Corporation in a Closed-loop Supply Chain Based on Remanufacturing

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Abstract: Remanufacturing is a new manufacture concept which is based on the theory of life cycle of products and focus on the objective to promote reuse of waste products and it has been more and more popular all over the world. This paper presents a theoretical model for remanufacturing system which is a closed-loop supply chain consists of one manufacturer, one dealer and one recycler. Game theory with respect to two pricing strategies is used to analyze the relationshpe between cooperation level and prices, sales quantity and core collection yield. The main results of the research prove that manufacturer will gain more profit by establishing partnership with dealer and recycler. Furthermore, the profits would increase along to the raise of cooperate level. At the same time, the cooperative relationships among manufacturer, dealer and recycler would generate mutual interaction in building sale and product return channel. At the end of this paper, we put forward some coordination suggestions.

Key–Words: Closed-loop supply chain; cooperative game; remanufacturing; stackelberg; Supply Chain Coordination

1 Introduction

Under certain conditions, remanufacturing can reduce costs and mitigate environmental threats. It has been proved by many researchers that remanufactured products could bring about more profits for manufacturers and the government would also prefer this new type of industrial development which can raise social outcome. In China, the government has gradually encouraged manufacturers to focus on and input more developing remanufacturing industry in nowadays.

With the help of remanufacturing, the less input may generate more economic benefits. During the twelfth five year plan, China will pay more attention on the development of the industry which calls for more savings and innovation. The success of the companies like Kodak, Caterpillar have not only given us a good example, but also inspire us to do some explorations in the field of remanufacture. At the same time, with the development of circulation economy, supply chain is no longer one-way but has changed into a closed-loop system by combining sales and recycle of the products.

Remanufacturing is a new manufacture idea which is based on the theory of ”Product Life Cycle” and focused on the objective to reuse waste products. The principles of remanufacturing are high quality, energy conservation, efficient, environmental friendly and so on. Remanufacturing is an advanced type of products recycle which can help utilize energy and materials efficiently and bring beneficial influence to the environment. Closed-loop supply chain is the combination of forward process and reverse process. It contains not only forward flow of ”resource-manufacturing-distribution-consumption” but also the reverse flow of ”waste-recycle-reuse-redistribution-consumption”. So the closed-loop supply chain is one of the most complicated system of all.

Savaskan achieved the integration of reverse supply chain and discussed the strategy of supply chain members in three types of recovery modes. Ferrer, Swaminathan established new products and remanufacturing products of two planning horizon model and spread it to multi-period horizon model. Based on a duopoly case of manufacturer, he found that the price of new products and remanufacturing products are the same. Then, based on a third party remanufacturing case, he analyzed manufacturer’s pricing strategies of new products and remanufacturing product. Afterwards these two researchers developed their study from the point of view that the differentiate pricing strategy to analyze the pricing and manufacturing strategy of manufacturers who both produce new and remanufactured products. According to the utility theory, we established a demand function of new products and remanufactured products, and gives the conditions that manufacturers would manufacture both new and remanufactured products. Chinese scholars also have developed the research in the field of closed-loop supply chain. Yuyin Yi analyzed all types of recovery mode based on game theory considering the influence of different leadership's type on the result of the game in supply chain. Zhongkai Xiong studied pointing from the profits and competition of remanufacturing closed-loop supply chain with remanufacturing. Guojun Ji studied the game between manufacturer and remanufacturer under the limitation of product recovery legislation. The result shows that it is the best to stimulate OEM by dependently taking the responsibility to recycle the core which means that under the responsibility of recovery OEM will tend to remanufacture more.

Chinese and foreign scholars have developed a lot of researches on the game of remanufacture based on closed-loop system. A lot of literatures analyzed this issue which are published in recently 10 years involve in single pricing strategy, differentiate strategy, different multi-period, multiple manufacturer, multiple dealer, stochastic demand and production, decentralized decision making and centralized decision making. Since there are decentralized and centralized decision making separately, we also consider the coordination in the game in this paper. Hao Sun and Qingli Da who have analyzed the amount of fixed payment given to the consumer who returns on unit of used product making in a Stackelberg game under the condition of centralized decision. The result shows that the result under decentralized condition is not the same with the situation under centralized condition. In the end, they establish a coordination mechanism to redistribute the income.

From the above review we may find that most of the researches about cooperative game in closed-loop supply chain is based on perfect cooperation. Considering the limitation of the cost, it is impossible for remanufacturer to establish the perfect cooperation with the other members of supply chain. Huiping Pan, Qiurong Chen studied the cooperation of the supply chain and found the relationship between cooperation level and profit of the supply chain. This paper will follow this research approach to study the influence of cooperation level in a closed-loop supply chain. We study a system consist of one manufacturer, one dealer and one recycler, and analyze the relation between cooperation level coefficient and price, production and profit under two types of pricing strategy. In the end, we will give some conclusions on the coordinate the closed-loop system.

