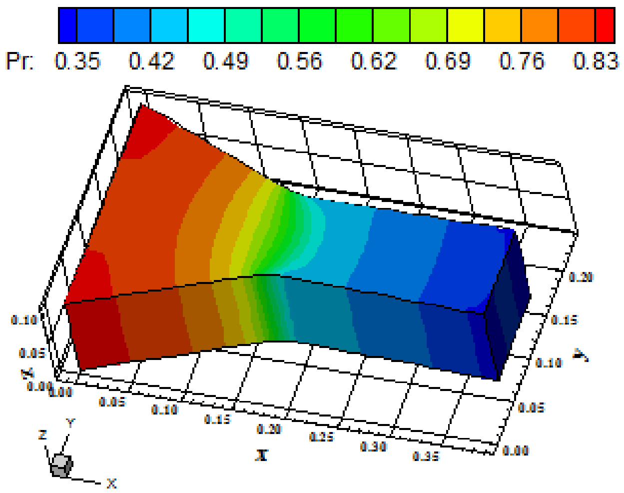

Figure 41. Pressure contours ([7]-Min2).

Figures 40 and 41 exhibit the pressure contours obtained by the [7] scheme, in its Min1 and Min2 variants, respectively. The most severe pressure field is due to the Min2 version of the [7] algorithm.

Figure 42 shows the wall pressure distribution obtained by the [7] in its two variants. They are compared with the experimental results of [42]. As can be observed, the reasonable solution is obtained by the [7] scheme using Min1 limiter. Hence, it is possible to conclude that for the laminar viscous results, the [7] scheme, in its Min1 version, provides the best solution.

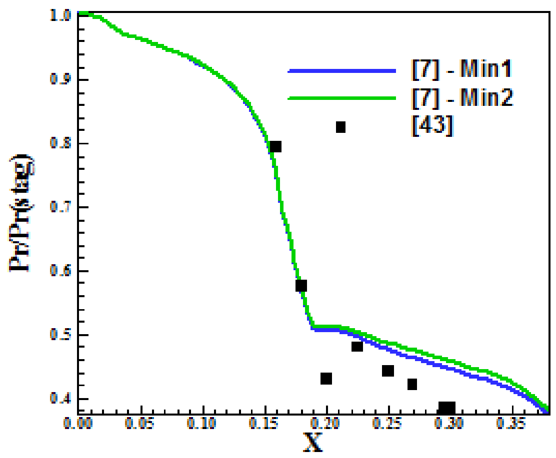

Figure 42. Wall pressure distribution ([7]).

11 Conclusion

In the present work, the [7] TVD symmetric, the [9] TVD symmetric, the [12] TVD, and the [13] TVD/ENO schemes are implemented, on a finite volume context and using a structured spatial discretization, to solve the Euler and Navier-Stokes equations in the three-dimensional space. With the exception of [7; 9], all others schemes are high resolution flux difference splitting ones, based on the concept of Harten's modified flux function. The [7; 9] TVD schemes are symmetric ones, incorporating TVD properties due to the appropriated definition of a limited dissipation function. All schemes are second order accurate in space. An implicit formulation is employed to solve the Euler equations, whereas a time splitting method, an explicit method, is used to solve the Navier-Stokes equations. An approximate factorization in Linearized Nonconservative Implicit LNI form is employed by the [12-13] schemes, whereas an approximate factorization ADI method is employed by the [7; 9] schemes. All algorithms are first order accurate in time. The algorithms are accelerated to the steady state solution using a spatially variable time step, which has demonstrated effective gains in terms of convergence rate ([30-31]). All schemes are applied to the solution of physical problems of the supersonic flows along a ramp and along a compression corner, in the inviscid case, whereas in the laminar viscous case, the supersonic flow along a convergent-divergent nozzle is solved.

The results have demonstrated that the [9] algorithm, with Min2 non-limiter, and [13], in its ENO version, has presented the best solutions in the inviscid ramp and compression corner problems; In the viscous problem, the [7] algorithm, in its Min1 variant, has presented the best solution in the viscous nozzle problem.

This work is the first part of this study, which compares different TVD and ENO algorithms. The next paper will treat more four numerical algorithms based on the Yee's and Yang's works.

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References:

[1] R. D. Richtmeyer, and K. W. Morton, *Difference Methods for Initial-Value Problem*, Interscience-Wiley, New York, 1967.

[2] A. Harten, On a Class of High Resolution Total Variation-Stable Finite Difference Schemes, *NYU Report*, New York University, New York, October, 1982.

