Explicit and Implicit TVD and ENO High Resolution Algorithms Applied to the Euler and Navier-Stokes Equations in Three-Dimensions – Turbulent Results

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Abstract: - In the present work, the Harten and Osher TVD/ENO and the Yee TVD symmetric schemes are implemented, on a finite volume context and using a structured spatial discretization, to solve the laminar/turbulent Navier-Stokes equations in the three-dimensional space. The Harten and Osher TVD/ENO schemes are flux difference splitting type, whereas the Yee TVD scheme is a symmetric one, which incorporates TVD properties due to the appropriated definition of a limited dissipation function. All three schemes are second order accurate in space. Turbulence is taken into account considering two algebraic models, namely: the Cebeci and Smith and the Baldwin and Lomax ones. A spatially variable time step procedure is also implemented aiming to accelerate the convergence of the algorithms to the steady state solution. The gains in convergence with this procedure were demonstrated in Maciel. The schemes are applied to the solution of the physical problem of the low supersonic flow along a ramp. The results have demonstrated that the most accurate results are obtained with the Harten and Osher ENO scheme. This paper is the third part of this work, TURBULENT RESULTS, considering the description of the turbulence models and the solutions obtained with them and compared with the laminar results.

Key-Words: - Harten and Osher algorithm, TVD/ENO formulations, Yee symmetric algorithm, TVD formulation, Euler and Navier-Stokes equations, Turbulence models, Explicit and implicit algorithms, Finite Volumes.

1 Introduction

In the present work, the [1] TVD/ENO and the [2] TVD symmetric schemes are implemented, on a finite volume context and using a structured spatial discretization, to solve the laminar/turbulent Navier-Stokes equations in the three-dimensional space. The [1] TVD/ENO schemes are flux difference splitting type, whereas the [2] TVD scheme is a symmetric one, which incorporates TVD properties due to the appropriated definition of a limited dissipation function. All schemes are second order accurate in space and their numerical implementation is based on the concept of [3]'s modified flux function. Turbulence is taken into account considering two algebraic models, namely: the [4-5] ones. The viscous simulations are treated with the explicit versions of the present algorithms, which employ a time splitting method ([6]). The schemes are accelerated to the steady state solution using a spatially variable time step procedure, which has demonstrated effective gains in terms of convergence rate ([7-8]). The algorithms are applied to the solution of the physical problem of the supersonic flow along a ramp. The results have demonstrated that the most accurate results are obtained with the [1] ENO scheme. The inviscid and laminar results are presented in [9].

The main contribution of the present work to the CFD (Computational Fluid Dynamics) community is the extension of the [1] TVD/ENO schemes, as well as the [2] TVD symmetric scheme, to three-dimensions, following a finite volume context, and their implementation coupled with two different turbulence algebraic models to simulate viscous turbulent flows, which characterizes an original contribution in the field of high resolution structured numerical algorithms. Details of the numerical implementation of the present algorithms are described in [6].

2 Turbulence Models

2.1 Turbulence model of [4]

The problem of the turbulent simulation is in the calculation of the Reynolds stress. Expressions involving velocity fluctuations, originating from the

and the best solution was again obtained by the [4] model. Figure 30 presents the wall pressure distribution obtained by the [2] TVD Min1 scheme to the three studied cases. The [4] model and the laminar solutions were closer to the theoretical result. Finally, Figure 31 presents better pressure distributions generated by the [2] TVD Min2 scheme in the laminar case and using the [4] model.

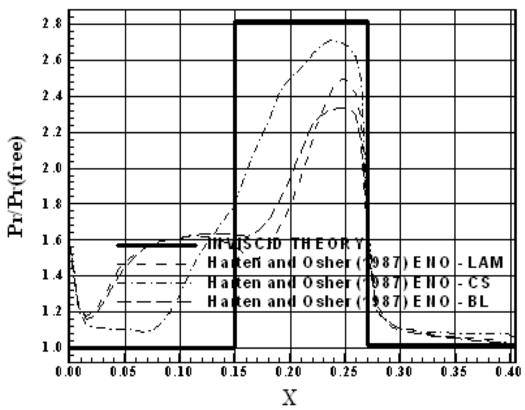

Figure 28 : Wall pressure distributions ([1]-ENO).

