Steady-state Accident Rate of a Plant Equipped with an Aging Single-channel Trip Device

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Abstract: - Protective devices are of utmost importance in industrial facilities for they have the capacity of monitoring important plant parameters and, if necessary, shut the plant down. One important feature in this case is when a protective channel undergoes aging, because this requires decisions on preventive maintenance in order to delay aging or even channel replacement, whichever is more cost-effective. We present in this paper the steady-state accident rate evaluation for a plant equipped with a single aging channel for comparing it with the transient analysis developed elsewhere by means of finite differences. The calculations are much simpler to perform and only one numerical integration is necessary. For the typical plant parameters used we concluded that the steady-state solution is not feasible because it generates quite conservative results when one considers typical proof test intervals. A sensitivity analysis on the results was also performed, which showed that the protective channel failure rate during its useful life is an important parameter. In this sense, it is not advisable to use steady-state parameters for making decisions regarding plant accident rates because the costs involved would be unsurmountable and it is concluded that finite-difference methods should be used and improved for more realistic decisions.

Key-Words: - Demand rate, Markovian reliability analysis, Supplementary variables, Plant accident rate, Steady-state behavior, Plant useful life.

1 Introduction

The importance of environmental and occupational accidents is related to the evolution of industrial activity and the relations of production and consumption over time. The evolution of the competitive nature of the industrial sector, combined with the growth of the world economy and the advancement of technology, has boosted the growth of industrial plants and the complexity of production processes. The social context has also been transformed and other issues, such as environmental pollution, safety and human health, have become a matter of concern to the public and to governments. Consequently, the industry has been obliged to examine the effects of its operations on the public and, in particular, to examine more carefully the possible hazards arising from its activities.

In nuclear power plants, one of the most complex and important systems is the reactor protection system, which is defined as the system that turns off the reactor and keeps it in a safe condition in the event of a transient or malfunction that may cause damage to the reactor core, mainly due to overheating [1]. The adequate functioning of this protection system is imperative for the optimal and safe operation of the nuclear reactor.

The reliability analysis of protection system components, such as the reactor protection system in nuclear power plants, is extremely important and widely studied. Despite this vast experience, an important issue needs to be addressed: the treatment of components that are no longer in their useful life, that is, they are aging.

To admit that a component is aging means that its failure times no longer follow exponential distributions, which assigns great complexity to the problem. It implies the use of models with increasing failure rates to represent them, such as the Weibull or lognormal distributions [2]. In this sense, the process becomes Non-markovian.

For the modeling of this problem, one must look for possible alternatives. By means of methods that use stochastic processes, one model stands out: the supplementary variables method [3]. This method has the advantage of being able to generate a general solution to the problem (of course, the intrinsic numerical error must be considered), and therefore does not require the use of optimization methods, unlike other methods, such as the method of stages [3].

The purpose of this paper is to obtain the steady-state solution for the accident rate for a plant equipped with a single-channel protective system and discuss its application, since to obtain the transient solution for the problem requires the use of finite differences [4].

The discussion of the reliability of a single-channel protective system was first addressed in [5], where a general expression was derived without considering repair. An improved approach was later published [6], where channel repair was considered and its effects on the plant accident rate were discussed. Later on, the discussion of redundancy (i.e., consideration of two protective channels) was derived and discussed in terms of repair policies [7]. Still for the case of a single protective channel, a sensitivity analysis on parameters was performed to investigate their influence on the plant accident rate by considering the generalized perturbation theory [8], originally developed for nuclear reactor physics applications. Investigations considering further channel logics were addressed for the case of beyond two channels also [9] and a detailed sensitivity analysis was performed for this case also by means of the generalized perturbation theory [10].

The consideration of aging effects was addressed in earlier papers by means of supplementary variables for some important cases [11, 12]. The consideration of this approach for a single protective channel was presented and discussed by alternative methods, like supplementary variables [13] and Monte Carlo simulation [14]

This paper is organized as follows: Section 2 discusses the concept of protective systems, while Section 3 is dedicated to the quantitative model for the evaluation of the plant accident rate. As the purpose of the paper is the steady-state solution, Section 4 presents the calculations for achieving this goal. Section 5 displays the results obtained and, finally, Section 6 is dedicated to the conclusions reached.

