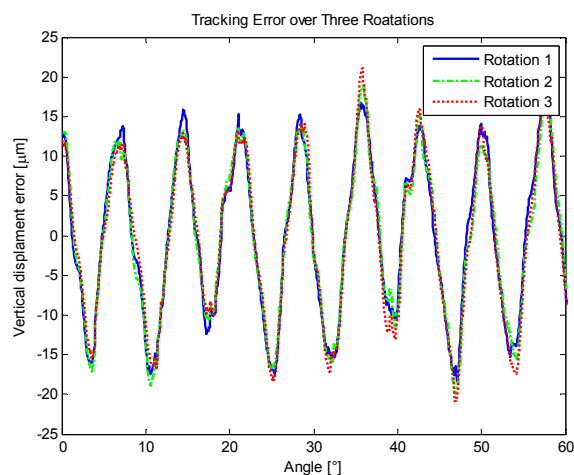


Figure 14: Tracking error plotted over three revolutions

In future works, the reference trajectory may be changed to reduce or eliminate the vertical oscillations during the non-cutting phase. In addition, a reference trajectory can be chosen so that the required acceleration is not discontinuous. Furthermore, Fourier analysis will be used to derive a systematic method for a further reduction of the tracking error. The results will be presented in upcoming publications.

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