[3] A. Harten, High Resolution Schemes for Hyperbolic Conservation Laws, *Journal of Computational Physics*, Vol. 49, No. 2, 1983, pp. 357-393.
[4] A. Harten, J. M. Hyman, and P. D. Lax, *Commun. Pure Appl. Math*, Vol. 29, 1976, p. 297.
[5] M. G. Crandall, and A. Majda, *Math. Comput.*, Vol. 34, No. 149, 1980, p. 1.
[6] P. L. Roe, Generalized Formulation of TVD Lax-Wendroff Schemes, *ICASE Report No. 84-53*, 1984.
[7] P. D. Lax, and B. Wendroff, Difference Schemes for Hyperbolic Equations with High Order of Accuracy, *Communications on Pure and Applied Mathematics*, Vol. XVII, 1964, pp. 381-398.
[8] S. F. Davis, TVD Finite Difference Schemes and Artificial Viscosity, *ICASE Report No. 84-20*, 1984.
[9] H. C. Yee, Construction of Explicit and Implicit Symmetric TVD Schemes and Their Applications, *Journal of Computational Physics*, Vol. 68, 1987, pp. 151-179.
[10] A. Harten, On a Class of High Resolution Total-Variation-Stable Finite-Difference Schemes", *SIAM Journal Numerical Analysis*, Vol. 21, 1984, p. 1.
[11] H. C. Yee, R. F. Warming, and A. Harten, In Proceedings of the AMS-SIAM Summer Seminar on Large-Scale Computation in Fluid Mechanics, edited by B. E. Engquist *et al.*, *Lectures in Applied Mathematics*, Vol. 22, 1983, p. 357.
[12] H. C. Yee, R. F. Warming, and A. Harten, Implicit Total Variation Diminishing (TVD) Schemes for Steady-State Calculations, *Journal of Computational Physics*, Vol. 57, No. 3, 1985, pp. 327-360.
[13] A. Harten, and S. Osher, Uniformly High Order Accurate Non-Oscillatory Schemes, *SIAM J. Num. Anal.*, Vol. 24, No. 2, 1987, pp. 279-309.
[14] A. Harten, B. Engquist, S. Osher, and S. R. Chakravarthy, Uniformly High Order Accurate Essentially Non-Oscillatory Schemes. II., *Preprint*, 1986.
[15] A. Harten, B. Engquist, S. Osher, and S. R. Chakravarthy, Some Results on Uniformly High Order Accurate Essentially Non-Oscillatory Schemes, *Advances in Numerical Analysis and Applied Mathematics*, edited by J. C. South, Jr. and M. Y. Hussaini, *ICASE Rept. 86-18*; also *Journal of Applied Num. Mathematics* Vol. 2, 1986, pp. 347-367.
[16] A. Harten, B. Engquist, S. Osher, and S. R. Chakravarthy, Uniformly High Order Accurate Essentially Non-Oscillatory Schemes. III., *ICASE Rept. 86-22*, 1986.
[17] S. K. Godunov, A Finite Difference Method for the Numerical Computation of Discontinuous Solutions of the Equations to Fluid Dynamics, *Mat. Sb*., Vol. 47, 1958, pp. 271-290.
[18] B. Van Leer, Towards the Ultimate Conservative Difference Scheme. V. A Second Order Sequel to Godunov's Method, *Journal of Computational Physics*, Vol. 32, No. 1, 1979, pp. 101-136.
[19] P. Colella, and P. R. Woodward, The Piecewise-Parabolic Method (PPM) for Gas-Dynamical Simulations, *Journal of Computational Physics*, Vol. 54, No. 1, 1984, pp. 174-201.
[20] S. Osher, and S. R. Chakravarthy, High Resolution Schemes and Entropy Conditions, *SIAM Journal of Numerical Analysis*, Vol. 21, No. 4, 1984, pp. 955-984.
[21] A. Harten, Preliminary Results on the Extension of ENO Schemes to Two-Dimensional Problems, *Proceedings of the International Conference on Hyperbolic Problems*, Saint-Etienne, France, 1986.
[22] H. C. Yee, A Class of High Resolution Explicit and Implicit Shock-Capturing Methods, *NASA TM-101088*, 1989.
[23] P. L. Roe, Characteristic-based Schemes for the Euler Equations, *Annual Review of Fluid Mechanics*, Vol. 18, 1986, pp. 337-365.
[24] G. Moretti, Computation of Flows with Shocks, *Annual Review of Fluid Mechanics*, Vol. 19, 1987, pp. 313-337.
[25] R. M. Beam, and R. F. Warming, An Implicit Factored Scheme for the Compressible Navier-Stokes Equations, *AIAA Journal*, Vol. 16, No. 4, 1978, pp. 393-402.
[26] J. Douglas, On the Numerical Integration of $u_{xx}+u_{yy}=u_t$ by Implicit Methods", Journal of the Society of Industrial and Applied Mathematics, Vol. 3, 1955, pp. 42-65.
[27] D. W. Peaceman, and H. H. Rachford, The Numerical Solution of Parabolic and Elliptic Differential Equations, *Journal of the Society of Industrial and Applied Mathematics*, Vol. 3, 1955, pp. 28-41.
[28] J. Douglas, and J. E. Gunn, A General Formulation of Alternating Direction Methods, Numerische Mathematiks, Vol. 6, 1964, pp. 428-453.
[29] N. N. Yanenko, *The Method of Fractional Steps*, Springer Verlag, N.Y., EUA, 1971.

