[16] turbulence model, as well the respective errors, shown in Tab. 17. As can be observed, the results of all three limiters employed with the [7] TVD scheme using the [16] model present the same values to the shock angle, with a percentage error of 0.53%.

Table 17. Shock angle of the oblique shock wave at the ramp and percentage error ([7]-Turbulent).

Scheme	β (°)	Error (%)
[7]-VL	37.9	0.53
[7]-VA	37.5	0.53
[7]-Min	37.9	0.53

As conclusion of the study analyzing the [7] TVD scheme in its three variants using the [16] model, the best variant is the [7] TVD scheme using VL limiter due to better accuracy in the determination of the shock angle and in the determination of the most severe pressure field.

As global conclusion, the most severe pressure field using the [16] model was obtained by the [6] scheme using VA limiter, as occurred in the laminar case. The best wall pressure distributions are obtained by the [4] TVD scheme in its three variants, namely: VL, VA and Min, and by the [5] TVD scheme in its two variants, namely: VA and Min. They present pressure plateau closer to the theoretical results than the others TVD schemes. The minimum extent of the separated flow region was detected by the [4] TVD scheme using Min limiter, as occurred in the laminar case.

Table 18 - Shock angle of the oblique shock wave at the ramp and percentage error (Best Results-Turbulent).

Scheme	β (°)	Error (%)
[4]-VA	37.8	0.27
[5]-VA	37.0	1.86
[5]-Min	37.0	1.86
[6]-VA	37.5	0.53
[7]-VL	37.9	0.53
[7]-VA	37.5	0.53
[7]-Min	37.9	0.53

Table 18 presents the best values of the shock angle of the oblique shock wave obtained by each scheme. The best result of all schemes, detecting more precisely the value of the shock angle, is due to [4] TVD scheme using VA limiter. As the [4] TVD scheme has presented the best value to the shock angle of the oblique shock wave with the VA variant, the best pressure distribution in this three variants and the minimum region of separated flow with the Min limiter, the best scheme in this viscous turbulent simulation using the [16] model is due to the [4] scheme using VA and Min limiters, although the [5] in its two variants also capture good pressure distributions.

2.3 Viscous Final Conclusions

As Final conclusion, the most severe pressure field was obtained by the [6] scheme using VA limiter. This behavior was observed in the laminar case and in the turbulent cases using the [15-16] models. The best wall pressure distributions are obtained by the [4] TVD scheme in its three variants, namely: VL, VA and Min, and by the [5] TVD scheme in its two variants, namely: VA and Min, in the laminar and in the turbulent case with the [16] model. They present the pressure plateau closer to the theoretical results than the other TVD schemes. The minimum extent of the separated flow region was detected by the [4] TVD scheme using Min limiter. This was observed in the laminar as well in the turbulent cases.

Table 19. Shock angle of the oblique shock wave at the ramp and percentage error (Best Results).

Scheme	β (°)	Error (%)
[7]-Min-Lam	37.6	0.27
[6]-VL-CS	37.7	0.00
[6]-Min-CS	37.7	0.00
[4]-VA-BL	37.8	0.27

Table 19 presents the best values of the shock angle of the oblique shock wave obtained by each scheme, considering laminar and turbulent cases. The best results are due to [6] using the [15] model and the VL and Min limiters.

As the [6] TVD scheme has presented the most severe pressure field using VA limiter in all cases, laminar and turbulent, and the best value to the shock angle of the oblique shock wave with the VL and Min variants, with an error of 0.00% using the [15] model, the best final scheme is the [6] TVD scheme in its three variants.