2 Protective Systems

As mentioned in the introduction, protection systems are key components for the proper functioning of a nuclear facility. Linked to the control of nuclear reactor status variables, these systems determine whether to shut down the entire facility. That is, in the event of any significant transient occurring, the protective system must act in order to shut down and maintain the reactor in a safe condition.

The operation of a nuclear reactor protection system from this simple model follows the order: sensors monitor the reactor control parameters, such as pressure, temperature, or neutron flux. When necessary, the logic of the protection system will decide whether to start the reactor shutdown. Several actuation logics are employed in the sensor channels of protection systems. An example is the 2-out-of-3 actuation logic. This logic defines that in order to activate the shutdown system, at least 2 of the 3 protection channels must detect a deviation in the parameter measured by them. The choice of the failure logic is related, among other issues, to the design characteristics to be met.

Once a transient is detected that can cause damage to the reactor core, the shutdown systems come into play. If these sensor channels fail, there is the possibility of manually shutting the nuclear reactor down.

Examples of protection systems are: tank filters and reliefs, governor check valves and mechanical disassembles, pressure relief valves, instrumentation disassembly systems, sprinkler systems, fire-fighting water systems, etc. [5].

The scope of this work is to model a protective system with only one sensor channel, subject to aging. The possible channel failures will be divided into revealed and unrevealed. This separation is made to ensure that the system status is known only if there is a demand, or if channel tests are performed. The inclusion of the demand rate in the system allows to analyze its implications when the values of this one are very high, a common fact in process facilities [6].

In this way, the channel may be in one of three different states, defined by the triplet $< i, j, k >$:

i = number of working channels;

j = number of failed channels with unrevealed failures;

k = number of failed channels with revealed failures.

For the case of a single channel, the values that the variables i, j and k can assume are 0 or 1 only. For example, $<0,1,0>$ means that the channel is failed and its failure has not been revealed. It is easy to see that the higher the number of channels, the greater the number of different states the system can be at.

In the context of protection systems, the reliability parameter of interest is the average unavailability of this system, U. This depends not only on the failure and repair rates, λ and μ, respectively, of the protective system channels, but also on the test and maintenance policies and the logic of their performance.

However, when it comes to the probabilistic safety analysis of the facility, the attribute that is effectively considered is the plant frequency of occurrence of accidents, η.

By definition, the frequency of occurrence of accidents is given by the product between the frequency of the initiating event, the so-called demand rate, ν, and the average unavailability of the protection system. We can define it as:

$$\eta = \nu \mathrm{U}(\lambda, \mu). \quad (1)$$

Considering that an integer number of test intervals is performed in a period of one year, we can state that the frequency of occurrence of accidents, applied to the case study of this work, will be given by:

$$\eta = \frac{\nu}{\tau_p} \int_0^{\tau_p} P_2(t)dt, \quad (2)$$

where $P_2(t)$ represents the probability that the system channel is failed and its failure has not been detected and τ_p is the interval between tests, or proof-test interval (in years, in general).

The use of Eq. (2) means that the only possible accident initiating events are those caused by a system demand while it is failed and its failure has not been detected, since it is assumed that during repair the plant is off. That is, only offline repairs are considered here.

3 Accident Rate by Supplementary Variables

Most reliability models assume that component failure times assume exponential distributions. This hypothesis leads to a Markovian model, with constant transition rates, which is easy to solve both analytically and numerically. However, when it is desired to treat problems where failure times no longer follow exponential distributions, as in the case of component aging, the process becomes non-Markovian and different approaches must be employed in order to solve the problem. One of these is the method of supplementary variables [3].

This method consists of adding additional variables to the model, taking into account, in this case, the age of components that do not have exponential failure times, in order to transform the initially Non-markovian model into a Markovian one. However, in order to obtain the reliability parameters of interest, a system of coupled partial and ordinary differential equations generated by the method must be solved with time-dependent boundary conditions, which makes it difficult to solve the problem. Usually this system is solved through the finite difference method, generating data of important applicability, such as the behavior of the system when it is in a transient state [7].

Next, we will illustrate the approach of the method, applying it to the case study of this paper: that of a protection system consisting of a single channel.

