A New Complete Irresoluteness Function

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Abstract: In this presentation, first of all we definite a new type of function by using delta-b-open sets. Then, we obtain some characterizations and some properties of this function. Besides, we give their relationships with other types of functions between topological spaces.

Key–Words: *δ-b-open sets, b-open sets, δ-semi-open sets, semi open sets, completely δ-b-irresolute functions* ■

1 Introduction

Of course, the notions of $continuous$ functions and a type of it's is called $irresolute$ functions are important subject in general topology. So, one can find several papers in literature related it's.

On the other hand, $open$ sets and it's modifications are studied very authors. One of these sets is b-$open$. It is defined by Andrijević [11] and El-Atik [10] independent of each other. It is well-known that b-$open$ set is weaker than $semi$-$open$ set which is a type of $open$ set.

Throughout this paper, we will denote topological spaces by (X, τ) and (Y, φ). For a subset A of a space (X, τ), the closure of A and the interior of A are denoted by $Cl(A)$ and $Int(A)$, respectively.

A subset A is said to be regular open (resp. regular closed) if $Int(Cl(A)) = A$ (resp. $Cl(Int(A)) = A$). The family of all regular open and regular closed sets of (X, τ) are denoted by RO (X, τ) and RC (X, τ), respectively. A subset A is said to be δ-open if for each $x \in A$ there exists a regular open set U such that $x \in U \subseteq A$. A point $x \in X$ is called a δ-cluster point of A if $A \cap Int(Cl(V)) \neq \varnothing$ for each open set V containing x. The set of all δ-cluster points of A is called the δ-closure of A and is denoted by $Cl_\delta(A)$. The set $\{x \in X \mid x \in U \subseteq A$ for some regular open set U of $X\}$ is called the δ-interior of A and is denoted by $Int_\delta(A)$.

A subset A of a space (X, τ) is called $preopen$ [16] (resp. b-$open$ [11] or γ-open [10], δ-b-open set [8]) if $A \subseteq Int(Cl(A))$ (resp. $A \subseteq Cl(Int(A)) \cup Int(Cl(A))$, $A \subseteq Cl(Int_\delta(A)) \cup Int(Cl(A))$). It is well known that a subset A of a space (X, τ) is called semi open (resp. δ-semi-open) if $A \subseteq Cl(Int(A))$ (resp. $A \subseteq Cl(Int_\delta(A))$). Besides, the complement of δ-b-open set is said δ-b-closed. The family of all δ-b-$open$ and δ-b-$closed$ sets of (X, τ) are denoted by δBO (X, τ) and δBC (X, τ), respectively.

2 Completely δ-b-irresolute Functions

In this section, we introduce the notion of completely δ-b-$irresolute$ functions and obtain some properties of them.

Definition 1 *A function $f : (X, \tau) \longrightarrow (Y, \varphi)$ is said to be completely δ-b-irresolute function if the inverse image of each δ-b-open set V in (Y, φ) is regular open set in (X, τ).*

Now, we give a characterization for completely δ-b-$irresolute$ functions.

Theorem 2 *Let $f : (X, \tau) \longrightarrow (Y, \varphi)$ be a function. f is completely δ-b-irresolute function if and only if the inverse image of each δ-b-closed set F in (Y, φ) is regular closed in (X, τ).*

Proof. The proof is obvious by considering the complement of Definition 1. ■

Definition 3 *A function $f : (X, \tau) \longrightarrow (Y, \varphi)$ is said to be completely irresolute [1] (resp. completely δ-semi-irresolute [2], completely b-irresolute) if $f^{-1}(V)$ is regular open set in (X, τ) for every semi open (resp. δ-semi-open, b-open) set V in (Y, φ).*

Remark 4 *For a function $f : (X, \tau) \longrightarrow (Y, \varphi)$, we have the following diagram by using Definitions 1 and 2.*

$completely\ irresolute \longrightarrow completely\ \delta$-$semi$-$irresolute$

$\uparrow \qquad \uparrow$

$completely\ b$-$irresolute \longrightarrow completely\ \delta$-$b$-$irresolute$

However, none of these implications is reversible as shown by in the following examples and [2].