3 Conclusions

In the present work, the [4-7] schemes are implemented, on a finite volume context and using a structured spatial discretization, to solve the Euler and the laminar/turbulent Navier-Stokes equations in the three-dimensional space. All schemes are flux vector splitting ones and in their original implementations are first order accurate. A MUSCL approach is implemented in these schemes aiming to obtain second order spatial accuracy. The Van Leer, the Van Albada and the Minmod nonlinear limiters are employed to guarantee such accuracy and TVD high resolution properties. These flux vector splitting schemes employ approximate factorizations in ADI form to solve implicitly the Euler equations. To solve the laminar/turbulent Navier-Stokes equations, an explicit formulation based on a dimensional splitting procedure is employed. All schemes are first order accurate in time in their implicit and explicit versions. Turbulence is taken into account considering two algebraic models, namely: the [14-15] ones. The algorithms are accelerated to the steady state solution using a spatially variable time step, which has demonstrated effective gains in terms of convergence rate ([20-21]. All four schemes are applied to the solution of the physical problems of the supersonic flow along a compression corner, in the inviscid case, and of the supersonic flow along a ramp, in the laminar and turbulent cases. The results have demonstrated that the most severe and most accurate results are obtained with the [6] TVD high resolution scheme.

In the implicit inviscid case, the most severe pressure field was obtained by the [6] scheme using VA limiter. The best wall pressure distributions obtained by each scheme is shown in [8]. The best wall pressure distribution among the distributions presented is due to [6] using Min limiter. [8] also presents the best values of the shock angle of the oblique shock wave obtained by each scheme. Except the [4] TVD scheme, all others schemes always present a variant with the correct value of the shock angle. As the [6] TVD scheme has presented the best wall pressure distribution using Min limiter and as it also presents the correct value of the shock angle of the oblique shock wave with this variant (the other two limiters too), the best scheme in the inviscid simulation is due to the [6] scheme using Min limiter.

In the viscous case, the most severe pressure field was obtained by the [6] scheme using VA limiter. This behavior was observed in the laminar case and in the turbulent cases using the [15-16] models. The best wall pressure distributions are obtained by the [4] TVD scheme in its three variants, namely: VL, VA and Min, and by the [5] TVD scheme in its two variants, namely: VA and Min, in the laminar and in the turbulent case with the [16] model. They present the pressure plateau closer to the theoretical results than the other TVD schemes. The minimum extent of the separated flow region was detected by the [4] TVD scheme using Min limiter. This was observed in the laminar as well in the turbulent cases.

Table 19 presents the best values of the shock angle of the oblique shock wave obtained by each scheme, considering laminar and turbulent cases. The best results are due to [6] using the [15] model and the VL and Min limiters.

As the [6] TVD scheme has presented the most severe pressure field using VA limiter in all cases, laminar and turbulent, and the best value to the shock angle of the oblique shock wave with the VL and Min variants, with an error of 0.00% using the [15] model, the best final scheme is the [6] TVD scheme in its three variants.

As final conclusion, the present author recommends the [6] TVD scheme, due to the best performance in the inviscid and viscous laminar and turbulent cases, to obtain more severe and accurate solutions in the three-dimensional space, which are desirable properties to the design and experimental phases of aerospace vehicles.

4 Acknowledgments

The present author acknowledges the CNPq by the financial support conceded under the form of a DTI (Industrial Technological Development) scholarship no. 384681/2011-5. He also acknowledges the infra-structure of the ITA that allowed the realization of this work.

References:

[1] P. Kutler, Computation of Three-Dimensional, Inviscid Supersonic Flows, *Lecture Notes in Physics*, Springer Verlag, Berlin, Vol. 41, 1975, pp. 287-374.

[2] J. L. Steger, Implicit Finite-Difference Simulation of Flow About Arbitrary Two-Dimensional Geometries, *AIAA Journal*, Vol. 16, No. 7, 1978, pp. 679-686.

[3] P. L. Roe, Approximate Riemann Solvers, Parameter Vectors, and Difference Schemes, *Journal of Computational Physics*, Vol. 43, 1981, 357-372.