2 Model instructions and assumptions

2.1 Instruction of the model

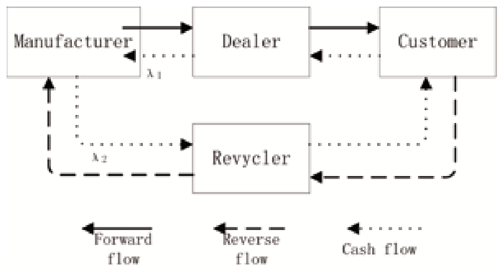

Figure 1: The closed loop system

In this paper, we develop a supply chain model that is shown in figure 1 above. Manufacturer yields new products and remanufactured products with the marginal cost of c_n and c_r respectively, w_n and w_r are the wholesale price of manufacturer, p_n and p_r are the sale price of dealer. If the price is undifferentiated then $w = w_n = w_r$, $p = p_n = p_r$. The third party recycler will determine its rate of used-product recycling(γ). The cost of recycle per unit is A, and manufacturer pay b to manufacturer for one unit of recycled products. In order to ensure that the manufacturers and recycling providers can achieve earnings, we assume that $0 \leq \gamma \leq 1$.

2.2 Variables and parameters

2.3 Assumptions

1) The manufacturers and dealers are both risk neutral. And manufacturers, according to market demand for production, do not exist surplus stock inventory.

2) To characterize the diminishing returns to investment, we use the structure $\gamma = \sqrt{I/E}$, where E is the scaling parameter(see, Literature [1])

3) Assuming that the market scale is normalized as 1. New products and remanufactured products can equally meet the consumers' needs, which means that they can be substituted by each other. When manufacturer take the undifferentiated strategy, its demand function is given by $q = 1 - p$. However, consumers' recognition of this two kinds of products are different, assuming that consumers' willingness payment is different, set α as consumers' acceptance parameter of remanufactured products, $\alpha \in (0, 1)$. v stands for willingness to pay for new products, and $v\alpha$ is consumers' willingness payment for the remanufactured products. Therefore, the utility function of consumers to buy new products and remanufactured products are $U_n(v) = v - p_n$, $U_r(v) = v - p_r$ respectively. If

Table 1: The values of Parameters and Descriptions

Parameter	Description
w_i	Wholesale price charged for product i. The subscripts are i=n(new) or r(remanufactured)
c_i	Marginal cost to produce a new product(c_n)and a remanufacturing product(c_r).
p_i	Price charged for product i.
q_i	Quantity demanded for product i.
γ	Core collection yield ($0 < \gamma < 1$), defined as the fraction of new products that is available for remanufacturing.
A	Unit cost of collecting and handling a returned unit for recycler.
b	Unit cost of returned unit paid by manufacturer to recycler.
E	The product collection effort.
α	Customers' tolerance for the remanufactured product,defined as a fraction of the utility provided by the product.
λ_1, λ_2	The cooperation level between manufacturer and dealer or recycler.

$U_n(v) \geq 0$, then the consumers are willing to buy new products. $U_r(v) \geq 0$ indicates that consumer is willing to buy remanufactured products. $U_n(v) > U_r(v)$ means consumers are more willing to buy new products rather than manufactured products. Conversely, $U_n(v) < U_r(v)$ means consumers are more willing to purchase remanufactured products rather than new products.

According to the result of computation of literature [7], we only considers the condition $\alpha \in (p_r/p_n, 1-(p_n-p_r))$. Among them, $q_n, q_r(q = q_n + q_r)$ denote the new and remanufactured-product production (sales quantity) respectively. To guarantee q_n and q_r are both non-negative, the value of parameter must meet $0 < c_r + b < c_n < 1, \frac{c_r+b}{c_n} < \alpha < 1 - c_n + (c_r + b)$. Therefore, the demand functions of new and manufactured products of are

$$q_n = 1 - \frac{p_n-p_r}{1-\alpha}$$
$$q_r = \frac{\alpha p_n - p_r}{\alpha(1-\alpha)}$$

4) λ_1 and λ_2 respectively represent the redistributing proportion of dealer and recycler from profit raising because of cooperation with manufacturer. The manufacturer need to make decision, the level of cooperation with dealer and recycler, which decide the outcome distribution among all members. The condition $\lambda_1 \in [0, 1](i = 1, 2)$ represents perfect cooperation between manufacturer and dealer or recycler. The condition $\lambda_1 \in (0, 1)$ represents the cooperation is imperfect. π^0_M, π^0_D and π^0_R stand for the game decision-making function of manufacturers, dealer and recycler respectively. π_C is the sum of the profits of manufacturers, dealers, recyclers, the profit of the entire supply chain (without considering the tax and inventory factors). In order to express more conveniently , it is assumed that profits of three parties are π_M, π_D , π_R respectively before distribution.