Example 1 *Let $X = \{a, b, c, d\}$ and $\tau = \{\varnothing, X, \{a\}, \{d\}, \{a, d\}, \{a, c\}, \{a, c, d\}\}$. Let $f : (X, \tau) \longrightarrow (X, \tau)$ be the identity function. Then f is completely δ-b-irresolute but it is not completely b-irresolute.*

Example 2 *Let $X = \{a, b, c, d\}$ and $\tau = \{\varnothing, X, \{d\}, \{a, b\}, \{a, b, d\}\}$. Let $f : (X, \tau) \longrightarrow (X, \tau)$ be the identity function. Then f is completely irresolute but it is not completely b-irresolute.*

Example 3 *Let $X = \{a, b, c, d\}$ and $\tau = \{\varnothing, X, \{a, b\}, \{a, b, c\}\}$. Let $f : (X, \tau) \longrightarrow (X, \tau)$ be the identity function. Then f is completely δ-$semi$-irresolute but it is not completely δ-b-irresolute.*

Now, we recall some types of functions to next theorem.

Definition 5 *A function $f : (X, \tau) \longrightarrow (Y, \varphi)$ is said to be*

(1) R-map [3] (resp. completely continuous [4]) if $f^{-1}(V)$ is regular open in (X, τ) for every regular open (resp. open) set V in (Y, φ).

(2) δ-b-continuous [5] if $f^{-1}(V)$ is δ-b-open in (X, τ) for every open set V in (Y, φ).

Now, we give the following theorem as not proof for compose functions:

Theorem 6 *Let $f : (X, \tau) \longrightarrow (Y, \varphi)$ and $g : (Y, \varphi) \longrightarrow (Z, \psi)$ be functions. The following properties hold:*

(1) If f is R-map and g is completely δ-b-irresolute, then gof is completely δ-b-irresolute.

(2) If f is completely δ-b-irresolute and g is δ-b-continuous, then gof is completely continuous.

It is known that the notion of restriction functions is important. So, we give the following lemma and theorem.

Lemma 7 *([6]) Let U be an open subset of a space (X, τ).*

(1) If A is regular open set in (X, τ), then $A \cap U$ is regular open in the subspace (U, τ_U).

(2) If $B \subset U$ is regular open in (U, τ_U), then there is a regular open set A in (X, τ) such that $B = A \cap U$.

Theorem 8 *If a function $f : (X, \tau) \longrightarrow (Y, \varphi)$ is completely δ-b-irresolute and U is open in (X, τ), then the restriction $f\mid_U : (U, \tau_U) \longrightarrow (Y, \varphi)$ is completely δ-b-irresolute.*

Proof. Let V be any δ-b-$open$ set in (X, τ). Since f is $completely\ \delta$-b-$irresolute$, $f^{-1}(V)$ is $regular\ open$ in (X, τ). By using Lemma 1(1), we have $f^{-1}(V) \cap U$ is $regular\ open$ in the subspace (U, τ_U). Since $f^{-1}(V) \cap U = (f\mid_U)^{-1}(V)$, we obtain that $f\mid_U$ is $completely\ \delta$-b-$irresolute$. ∎

It is well known that every open set is preopen but the converse is not true in generally. Really, let $\mathbb{R}$ is real number and τ is usual topology on $\mathbb{R}$. Then, $\mathbb{Q} \subset \mathbb{R}$ is a preopen set but it is not an open set.

Lemma 9 *([15]) Let A be a preopen subset of X. Then, $(A \cap U)$ is a regular open in A for each regular open subset U of X.*

If we take preopen set instead of open set in Theorem 3, we obtain the next theorem by using Lemma 2.

Theorem 10 *If a function $f : (X, \tau) \longrightarrow (Y, \varphi)$ is completely δ-b-irresolute and U is preopen in (X, τ), then the restriction $f\mid_U : (U, \tau_U) \longrightarrow (Y, \varphi)$ is completely δ-b-irresolute.*

3 Some Non-preservation Properties Via Completely δ-b-irresolute Functions

Lemma 11 *In this section, we investigate some separation axioms via $completely\ \delta$-b-$irresolute$ functions such as b-normal, b-T_2, r-$connected$, $hyperconnected$ and $nearly\ compact$. Then, we give a type of graph function is called r-δ_b-$graph$ and obtain some properties it's. Finally, we consider a relation between product spaces and $completely\ \delta$-b-$irresolute$ functions. For γ-$sets$ modification of this notion is defined as follows:*

It is known that the notion of normal spaces is one of separation axioms in topological spaces. A space (X, τ) is said to be γ-normal [15] if for every disjoint closed sets A and B of (X, τ), there exist disjoint sets $U, V \in BO(X)$ such that $A \subset U$ and $B \subset V$. The notion of γ-normal spaces is restated in [8] by using δ-b-$open$ sets.

Lemma 12 *([8]) For a space (X, τ), the following properties are equivalent:*

(1) (X, τ) is γ-normal,

(2) Every pair of nonempty disjoint closed sets can be separeted by disjoint δ-b-open sets.

Now, we give the next theorem which is related to $completely$ δ-b-$irresolute$ functions and γ-$normal$ spaces.