[4] J. L. Steger, R. F. Warming, Flux Vector Splitting of the Inviscid Gasdynamic Equations with Application to Finite-Difference Methods,

Journal of Computational Physics, Vol. 40, 1981, pp. 263-293.
[5] B. Van Leer, Flux-Vector Splitting for the Euler Equations, *Lecture Notes in Physics*, Vol. 170, 1982, pp. 507-512, Springer Verlag, Berlin.
[6] M. Liou, C. J. Steffen Jr., A New Flux Splitting Scheme, *Journal of Computational Physics*, Vol. 107, 1993, pp. 23-39.
[7] R. Radespiel, N. Kroll, Accurate Flux Vector Splitting for Shocks and Shear Layers, *Journal of Computational Physics*, Vol. 121, 1995, pp. 66-78.
[8] E. S. G. Maciel, TVD Flux Vector Splitting Algorithms Applied to the Solution of the Euler and Navier-Stokes Equations in Three-Dimensions – Part I, WSEAS Transactions on Fluid Mechanics (under revision).
[9] C. Hirsch, *Numerical Computation of Internal and External Flows – Computational Methods for Inviscid and Viscous Flows*, John Wiley & Sons Ltd, 691p, 1990.
[10] R. M. Beam, R. F. Warming, An Implicit Factored Scheme for the Compressible Navier-Stokes Equations, *AIAA Journal*, Vol. 16, No. 4, 1978, pp. 393-402.
[11] J. Douglas, On the Numerical Integration of $u_{xx}+u_{yy}=u_t$ by Implicit Methods, *Journal of the Society of Industrial and Applied Mathematics*, Vol. 3, 1955, pp. 42-65.
[12] D. W. Peaceman, H. H. Rachford, The Numerical Solution of Parabolic and Elliptic Differential Equations, *Journal of the Society of Industrial and Applied Mathematics*, Vol. 3, 1955, pp. 28-41.
[13] J. Douglas, J. E. Gunn, A General Formulation of Alternating Direction Methods, *Numerische Mathematiks*, Vol. 6, 1964, pp. 428-453.
[14] N. N. Yanenko, *The Method of Fractional Steps*, Springer Verlag, NY, EUA, 1971.
[15] T. Cebeci, A. M. O. Smith, A Finite-Difference Method for Calculating Compressible Laminar and Turbulent Boundary Layers, *Journal of Basic Engineering*, Trans. ASME, Series B, Vol. 92, No. 3, 1970, pp. 523-535.
[16] B. D. Baldwin, H. Lomax, Thin Layer Approximation and Algebraic Model for Separated Turbulent Flows, *AIAA Paper 78-257*, 1978.
[17] E. S. G. Maciel, Turbulent Flow Simulations Using the MacCormack and the Jameson and Mavriplis Algorithms Coupled with the Cebeci and Smith and the Baldwin and Lomax Models in Three-Dimensions, *Engineering Applications of Computational Fluid Mechanics*, China, Vol. 1, No. 3, 2007, pp. 147-163.
[18] R. W. MacCormack, The Effect of Viscosity in Hypervelocity Impact Cratering, *AIAA Paper 69-354*, 1969.
[19] A. Jameson, D. J. Mavriplis, Finite Volume Solution of the Two-Dimensional Euler Equations on a Regular Triangular Mesh, *AIAA Journal* Vol. 24, No. 4, 1986, pp. 611-618.
[20] E. S. G. Maciel, Analysis of Convergence Acceleration Techniques Used in Unstructured Algorithms in the Solution of Aeronautical Problems – Part I, *Proceedings of the XVIII International Congress of Mechanical Engineering (XVIII COBEM)*, Ouro Preto, MG, Brazil, 2005. [CD-ROM]
[21] E. S. G. Maciel, Analysis of Convergence Acceleration Techniques Used in Unstructured Algorithms in the Solution of Aerospace Problems – Part II, *Proceedings of the XII Brazilian Congress of Thermal Engineering and Sciences (XII ENCIT)*, Belo Horizonte, MG, Brazil, 2008. [CD-ROM]
[22] R. W. Fox, A. T. McDonald, *Introdução à Mecânica dos Fluidos*, Ed. Guanabara Koogan, Rio de Janeiro, RJ, Brazil, 632p, 1988.
[23] J. D. Anderson Jr., *Fundamentals of Aerodynamics*, McGraw-Hill, Inc., EUA, 563p, 1984.