Figure 1 shows the Markovian transition diagram that represents the transition logic between the possible states of the system. Although one of the transition rates is a function of time, the model can be considered Markovian by inserting a supplementary variable, which takes into account the age of the channel. If x is the supplementary variable, the system of integral-differential equations [4] associated to the state transition diagram will be:

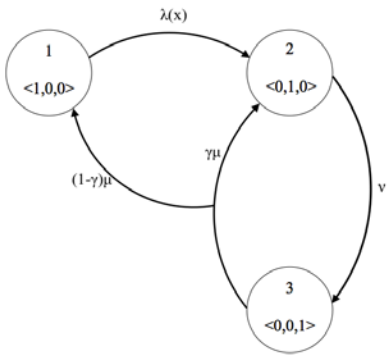

Fig. 1 Transition state diagram for an aging channel by the method of supplementary variables

$$\frac{\partial p_1(x,t)}{\partial x} + \frac{\partial p_1(x,t)}{\partial t} = -\lambda(x)\, p_1(x,t) \quad (3)$$

$$\frac{dP_2(t)}{dt} = \int_0^{\infty} \lambda(x)\, p_1(x,t)dx - \nu P_2(t) + \gamma\mu P_3(t) \quad (4)$$

$$\frac{dP_3(t)}{dt} = \nu P_2(t) - \mu P_3(t) \tag{5}$$

where $\lambda(x)$ represents the failure rate of the system, its mathematical definition is made later. t represents the time calendar. μ and ν represent, respectively, the rate of repair and the rate of demand of the system. For the case studied in this work, the two are considered constant. The probability of human error is represented by the constant γ.

It should be noted that x and t vary in exactly the same way, however, only while the system is running. When a repair occurs, the variable x remains constant in relation to t [12]. Both vary from 0 to ∞.

The states presented in the diagram have the following meanings: state 1 represents the system in operation, and is modeled by means of a probability density $p_1(x, t)$, which is interpreted as the probability of the system being in state 1 between the instants of time t and $t + \Delta t$ with age between x and $x + \Delta x$.

Thus, the probability of the system being in state 1 at t will be given by:

$$P_1(t) = \int_0^\infty p_1(x,t)dx \tag{6}$$

State 2, in turn, represents the system when the channel is failed and its failure has not been revealed. The probability of the system being in state 2 is given by $P_2(t)$.

Finally, state 3 represents the failed system with the failure revealed. Similarly, the probability of the system being in this state is given by $P_3(t)$.

For the solution of this system of equations, we need initial conditions. The first equation will be referred to as a boundary condition, since Eq. (3) refers to the first-order wave equation. This condition will be:

$$p_1(x,0) = f(x) \tag{7}$$

where $f(x)$ represents the probability density function of the channel failure times. In this work, it is assumed that the failure times follow a three-parameter Weibull distribution [2], so we will have for $\lambda(x)$ and $f(x)$, respectively:

$$\lambda(x) = \begin{cases} \lambda_0, & x < x_0 \\ \lambda_0 + \left(\frac{m}{\theta}\right)\left(\frac{x - x_0}{\theta}\right)^{m-1}, & x \geq x_0 \end{cases} \tag{8}$$

and

$$f(x) = \begin{cases} \lambda_0 e^{-\lambda_0 x}, & x < x_0 \\ \left(\lambda_0 + \left(\frac{m}{\theta}\right)\left(\frac{x - x_0}{\theta}\right)^{m-1}\right) e^{-\lambda_0 x_0 - \left(\frac{x - x_0}{\theta}\right)^m}, & x \geq x_0 \end{cases} \tag{9}$$

where m represents the shape parameter and $m > 1$ so that we consider that the system ages, θ represents the scale parameter of the Weibull distribution, and λ_o represents the failure rate of the system before it ages and, therefore, is constant.

In this model, it is also assumed that the channel repair returns it to an as good as new condition. Of course, this does not always portray the actual behavior of protection systems.

The condition imposed by Eq. (7) is not enough to solve the problem. Besides it, we also have:

$$p_1(x_0,t) = (1-\gamma)\mu P_3(t) \tag{10}$$

$$P_1(x_0) = a \tag{11}$$

$$P_2(x_0) = b \tag{12}$$

$$P_3(x_0) = c \tag{13}$$

where a, b, c are obtained by solving the problem for the useful life period from $t = 0$ until $t = x_o$.