[30] E. S. G. Maciel, Analysis of Convergence Acceleration Techniques Used in Unstructured Algorithms in the Solution of Aeronautical Problems – Part I, *Proceedings of the XVIII International Congress of Mechanical Engineering (XVIII COBEM)*, Ouro Preto, MG, Brazil, 2005. [CD-ROM]
[31] E. S. G. Maciel, Analysis of Convergence Acceleration Techniques Used in Unstructured Algorithms in the Solution of Aerospace Problems – Part II, *Proceedings of the XII Brazilian Congress of Thermal Engineering and Sciences (XII ENCIT)*, Belo Horizonte, MG, Brazil., 2008 [CD-ROM]
[32] R. W. Fox, and A. T. McDonald, *Introdução à Mecânica dos Fluidos*, Ed. Guanabara Koogan, Rio de Janeiro, RJ, Brazil, 632p, 1988.
[33] E. S. G. Maciel, Simulação Numérica de Escoamentos Supersônicos e Hipersônicos Utilizando Técnicas de Dinâmica dos Fluidos Computacional, *Doctoral Thesis*, ITA, CTA, São José dos Campos, SP, Brazil, 258p, 2002.
[34] E. S. G. Maciel, Relatório ao Conselho Nacional de Pesquisa e Desenvolvimento Tecnológico (CNPq) sobre as Atividades de Pesquisa Desenvolvidas no Terceiro Ano de Vigência da Bolsa de Estudos para Nível DCR-IF Referente ao Processo No. 304318/2003-5, *Report to the National Council of Scientific and Technological Development (CNPq)*, Recife, PE, Brazil, 52p, 2006. [available in the website www.edissonsavio.eng.br]
[35] P. L. Roe, Approximate Riemann Solvers, Parameter Vectors, and Difference Schemes, *Journal of Computational Physics*, Vol. 43, No. 2, 1981, pp. 357-372.
[36] E. S. G. Maciel, Explicit and Implicit TVD and ENO High Resolution Algorithms Applied to the Euler and Navier-Stokes Equations in Three-Dimensions – Theory, *Proceedings of the 20th International Congress of Mechanical Engineering (XX COBEM)*, Gramado, RS, Brazil. [CD-ROM]
[37] P. L. Roe, In Proceedings of the AMS-SIAM Summer Seminar on Large-Scale Computation in Fluid Mechanics, edited by B. E. Engquist *et al.*, *Lectures in Applied Mathematics*, (Amer. Math. Soc., Providence, R. I., 1985), Vol. 22, 1983, p. 163.
[38] B. Van Leer, Towards the Ultimate Conservative Difference Scheme. II. Monotonicity and Conservation Combined in a Second-Order Scheme, *Journal of Computational Physics*, Vol. 14, 1974, pp. 361-370.
[39] H. C. Yee, R. F. Warming, and A. Harten, A High-Resolution Numerical Technique for Inviscid Gas-Dynamic Problems with Weak Solutions, *Proceedings of the 8th International Conference on Numerical Methods in Fluid Dynamics*, E. Krause, Editor, Lecture Notes in Physics, Springer-Verlag, Berlim, Germany, Vol. 170, 1982, pp. 546-552.
[40] A. Jameson, and D. Mavriplis, Finite Volume Solution of the Two-Dimensional Euler Equations on a Regular Triangular Mesh, *AIAA Journal*, Vol. 24, 1986, pp. 611-618.
[41] J. T. Batina, Implicit Upwind Solution Algorithms for Three-Dimensional Unstructured Meshes, *AIAA Journal*, Vol. 31, No. 5, 1993, pp. 801-805.
[42] J. D. Anderson Jr., *Fundamentals of Aerodynamics*, McGraw-Hill, Inc., EUA, 563p, 1984.
[43] M. L. Mason, L. E. Putnam, and R. J. Re, The Effect of Throat Contouring on Two-Dimensional Converging-Diverging Nozzles at Sonic Conditions, *NASA Technical Paper 1704*, 1980.