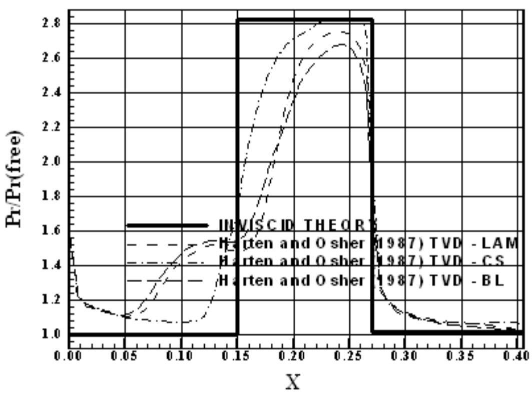

Figure 29 : Wall pressure distributions ([1]-TVD).

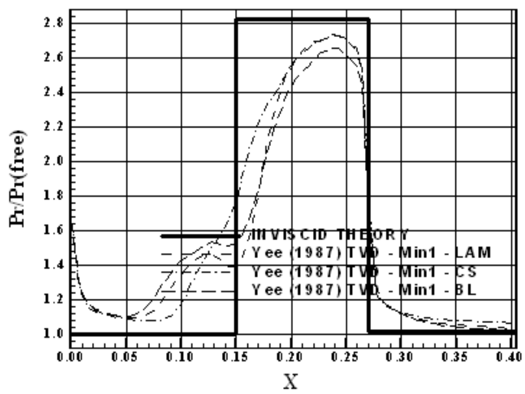

Figure 30 : Wall pressure distributions ([2]-Min1).

Table 3 presents the points of detachment and reattachment involving the laminar and turbulent cases to the four algorithms tested in these viscous simulations.

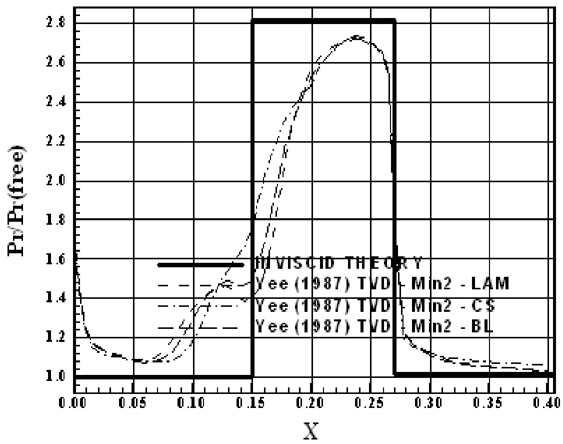

Figure 31 : Wall pressure distributions ([2]-Min2).

Analysing this table, it is possible to highlight that the [2] TVD Min2 scheme presents the minimum region of separated flow in comparison with the other schemes in all three possible cases studied in this work. As the behaviour expected for the turbulence model is to reduce the extension of the separation region due to better equilibrium characteristics inherent to turbulent boundary layers, the [2] TVD Min2 scheme reaches such goal satisfactorily in theoretical terms.

Table 3 : Boundary layer detachment and reattachment points.

Laminar	[1] ENO	[1] TVD	[2] Min1	[2] Min2
Det.(1) (m)	0.05	0.08	0.09	0.10
Reat.(2)(m)	0.25	0.20	0.20	0.20
[4]	[1] ENO	[1] TVD	[2] Min1	[2] Min2
Det. (m)	0.10	0.12	0.12	0.12
Reat. (m)	0.20	0.18	0.18	0.18
[5]	[1] ENO	[1] TVD	[2] Min1	[2] Min2
Det. (m)	0.08	0.09	0.09	0.10
Reat. (m)	0.22	0.20	0.20	0.20

(1): Detachment; (2): Reattachment.

Table 4 : Shock angle obtained in the laminar and turbulent cases.

Laminar	[1] ENO	[1] TVD	[2] Min1	[2] Min2
β (°)	48.00	54.20	53.00	52.80
Error (%)	9.43	2.26	0.00	0.38
[4]	[1] ENO	[1] TVD	[2] Min1	[2] Min2
β (°)	52.20	54.50	53.90	54.00
Error (%)	1.51	2.83	1.70	1.89
[5]	[1] ENO	[1] TVD	[2] Min1	[2] Min2
β (°)	52.00	54.00	51.40	55.00
Error (%)	1.89	1.89	3.02	3.77

Aiming a global comparison involving the shock angle of the oblique shock waves estimated by the schemes in these viscous simulations, Tab. 4 exhibits the values of these angles and respective errors. As can be observed, the [1] ENO scheme presents more accurate values of the angle of the oblique shock wave in two of the three studied cases (in both turbulent cases). The global error was less than 4.0% to all schemes, except to the [1] ENO scheme in the laminar case.

Table 5 shows the computational data of the numerical simulations in the viscous laminar and turbulent cases in the ramp problem. All four schemes to the viscous laminar and turbulent cases used an explicit formulation to the simulations. As observed, the fastest scheme is due to [1] TVD algorithm in all cases.