The boundary condition, Eq. (10), represents the state of the system at age x_o, where x_o is defined as the age of the equipment as it starts aging. Equations (11) to (13), initial conditions, represent the probabilities of the protection system to be found in each state, 1, 2 or 3, at the beginning of its operation ($t = 0$), that is, the system is working. The values of the probabilities $P_1(t)$, P_2(t) and $P_3(t)$ at this point are known, because before aging the failure times are exponential and hence analytically calculable. What guarantees the use of these values is an important property of Markov chains: lack of memory.

One last equation remains to be presented, namely the sum of the probabilities of each state at any instant of time:

$$\sum_{i=1}^{3} P_i = 1 \tag{14}$$

Eq. (14) is of paramount importance for solving the steady-state problem.

Thus, it is clear that we have a well-postulated problem, since we have a system of three integral-differential equations, three unknowns, three initial conditions, and two boundary conditions to solve them.

At this point, it is relevant to mention that from the definition of the frequency of occurrence of accidents, Eq. (2), we note that we are interested only on the probability $P_2(t)$ over time, since this is the only one used in the calculations of the plant accident rate, after all, as already mentioned, we consider in this work that only offline repairs are performed.

4 Steady-state Solution

In this section we will seek the solution of the problem studied in the steady state, that is, it no longer varies with the time calendar, *t*.

Knowing the steady-state solution is an important tool for assessing whether it is necessary to calculate the probabilities in the transient state, since if they converge rapidly to the steady-state values, very little depends on the transient state results, making the effort to calculate them unnecessary. It should be rememberd that the transient solution demands the use of finite diferences and considerable effort [4, 17].

The following calculations are performed following the guidelines of Ref. [3].

Thus, with t going to infinity, the derivatives with respect to t go to zero and Eqs. (3) to (5) become:

$$\frac{dp_1(x,\infty)}{dx} = -\lambda(x)\, p_1(x,\infty) \tag{15}$$

$$0 = \int_0^\infty \lambda(x)\, p_1(x,\infty)dx - \nu P_2(\infty) + \gamma\mu P_3(\infty) \tag{16}$$

$$0 = \nu P_2(\infty) - \mu P_3(\infty) \tag{17}$$

Eq. (15) has the following general solution:

$$p_1(x,\infty) = C \times \exp\left[-\int_{x_0}^{x} \lambda(w)dw \right] \tag{18}$$

where C is an integration constant.

Solving the integral, we have:

$$p_1(x,\infty) = C \times \exp\left[-\lambda_0(x-x_0) - \left(\frac{x-x_0}{\theta}\right)^m \right],\ x \geq x_0 \tag{19}$$

In order to obtain the value of this integration constant, we use the boundary condition, Eq. (10), which in the steady state becomes:

$$p_1(x_0,\infty) = (1-\gamma)\mu P_3(\infty) \tag{20}$$

We do not know the values of $p_1(x_o,\infty)$ and $P_3(\infty)$ in Eq. (20). In order to find them, we will write $P_1(\infty)$ and $P_2(\infty)$ as a function of $P_3(\infty)$ and use Eq. (14) so as to find the solution of $P_3(\infty)$ and, from it, the solutions to the other probabilities and to the frequency of occurrence of accidents.

Thus, Eq. (19) at $x = x_o$ is:

$$p_1(x_0,\infty) = C, \tag{21}$$

from which one can write down:

$$C = p_1(x_0,\infty) = (1-\gamma)\mu P_3(\infty) \tag{22}$$

Putting Eq. (22) in Eq. (19), one has for $x \geq x_o$:

$$p_1(x_0,\infty) = (1-\gamma)\mu P_3(\infty).\exp\left[-\lambda_0(x-x_0) - \left(\frac{x-x_0}{\theta}\right)^m \right] \tag{23}$$

By definition:

$$P_1(\infty) = \int_0^\infty p_1(x,\infty)dx \tag{24}$$

So that,

$$p_1(\infty) = (1-\gamma)\mu P_3(\infty)\int_{x_0}^{\infty} \exp\left[-\lambda_0(x-x_0) - \left(\frac{x-x_0}{\theta}\right)^m \right]dx \tag{25}$$