(5) The manufacturer is the Stackelberg leader and has enough power to influence dealer and recycler in the supply chain. The quantity of recycle can meet the need of remanufacturing. The game is divided into two stages. In the first stage, the manufacturer as the leader of the supply chain firstly identify wholesale price per unit of new products and remanufactured products and pay recycler for return used-product. In the second stage, dealer decides the sale price and recycler decides the payment for returned products per unit to consumer (A).

3 Model solution and analysis

3.1 Single-pricing models

If manufacturer adopts the non-differentiated strategy , then its demand function will be given by $q = 1 - p$. The three parties all set profit maximization as the goal in game. The profit of the manufacturer includes the income by wholesaling products to dealer and the share of total supply chain profit raising. As the Stackelberg leader, manufacturer would make decision at the first place, then dealer and recycler would decide their strategy according to manufacturer. Therefore, the single-pricing strategy model is:

$$\begin{aligned} \pi^0_M &= [w - c_n + \gamma(c_n - c_r - b)]q \\ &\quad +\lambda_1\pi_D + \lambda_2\pi_R \end{aligned} \quad (1)$$

$$\pi_D = (1 - \lambda_1)(p - w)q \quad (2)$$

$$\pi_R = (1 - \lambda_2)(b - A)\gamma q - E\gamma^2 \quad (3)$$

Because the manufacturer is the Stackelberg leader, the manufacturer firstly makes its own decision firstly, then dealer and recycler will make decision according to the manufacturer's decision to decide the price and products return rate. At the first stage, according to the equation $\frac{\partial \pi_R}{\partial \gamma} = 0$.

$$p = \frac{1 + w}{2} \quad (4)$$

And with the equation $\frac{\partial \pi_D}{\partial p} = 0$

$$\gamma = \frac{(b-A)(1-W)}{4E} \tag{5}$$

Put (4) and (5) into (1), under the extreme conditions of $\frac{\partial \pi_M}{\partial w} = 0$, the manufacturer's optimal wholesale price:

$$w = 1 - \frac{4E(1-c_n)}{f} \tag{6}$$

where $f = 8E - 2(b-A)(c_n - c_r - b) - 4E\lambda_1 - (b-A)^2$. Put (6) into the model, and it can export the outcome of the game (see Table 2).

Table 2: The values of single strategy calculations

p	$1 - \frac{2E(1-c_n)}{8E-2(b-A)(c_n-c_r-b)-4E\lambda_1-(b-A)^2\lambda_2}$
q	$\frac{2E(1-c_n)}{8E-2(b-A)(c_n-c_r-b)-4E\lambda_1-(b-A)^2}$
γ	$\frac{(b-A)(1-c_n)}{8E-2(b-A)(c_n-c_r-b)-4E\lambda_1-(b-A)^2}$
π_M	$\frac{E(1-c_n)^2[8E-2(b-A)(c_n-c_r-b)-8E\lambda_1-2(b-A)^2\lambda_2]}{[8E-2(b-A)(c_n-c_r-b)-4E\lambda_1-(b-A)^2]^2}$
π_D	$\frac{4E^2(1-c_n)^2}{[8E-2(b-A)(c_n-c_r-b)-4E\lambda_1-(b-A)^2]^2}$
π_R	$\frac{4E^2(1-c_n)^2(b-A)^2}{[8E-2(b-A)(c_n-c_r-b)-4E\lambda_1-(b-A)^2]^2}$
π_C	$\frac{E^2(1-c_n)^2}{8E-2(b-A)(c_n-c_r-b)-4E\lambda_1-(b-A)^2} + \frac{E(1-c_n)^2[4E(1-\lambda_1)+(b-A)^2(1-\lambda_1)]}{[8E-2(b-A)(c_n-c_r-b)-4E\lambda_1-(b-A)^2\lambda_2]^2}$

3.2 Differentiated pricing models

As is assumed in hypothesis, if the prices of new products and remanufactured products are different, the demand function are different.

