Fast Interpolation and Approximation of Scattered Multidimensional and Dynamic Data Using Radial Basis Functions

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Abstract: Interpolation or approximation of scattered data is very often task in engineering problems. The Radial Basis Functions (RBF) interpolation is convenient for scattered (un-ordered) data sets in k-dimensional space, in general. This approach is convenient especially for a higher dimension $k > 2$ as the conversion to an ordered data set, e.g. using tessellation, is computationally very expensive. The RBF interpolation is not separable and it is based on distance of two points. It leads to a solution of a Linear System of Equations (LSE) $\boldsymbol{Ax} = \boldsymbol{b}$. There are two main groups of interpolating functions: 'global" and "local". Application of "local" functions, called Compactly Supporting RBF (CSFBF), can significantly decrease computational cost as they lead to a system of linear equations with a sparse matrix.

In this paper the RBF interpolation theory is briefly introduced at the "application level" including some basic principles and computational issues and an incremental RBF computation is presented and approximation RBF as well.

The RBF interpolation or approximation can be used also for image reconstruction, inpainting removal, for solution of Partial Differential Equations (PDE), in GIS systems, digital elevation model DEM etc.

Key-Words: - Radial basis function, RBF interpolation, image reconstruction, incremental computation, RBF approximation, fast matrix multiplication

1 Introduction

Interpolation and approximation are probably the most frequent operations used in computational techniques. Several techniques have been developed for data interpolation, but they expect some kind of data "ordering", e.g. structured mesh, rectangular mesh, unstructured mesh etc. The typical example is a solution of partial differential equations (PDE) where derivatives are replaced by differences and rectangular or hexagonal meshes are used in the vast majority of cases. However in many engineering problems, data are not ordered and they are scattered in *k* -dimensional space, in general. Usually, in technical applications the scattered data are tessellated using triangulation but this approach is quite prohibitive for the case of *k*-dimensional data interpolation because of the computational cost.

An interesting technique is *k*-dimensional data interpolation using Radial Basis Functions (RBF). The RBF interpolation is computationally more expensive because interpolated data are not ordered, but offers quite interesting applications at acceptable computational cost, e.g. solution of partial differential equations, image reconstruction, neural networks, fuzzy systems, GIS systems, optics and interferometry etc.

2 Problem Formulation

Interpolation is very often used and mostly linear interpolation is used in technical applications. Let us analyze first different types of data to be processed. Also there is a question whether the Euclidean space representation is the best for computing and engineering applications. It is well known that the division operation is very dangerous in numerical computations and causes severe problems in numerical methods. Also it is known that computations can be made in the projective extension of the Euclidean space [20] [21] [23] [27]. The projective formulation of numerical problems leads to very interesting questions, e.g. an explicit solution of LSE is equivalent to the cross-product, like why the division operation in the Gauss-Seidel or similar methods is needed [21]? The projective space representation and the principle of duality also help to solve some problems more efficiently [19] [20] [25]. Also Non-rational uniform B-Splines