The integral in Eq. (25) has no analytical solution. Simpson's compound method [16] will be used to solve it. Defining:

$$I^\infty = \int_{x_0}^{\infty} \exp\left[-\lambda_0(x-x_0) - \left(\frac{x-x_0}{\theta}\right)^m \right]dx \quad , x \geq x_0 \tag{26}$$

Eq. (25) will become:

$$P_1(\infty) = (1-\gamma)\mu P_3(\infty) I^{\infty} \tag{27}$$

With $P_1(\infty)$ as a function of $P_3(\infty)$, we return to Eq. (17):

$$P_2(\infty) = \frac{\mu}{\nu} P_3(\infty) \tag{28}$$

Recalling that the sum of the probabilities at any instant of time is equal to one [Eq. (14)] and using Eqs. (27) and (28), we obtain, for $P_3(\infty)$:

$$P_3(\infty) = \left(1 + (1-\gamma)\mu I^{\infty} + \frac{\mu}{\nu}\right)^{-1} \tag{29}$$

Therefore, due to Eq. (28), $P_2(\infty)$ can be written as:

$$P_2(\infty) = \frac{\mu}{\nu}\left(1 + (1-\gamma)\mu I^{\infty} + \frac{\mu}{\nu}\right)^{-1} \tag{30}$$

And finally, we may write $P_1(\infty)$ as:

$$P_1^{\infty} = 1 - \left(1 + \frac{\mu}{\nu}\right)\left(1 + (1-\gamma)\mu I^{\infty} + \frac{\mu}{\nu}\right)^{-1} \tag{31}$$

As the quantitiy of interest in this work is the frequency of occurrence of accidents, using Eq. (2) we will have

$$\eta^{\infty} = \nu P_2(\infty) = \mu\left(1 + (1-\gamma)\mu I^{\infty} + \frac{\mu}{\nu}\right)^{-1} \tag{32}$$

5 Results and Discussion

In order to evaluate the feasibility of the steady-state approach, we take into account the results of [4], which were obtained considering transient solutions by means of finite differences.

Table 1 displays the parameters used for the calculations [4]. These parameters are typical of process industries, which include the nuclear industry. It should be noted that the values used are quite representative and, for example, it is not common to have a proof test interval higher than one year [5].

Table 1. Parameters used for the steady-state analysis [4]

Parameter	Value
λ_o (yr^{-1})	1 10
x_o (yr)	1
m	2.5
θ (yr)	1
γ	0.1
μ (yr^{-1})	52
ν (yr^{-1})	0.5 10.0 100.0
τ_P (yr)	1.0

The results obtained are displayed in Table 2, where the last column displays the relative error for the calculations of η^{∞}.

Table 2. Results obtained from the steady-state approach

λ_o (yr^{-1})	ν (yr^{-1})	η (yr^{-1})	η^{∞} (yr^{-1})	Rel. er. (%)
10.0	0.5	0.07	0.47	571
	10.0	0.21	4.80	2,186
	100.0	0.22	8.45	3,740
1.0	0.5	0.23	0.40	74
	10.0	0.93	1.61	73
	100.0	1.07	1.88	75

It is clear, from Table 2, that the use of the steady-state solution – that is, $P_2(\infty)$ – is not feasible, because as can be seen from Eq. (2), we need to evaluate the integral of $P_2(t)$ in the interval from $t = 0$ to $t = \tau_P$. If we approximate this integral with the asymptotic value of $P_2(t)$ we will be in error because the $P_2(t)$ behavior is quite smooth [4] and its approximation by a rectangle is not adequate.

It can also be seen from Table 2 that the relative errors are much higher for higher failure rates, because P_2 is much less steeper in the transient phase than for lower failure rates. On the other hand, the higher the demand rate, the higher the plant accident rate for higher failure rates. For lower failure rates, the steady-state plant accident rate is quite constant.

We performed a sensitivity analysis on the parameters involved to check their influence upon the plant accident rate. Table 3 displays the results obtained for η^{∞} [Eq. (32)].