Table 5 : Computational data of the explicit algorithms to the ramp viscous cases.

	Laminar		[4]		[5]	
Scheme	CFL	Iter.(1)	CFL	Iter.	CFL	Iter.
[1] ENO	0.9	3,515	0.9	3,141	0.9	4,160
[1] TVD	0.9	1,755	0.9	955	0.9	2,094
[2] TVD – Min1	0.5	3,374	0.5	4,632	0.5	4,342
[2] TVD – Min2	0.5	3,026	0.5	4,657	0.5	3,438

(1): Iterations.

Table 6 exhibits the computational costs of the numerical algorithms obtained in the viscous laminar and turbulent cases. The cheapest algorithm in the laminar case is due to [2] TVD Min2, while the most expensive is due to [1] ENO. In the turbulent case, using the [4] model, the cheapest algorithm is due to [2] TVD Min1, while the most expensive is again due to [1] ENO. Finally, using the [5] model, the cheapest algorithm is the [2] TVD Min2 scheme, while the most expensive is again the [1] ENO scheme. As conclusion, in general the [2] TVD Min2 scheme yields the cheapest one in terms of viscous laminar and turbulent simulations.

Table 6. Computational cost of the numerical algorithms (laminar and turbulent cases).

	Computational Cost(1)		
Scheme	Laminar	[4]	[5]
[1] ENO	0.0000779	0.0001612	0.0000915
[1] TVD	0.0000744	0.0001548	0.0000885
[2] TVD Min1	0.0000694	0.0001523	0.0000843
[2] TVD Min2	0.0000692	0.0001524	0.0000841

(1): Measured in seconds/per cell/per iterations.

4 Conclusions

In the present work, the [1] TVD/ENO and the [2] TVD symmetric schemes are implemented, on a finite volume context and using a structured spatial discretization, to solve the laminar/turbulent Navier-Stokes equations in the three-dimensional space. The [1] TVD/ENO schemes are flux difference splitting type, whereas the [2] TVD scheme is a symmetric one, which incorporates TVD properties due to the appropriated definition of a limited dissipation function. All schemes are second order accurate in space and their numerical implementation is based on the concept of [3]'s modified flux function. Turbulence is taking into account considering two algebraic models, namely: the [4-5] ones. The viscous simulations are treated with the explicit versions of the present algorithms, which employ a time splitting method ([6]). The schemes are accelerated to the steady state solution using a spatially variable time step procedure, which has demonstrated effective gains in terms of convergence rate ([7-8]). The algorithms are applied to the solution of the physical problem of the low supersonic flow along a ramp.

The results have demonstrated that the most accurate results are obtained with the [1] ENO scheme. In the inviscid case ([9]), it is possible to highlight that the [2] TVD VL scheme yields the best pressure distribution along the nozzle lower wall. In the compression corner, the [1] ENO and TVD schemes present better pressure distributions than those generated by the [2] TVD schemes. The [2] TVD Min1, Min2, Min3 and VL variants present oscillations in the pressure distributions. The shock angle of the oblique shock wave that is formed at the compression corner is best estimated by the [1] ENO and [2] TVD VL algorithms. The most expensive tested implicit scheme was due to [1] ENO scheme, while the cheapest was the [2] TVD Min2 scheme. The former is approximately 172.91% more expensive than the latter.

In the ramp viscous case, the laminar results ([9]) present the [2] TVD Min1 scheme as yielding the best value to the shock angle at the ramp. In the turbulent case, present paper, the [4] model presents the [1] ENO scheme as yielding the best estimation, while in the [5] model, the [1] ENO and TVD schemes produce the best values to the shock angle. In the laminar case, the [1] ENO scheme presents the biggest separation region with the formation of a circulation bubble. The best pressure distribution, closest with the inviscid solution - true solution considering the boundary layer theory -, was obtained by the [1] TVD scheme. Employing the [4] model, all tested algorithms present the minimum separation region, with this model energizing the boundary layer sufficiently to guarantee the minimum circulation region. Again, the [1] TVD scheme presents the pressure distribution at the wall closest with inviscid solution. With the [5] model, the best pressure distribution is obtained by the [2] TVD Min2 algorithm - in relation to the inviscid solution). The minimum region of separation was obtained by the [2] TVD Min2 scheme in all three cases (laminar and the two turbulent cases). Considering the values estimated by the shock angle of the oblique shock wave, the [1] ENO algorithm presents the best values to this parameter in two of the three cases – in the two turbulent cases. As general conclusion in terms of viscous simulations, all algorithms present the best solution considering wall pressure distribution as using the [4] model.

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