Theorem 13 *If $f : (X, \tau) \longrightarrow (Y, \varphi)$ is completely δ-b-irresolute closed injection and (Y, φ) is γ-normal, then (X, τ) is normal.*

Proof. Let F_1 and F_2 be disjoint nonempty $closed$ sets in (X, τ). Since f is $injective$ and $closed$, $f(F_1)$ and $f(F_2)$ are disjoint $closed$ sets in (Y, φ). Besides, (Y, φ) is γ-$normal$, there exist δ-b-$open$ sets V_1 and V_2 in (X, τ) such that $f(F_1) \subset V_1$ and $f(F_2) \subset V_2$ and $V_1 \cap V_2 = \varnothing$ by using Lemma 3. Since f is $completely$ δ-b-$irresolute$, we have $f^{-1}(V_1)$, $f^{-1}(V_2)$ are regular open sets in (X, τ). Of course in this case $F_1 \subset f^{-1}(V_1)$, $F_2 \subset f^{-1}(V_2)$ and $f^{-1}(V_1) \cap f^{-1}(V_2) = \varnothing$. This shows that (X, τ) is $normal$. ■

Of course, another separation axiom is a modification of T_2-spaces. For b-$open$ sets is called b-T_2 by Park [7]. Recall that a topological space (X, τ) is said to be b-T_2 [7] if for each distinct points $x, y \in X$, there exist b-$open$ sets U_1 and U_2 containing x and y, respectively, such that $U_1 \cap U_2 = \varnothing$.

One can see the following lemma which is related to b-T_2 spaces which contain δ-b-$open$ sets.

Lemma 14 *([8]) For a topological space (X, τ), the following properties are equivalent:*

(1) (X, τ) is b-T_2,

(2) For each distinct points $x, y \in X$, there exist $U_1, U_2 \in \delta BO(X)$ containing x and y, respectively, such that $U_1 \cap U_2 = \varnothing$.

In the view of definition of notion of b-T_2 spaces and Lemma 4, we give the following theorem.

Theorem 15 *If $f : (X, \tau) \longrightarrow (Y, \varphi)$ is completely δ-b-irresolute injection and (Y, φ) is b-T_2, then (X, τ) is T_2.*

Proof. Let x, y be any distinct points of X. By hypothesis, $f(x) \neq f(y)$. Since (Y, φ) is b-T_2, there exist δ-b-$open$ sets U_1 and U_2 in (Y, φ) containing $f(x)$ and $f(y)$, respectively, such that $U_1 \cap U_2 = \varnothing$ by Lemma 4. At the same time, since f is $completely$ δ-b-$irresolute$ $f^{-1}(U_1)$, $f^{-1}(U_2) \in RO(X)$ containing x and y, respectively, such that $f^{-1}(U_1) \cap f^{-1}(U_2) = \varnothing$. This shows that (X, τ) is T_2. ■

Each of the next two theorem is a property of a function $completely$ δ-b-$irresolute$ such that range space is b-T_2.Let (Y, φ) be a b-T_2. If $f, g : (X, \tau) \longrightarrow (Y, \varphi)$ are completely δ-b-irresolute functions, then the set $A = \{x \in X \mid f(x) = g(x)\}$ is δ-closed in (X, τ).

Proof. Assume that $x \notin A$. Then, we have $f(x) \neq g(x)$. Since (Y, φ) is b-T_2, there exists δ-b-$open$ sets V_1 and V_2 in (Y, φ) such that $f(x) \in V_1$ and $g(x) \in V_2$ and $V_1 \cap V_2 = \varnothing$ by Lemma 4. Besides, since f and g are completely δ-b-irresolute functions then $f^{-1}(V_1)$ and $g^{-1}(V_2)$ are regular open sets in (X, τ). If we take $U = f^{-1}(V_1) \cap g^{-1}(V_2)$, then U is a regular open set containing x and $U \cap A = \varnothing$. Therefore, we have $x \notin Cl_\delta(A)$. Consequently, this proof is completed. ■

Theorem 16 *If $f : (X, \tau) \longrightarrow (Y, \varphi)$ is completely δ-b-irresolute injection and (Y, φ) is b-T_2, then $E = \{(x, y) \mid f(x) = f(y)\}$ is δ-closed in $X \times X$.*

Proof. This proof is obtained similar to Theorem 7. ■

The other separation axiom is $connected$ spaces. Of course, this notion is applied to b-$open$ (resp. δ-b-$open$, $regular$ $open$) sets, respectively, as follows:

Definition 17 *A topological space (X, τ) is said to be γ-$connected$ [10] (resp. δ-b-$connected$ [8], r-$connected$ [9]) if it cannot be expressed as the union of two non-empty disjoint b-open (resp. δ-b-open, regular open) sets.*

Lemma 18 *([8]) For a topological space (X, τ), the following properties are equivalent:*

(1) (X, τ) is γ-$connected$,

(2) X cannot be expressed as the union of two nonempty disjoint δ-b-open sets.