We kept the failure rate during useful life with the values used before, that is, 10/yr and 1/yr, which represent mean times to failure of 0.1 yr and 1 yr, respectively. The useful life considered, x_o, had been initially assumed 1 yr and then we considered higher periods up to 4 yr. A similar approach was adopted for the characteristic life, θ, but we varied it up to 6 yr. Finally, the shape parameter, m, was considered in the range from 1.5 to 3.5.

Table 3. Sensitivity analysis on η^{∞} (μ = 52 yr^{-1}, ν = 10 yr^{-1})

λ_o (yr^{-1})	x_o (yr)	θ (yr^{-1})	m	η^{∞}
10.0	1.0	1.0	1.5	4.86
			2.5	4.80
			3.5	4.79
	2.0	4.0	1.5	4.79
			2.5	4.78
			3.5	4.78
	4.0	6.0	1.5	4.79
			2.5	4.78
			3.5	4.78
1.0	1.0	1.0	1.5	1.68
			2.5	1.61
			3.5	1.57
	2.0	4.0	1.5	1.11
			2.5	1.05
			3.5	1.03
	4.0	6.0	1.5	1.05
			2.5	1.01
			3.5	1.00

It may be inferred from Table 3 that for higher failure rates, the steady-state frequency of accident is quite insensible to all other parameters, inasmuch as the variation from the highest value to the lowest is of 1.6%. On the other hand, when one considers the lower failure rate, this relative variation goes up to 40.5%. The reason for this behavior is the P_2 behavior, as discussed earlier.

Sensitivity calculations performed considering different repair rates and demand rates showed no significant variations from the ones displayed in Table 3.

6 Conclusion

The use of steady-state solutions when solving systems of differential equations has the purpose of avoiding sophisticated mathematical and/or numerical methods. The case presented in this paper is typical. Mathematically, one has to solve a set of differential equations, in fact a set of integral-differential equations with coupled boundary conditions. In order to obtain the time-dependent behavior for the plant accident rate, it is necessary to define a mesh in terms of calendar time and the channel age and use finite difference methods. This is true even if we consider other channel arrangements, as for example, a set of three channels, in order to avoid spurious plant trips [16]. In this sense, the problem requires a great computational effort. As the channel probability failure has an asymptotic behavior, one could imagine that the analysis of this steady-state behavior could be used instead of the transient behavior. However, if calculations turn quite easier, as we have seen, where the only shortcoming was the calculation of an integral by numerical methods because the function to be integrated comes from a Weibull distribution, the results are not adequate for plant decision making, as next explained.

The use of the steady-state calculation for the accident rate of a plant equipped with an aging protective channel is not adequate because typical proof test intervals are equal or less than 1 yr. This is because the channel failure probability typically requires much more than one year to reach its steady-state behavior. Even for those cases where conservative plant accident rates could be used, this would not be feasible because the relative errors can be as high as 4,000%. Any decisions based upon higher plant accident rates could turn economically inviable.

The use of higher proof test intervals could significantly reduce these relative errors but would not be sustained on practical grounds, because channel-aging effects could influence plant availability. This is explained by the fact that as the channel is aging, plant decisions must consider system replacement or, alternatively, preventive maintenance policies. This optimization problem should take into account plant performance and associated costs.

The conclusion is that the use of steady-state solutions is not feasible for the problem at hand and research should be conducted in order to improve methods for the transient analysis of the plant accident rate. For example, by employing other finite difference methods, like the Lax-Friedrichs method [18, 19] and by investigating different numerical integration methods for obtaining the probability of being in state 1 (that is, Eq. 6) [16, 19]. Even is the steady-state solutions is pursued for higher channel configurations, it is not expected to reach errors lower than those obtained in this paper.