Based on the condition that all members seek for maximizing profits the model will be

$$\pi_M^0 = \pi_M + \lambda_1\pi_D + \lambda_2\pi_R \tag{7}$$

$$\pi_D = (1-\lambda_1)[(p_n - w_n)q_n + (p_r - w_r)q_r] \tag{8}$$

$$\pi_R = (1-\lambda_2)[(b-A)\gamma(q_n+q_r) - E\gamma^2] \tag{9}$$

When the Hessian matrix is negative definite the profit function is given by π_M and shown to be concave in the decision variable p_n, p_r. We can easily judge that the Hessian matrix is negative, $\frac{4(1+\alpha)}{\alpha^2(1-\alpha)} > 0$. According to reverse induction method: $\frac{\partial \pi_D}{\partial p_n} = 0$ and $\frac{\partial \pi_D}{\partial p_r} = 0$ are simultaneously true, then it holds that

$$\begin{cases} p_n = \frac{1+w_n}{2} \\ p_r = \frac{\alpha+w_r}{2} \end{cases} \tag{10}$$

$\frac{\partial \pi_R}{\partial \gamma} = 0$ leads to

$$\gamma = \frac{(b-A)(\alpha - w_r)}{4E\alpha} \tag{11}$$

Put (10) and (11) into equation (7) by $\frac{\partial \pi_M}{\partial w_n} = 0$ and $\frac{\partial \pi_M}{\partial w_r} = 0$ simultaneously, the solutions are

$$\begin{cases} w_n' = \frac{(\alpha-c_r)(b-A)(2b-\lambda_2 b+\lambda_2 A)}{g} + \frac{1-\lambda_1+c_n}{2-\lambda_1} \\ w_r' = \alpha - \frac{4E\alpha(\alpha-c_r)}{g} \end{cases} \tag{12}$$

where $g = [4E\alpha(2-\lambda_1) + (b-A)(2b-\lambda_2 b + \lambda_2 A)](2-\lambda_1)$. The final solution is shown in table 3.

Table 3: The values of differentiated strategy calculations

p_n'	$\frac{3-2\lambda_1+c_n}{2(2-\lambda_1)} + \frac{(\alpha-c_r)(b-A)(2b-\lambda_2 b+\lambda_2 A)}{2[4E\alpha(2-\lambda_1)+(b-A)(2b-\lambda_2 b+\lambda_2 A)](2-\lambda_1)}$
p_r'	$\alpha - \frac{2E\alpha(\alpha-c_r)}{4E\alpha(2-\lambda_1)+(b-A)(2b-\lambda_2 b+\lambda_2 A)}$
q_n'	$\frac{1}{2(2-\lambda_1)} - \frac{c_n-c_r}{2(1-\alpha)(2-\lambda_1)}$
q_r'	$\frac{\alpha c_n-c_r}{2\alpha(1-\alpha)(2-\lambda_1)} - \frac{(\alpha-c_r)(b-A)(2b-\lambda_2 b+\lambda_2 A)}{2\alpha[4E\alpha(2-\lambda_1)+(b-A)(2b-\lambda_2 b+\lambda_2 A)(2-\lambda_1)]}$
γ'	$\frac{(b-A)(\alpha-c_r)}{4E\alpha(2-\lambda_1)+(b-A)(2b-\lambda_2 b+\lambda_2 A)}$

3.3 Conclusion and analysis

We can draw serval conclusions from the formula above

Conclusion 1. The wholesale price w, w_n , w_r and sale price p , p_n , p_r will decrease under the situation of the increasing of the corporation level λ_1 ,λ_2 , thus costumers would gain more surplus.

Proof of conclusion 1:

1. Solve the first-order partial derivatives of w on λ_1 and λ_2.

$$\frac{\partial w}{\partial \lambda_1} = -\frac{16E^2(1-c_n)}{[8E-2(b-A)(c_n-c_r-b)-4E\lambda_1-(b-A)^2\lambda_2]^2} < 0$$
$$\frac{\partial w}{\partial \lambda_2} = -\frac{16E^2(1-c_n)}{[8E-2(b-A)(c_n-c_r-b)-4E\lambda_1-(b-A)^2\lambda_2]^2} < 0$$

2. Solve the first-order partial derivatives of w_n' on λ_1 and λ_2 .

$$\frac{\partial w_n'}{\partial \lambda_1} = -\frac{1-c_n-\alpha+c_r}{(2-\lambda_1)^2} - \frac{16E^2\alpha^2(\alpha-c_r)}{[8E-2(b-A)(c_n-c_r-b)-4E\lambda_1-(b-A)^2\lambda_2]^2}$$