It is obvious that Lemma 5 states a topological space is γ-$connected$ if and only if it is δ-b-$connected$. Now, we have the following theorem.

Theorem 19 *If $f : (X, \tau) \longrightarrow (Y, \varphi)$ is completely δ-b-irresolute surjection and (X, τ) is r-$connected$, then (Y, φ) is γ-$connected$.*

Proof. Assume that (Y, φ) is not γ-$connected$. There exist nonempty δ-b-$open$ sets U_1 and U_2 in (Y, φ) such that $Y = U_1 \cup U_2$ and $U_1 \cap U_2 = \varnothing$ by using Lemma 5. Since f is $completely$ δ-b-$irresolute$, $f^{-1}(U_1)$ and $f^{-1}(U_2)$ are nonempty $regular$ $open$ sets in (X, τ) such that $X = f^{-1}(U_1) \cup f^{-1}(U_2)$ and $f^{-1}(U_1) \cap f^{-1}(U_2) = \varnothing$. Thus, (X, τ) is not r-$connected$. But, this is contradiction with hypothesis. Consequently, (Y, φ) is γ-$connected$. ■

In 1970, the notion of $hyperconnected$ $spaces$ which is stronger than $connected$ $spaces$ is defined by Steen and Seebach [12] as the following: "A topological space (X, τ) is said to be $hyperconnected$ [12] if every $open$ subset in (X, τ) is $dense$."

Now we give the following theorem. It is important, because it denotes that the notion of $hyperconnected\ spaces$ isn't preservate under $completely$ δ-b-$irresolute$ functions.

Theorem 20 *Let $f : (X, \tau) \longrightarrow (Y, \varphi)$ is completely δ-b-irresolute function. If (X, τ) is hyperconnected space, then (Y, φ) is γ-$connected$.*

Proof. Suppose that V is proper δ-b-$clopen$, i.e. both δ-b-$open$ and δ-b-$closed$, subspace of (Y, φ). Since f is $completely$ δ-b-$irresolute$ function, $U = f^{-1}(V)$ is both $regular\ open$ and $regular\ closed$. This contradicts to (X, τ) is $hyperconnected$. So, (Y, φ) is γ-$connected$. ■

In the end of this section, we consider some of the notions of compactness related to this subject.

Recall that a topological space (X, τ) is said to be $nearly\ compact$ [13] (resp. δ-b-$compact$ [5]) if every $regular\ open$ (resp. δ-b-$open$) cover of (X, τ) has a finite subcover.

Theorem 21 *If $f : (X, \tau) \longrightarrow (Y, \varphi)$ is completely δ-b-irresolute surjection and (X, τ) is nearly compact, then (Y, φ) is δ-b-compact.*

Proof. Let $\{V_\alpha : \alpha \in \Delta\}$ be a δ-b-$open$ cover of (Y, φ). Since f is $completely$ δ-b-$irresolute$ $\{f^{-1}(V_\alpha) : \alpha \in \Delta\}$ is a $regular\ open$ cover of (X, τ). By hypothesis, since (X, τ) is $nearly\ compact$ there exists a finite subset Δ_0 of Δ such that $X = \{f^{-1}(V_\alpha) : \alpha \in \Delta_0\}$. Since f is $surjection$, we obtain that $Y = \{V_\alpha : \alpha \in \Delta_0\}$. This shows that (Y, φ) is δ-b-$compact$. ■

4 Some properties of graphic functions

In this section, firstly we obtain relation between a function and it's graph function to be completely δ-b-irresolute. Secondly, we define the notion of r-δ_b-$graph$. Then, we investigate some properties of it.

Recall that for a function $f : (X, \tau) \longrightarrow (Y, \varphi)$, the subset $\{(x, f(x)) : x \in X\}$ of $X \times Y$ is called the $graph\ of\ f$ and is denoted by G_f.