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References:

[1] S. Glasstone and A. Sesonske, *Nuclear Reactor Engineering*, Van Nostrand Reinhold, New York, 1981.
[2] D. Kececioglu, *Reliability Engineering Handbook*, Prentice Hall, Englewood Cliffs, NJ, 1991.
[3] C. Singh and R. Billinton, *System Reliability Modelling and Evaluation*, Hutchinson, London, 1977.
[4] L. G. Oliveira, D. G. Teixeira, and P. F. Frutuoso e Melo, "Accident Frequency of an Industrial Facility Equipped with a Single-Channel Protective System under Aging", accepted for presentation, Nat. Conf. of Appl. and Comp. Math., Campinas, SP, Brazil, September 2018.
[5] F. P. Lees, "A General Relation for the Reliability of a Single-Channel Trip System", *Reliab. Eng.*, Vol. 3, No. 1, 1982, pp. 1-5.
[6] L. F. Oliveira, J. D. Amaral Netto, "Influence of the Demand Rate and Repair Rate on the Reliability of a Single-channel Protective System", *Reliab. Eng.*, Vol. 17, 1987, pp. 267-276.
[7] L. F. Oliveira, R. Youngblood, P. F. Frutuoso e Melo, "Hazard Rate of a Plant Equipped with a Two-channel Protective System Subject to a High Demand Rate", *Reliab. Eng. & Syst. Saf.*, Vol. 28, 1990, pp. 35-58.
[8] P. F.Frutuoso e Melo, F. C. Silva, A. C. M., Alvim, "Sensitivity Analysis on the Accident Rate of a Plant Equipped with a Single Protective Channel by Generalized Perturbation Methods", *An. Nucl. En.*, Vol. 25, No. 15, 1998, pp. 1191-1207.
[9] P. F. Frutuoso e Melo, L. F. Oliveira, R. Youngblood, "A Markovian Model for the Reliability Analysis of Multichanneled Protective Systems Considering Revealed Failures and Common-Cause Failures by the Alpha Model., *Proc. 9th Braz. Meet. On Reac. Phys. & Ther. Hyd.*, Caxambu, Brazil, Brazilian Association for Nuclear Energy, 1993, pp. 440-446.
[10] E. F. Lima, D. G. Teixeira, P. F. Frutuoso e Melo, F. C. Silva, A. C. M. Alvim, "Sensitivity Analysis of the Accident Rate of a Plant by the Generalized Perturbation Theory", *Int. J. Math. Mod. & Met. App. Sci.*, Vol. 10, 2016, pp. 309-316.
[11] M. O. Pinho, H. C. N. Fernandez, A. C. M. Alvim, P. F. Frutuoso e Melo, "Availability of a component subject to na Erlangian failure mode under wearout by supplementary variables", *J. Braz. Soc. Mech. Sci.*, Vol. 21, 1999, pp. 109-122.
[12] E. A. Oliveira, A. C. M. Alvim, P. F. Frutuoso e Melo, "Unavailability Analysis of Safety Systems under Aging by Supplementary Variables with Imperfect Repair", *An. Nucl. En.*, Vol. 32, 2005, pp. 241-252.
[13] P. F. Frutuoso e Melo, D. G. Teixeira, A. C. M. Alvim, "A Monte Carlo Evaluation of the Accident Rate of a Plant Equipped with an Aging Single-channel Trip Device", *14th Int. Conf. on Num. Anal. and Apl. Math 2016 (ICNAAM 2016), Am. Inst. of Phys. Conf. Proc.*, Rhodes, Greece, 19-25 September 2016.
[14] P. F. Frutuoso e Melo, D. G. Teixeira, A. C. M. Alvim, "Accident Rate of a Plant Equipped with an Aging Single-channel Trip Device with Non-exponential Demand and Repair Times", *1st Int. Conf. Appl. Math. and Comp. Sci., Am. Inst. Phy. Conf. Proc.*, Rome, Italy, 27-29 January 2017, p. 0200083.
[15] F. P. Lees, *Loss Prevention in the Process Industries – Hazards Identification, Assessment and Control*, Butterworth-Heinemann, Oxford, 1980.
[16] S. Nakamura, *Computational Methods in Engineering and Science with Applications to Fluid Dynamics and Nuclear Systems*, John Wiley & Sons, New York 1977.
[17] E. E. Lewis, *Introduction to Reliability Engineering*, John Wiley & Sons, New York, 1996.
[18] C. Hirsch, *Numerical Computation of Internal & External Flows*, John Wiley & Sons, New York, 2007.
[19] W. H. Press, S. A. Teukolsky, W. T. Vetterling, B. P. Flannery, N*umerical Recipes: The Art of Scientific Computing*, Cambridge University Press, New York, 2007.