Theorem 22 *A function $f : (X, \tau) \longrightarrow (Y, \varphi)$ is completely δ-b-irresolute if the graph function $G_f : X \longrightarrow X \times Y$ is completely δ-b-irresolute.*

Proof. Let $x \in X$ and V be a δ-b-$open$ set containing $f(x)$. Then, $X \times V$ is a δ-b-$open$ set of $X \times Y$ containing $G_f(x)$. So, $G_f^{-1}(X \times V) = f^{-1}(V)$ is a $regular\ open$ set containing x. This shows that f is $completely$ δ-b-$irresolute$. ■

Let $\{X_\alpha : \alpha \in \Delta\}$ and $\{Y_\alpha : \alpha \in \Delta\}$ be two families of topological spaces with the same index set Δ. The product space of $\{X_\alpha : \alpha \in \Delta\}$ is denoted by $(\Pi X_\alpha)_{\alpha \in \Delta}$. Let $f_\alpha : X_\alpha \longrightarrow Y_\alpha$ be a function for each $\alpha \in \Delta$. The product function $f : (\Pi X_\alpha)_{\alpha \in \Delta} \longrightarrow (\Pi Y_\alpha)_{\alpha \in \Delta}$ is denoted by $f((x_\alpha)) = (f(x_\alpha))$ for each $(x_\alpha) \in (\Pi X_\alpha)_{\alpha \in \Delta}$.

Now, we consider the following theorem.

Theorem 23 *If a function $f : (\Pi X_\alpha)_{\alpha \in \Delta} \longrightarrow (\Pi Y_\alpha)_{\alpha \in \Delta}$ is completely δ-b-irresolute, then $f_\alpha : X_\alpha \longrightarrow Y_\alpha$ is completely δ-b-irresolute for each $\alpha \in \Delta$.*

Proof. Let γ be an arbitrary fixed index and V_γ any δ-b-$open$ set of Y_γ. Then, $(\Pi Y_\beta \times V_\gamma)$ is δ-b-$open$ in $(\Pi Y_\alpha)_{\alpha \in \Delta}$, where $\beta \in \Delta$ and $\beta \neq \gamma$, and hence $f^{-1}(\Pi Y_\beta \times V_\gamma) = \left(\Pi Y_\beta \times f_\gamma^{-1}(V_\gamma)\right)$ is $regular\ open$ in $(\Pi X_\alpha)_{\alpha \in \Delta}$. So, $f_\gamma^{-1}(V_\gamma)$ is $regular\ open$ in X_γ and hence f_γ is $completely$ δ-b-$irresolute$. ■

Definition 24 *A graph function G_f of a function $f : (X, \tau) \longrightarrow (Y, \varphi)$ is called r-δ_b-graph if for each $(x, y) \in (X \times Y) \backslash G(f)$, there exist regular open set U in (X, τ) containing x and a δ-b-open V in (Y, φ) containing y such that $(U \times V) \cap G(f) = \varnothing$.*

We give the following lemma as a characterization of r-δ_b-$graph$.

Lemma 25 *Let $f : (X, \tau) \longrightarrow (Y, \varphi)$ be a function and $G(f)$ be a graph of f. Then, we have the following property:*

"$G(f)$ is r-δ_b-graph if and only if for each $(x, y) \in (X \times Y) \backslash G(f)$, there exist regular open set U in (X, τ) containing x and a δ-b-open set V in (Y, φ) containing y such that $f(U) \cap V = \varnothing$."

Theorem 26 *If If $f : (X, \tau) \longrightarrow (Y, \varphi)$ is completely δ-b-irresolute and (Y, φ) is b-T_2, then $G(f)$ is r-δ_b-graph in $(X \times Y)$.*

Proof. Let $(x, y) \in (X \times Y) \backslash G(f)$ and (Y, φ) is b-T_2. Then, $f(x) \neq y$. Since (Y, φ) is b-T_2, there exist δ-b-$open$ sets V_1 and V_2 containing $f(x)$ and y, respectively, such that $V_1 \cap V_2 = \varnothing$ by using Lemma 4. Since f is $completely$ δ-b-$irresolute$, $f^{-1}(V_1) = U$ is a $regular\ open$ set containing x. Therefore, $f(U) \cap V_2 = \varnothing$ and $G(f)$ is r-δ_b-$graph$ in $(X \times Y)$. ■

Theorem 27 *Let a function* $f : (X, \tau) \longrightarrow (Y, \varphi)$ *has the r-*δ_b*-graph. If f is injective, then* (X, τ) *is* T_1*.*

Proof. Let x and y be any two distinct points of (X, τ). Then, we have $(x,\ f(y)) \in (X \times Y) \setminus G(f)$. Then, there exist a $regular\ open$ set U in (X, τ) containing x and a δ-b-$open$ set V in (Y, φ) containing $f(y)$ such that $f(U) \cap V = \varnothing$ by using Lemma 6. Hence, we obtain $U \cap f^{-1}(V) = \varnothing$ and $y \notin U$. This implies that (X, τ) is T_1. ■

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