$0 < \lambda_1 < 1$, $0 < \lambda_2 < 1$

Known by the conditions in assumption 4:

$$\frac{c_r+b}{c_n} < \alpha < 1-c_n+(c_r+b)$$

Then $\alpha - c_r > 0$ According to the requisition $\alpha \in (p_r/p_n, 1-(p_n-p_r))$,

$$\alpha < 1-(P_n-p_r) = \alpha + \frac{1-c_n-\alpha+c_r}{2(2-\lambda_1)}$$
$$\Rightarrow 1-c_n-\alpha+c_r > 0$$
$$\Rightarrow \frac{\partial w_n'}{\partial \lambda_1} < 0$$
$$\frac{\partial w_n'}{\partial \lambda_2} = -\frac{4E\alpha(\alpha-c_r)(b-A)^2}{[4E\alpha(2-\lambda_1)+(b-A)(2b-\lambda_2 b+\lambda_2 A)]^2} < 0$$

3. Solve the first-order partial derivatives of w_r' on λ_1 and λ_2 .

$$\frac{\partial w_r'}{\partial \lambda_1} = -\frac{16E^2\alpha^2(\alpha-c_r)}{[4E\alpha(2-\lambda_1)+(b-A)(2b-\lambda_2 b+\lambda_2 A)]^2} < 0$$
$$\frac{\partial w_r'}{\partial \lambda_2} = -\frac{4E(b-A)^2\alpha(\alpha-c_r)}{[4E\alpha(2-\lambda_1)+(b-A)(2b-\lambda_2 b+\lambda_2 A)]^2} < 0$$

So w w_n w_r are all decreasing function about λ_1 , λ_2 under the two conditions of pricing strategies, the wholesale price of new products and manufactured products decrease with the increasing of cooperation level.

Same reason: the wholesale price, sale price of new products and remanufactured products reduce with the increasing of the cooperation level. That is conducive to product production and sales. So the increasing of cooperation level can do favor to consumer.

Conclusion 2. The increase of λ_1 and λ_2 will promote q and q'.

Proof of conclusion 2:

1. Solve the first-order partial derivatives of q on λ_1 and λ_2 .

$$\frac{\partial q}{\partial \lambda_1} = \frac{8E^2(b-A)(1-c_n)}{[8E-2(b-A)(c_n-c_r-b)-4E\lambda_1-(b-A)^2\lambda_2]^2} > 0$$
$$\frac{\partial q}{\partial \lambda_2} = \frac{2E(b-A)^2(1-c_n)}{[8E-2(b-A)(c_n-c_r-b)-4E\lambda_1-(b-A)^2\lambda_2]^2} > 0$$

2. Solve the first-order partial derivatives of q' on λ_1 and λ_2 .

$$q' = q_n' + q_r' = \frac{2E(\alpha-c_r)}{4E\alpha(2-\lambda_1)+(b-A)(2b-\lambda_2 b-\lambda_2 A)}$$
$$\frac{\partial q'}{\partial \lambda_1} = \frac{8E^2\alpha(\alpha-c_r)}{[4E\alpha(2-\lambda_1)+(b-A)(2b-\lambda_2 b+\lambda_2 A)]^2} > 0$$
$$\frac{\partial q'}{\partial \lambda_2} = \frac{2E(b-A)^2(\alpha-c_r)}{[4E\alpha(2-\lambda_1)+(b-A)(2b-\lambda_2 b+\lambda_2 A)]^2} > 0$$

Hence, q and q' are reduction function of λ_1 and λ_2 , that means under the conditions of the two pricing strategies, the total amount of sales of new products and manufactured products increase with the increasing of cooperation level.

In addition, we must point out that under the differentiate condition despite that the quantity of whole demand would increase with the raise of λ_1 the demand of remanufactured parts (q_r') would go down when

$$\lambda_1 > 2 - \frac{(b-A)(2b-\lambda_2 b+\lambda_2 A)}{4E(\sqrt{\frac{\alpha(1-\alpha)(\alpha-cr)}{1-\alpha-c_n+c_r}}-\alpha)} = \lambda_1^*.$$

This is because that while the manufacturer raising the level of cooperation with dealer in building product supply channel the price of remanufacturing parts would go down as the customers' tolerance of this two products are very different. So when the price of the new products decrease below a certain degree, it is possible that more customers choose to buy cheaper new products instead of cheaper remanufacturing products. This phenomenon can be called as "extrusion effect". In the same way we can find the relation between λ_1^* and λ_2 which is λ_1^* will increase with λ_2 , meaning that the cooperation between remanufacturer and recycler will remit this effect.

Conclusion 3. The increase of λ_1 and λ_2 will promote γ and γ' , which will turn out to dwindle waste and pollution, doing good to our environment.

Proof of conclusion 3:

1. Solve the first-order partial derivatives of γ on λ_1 and λ_2 .

$$\frac{\partial \gamma}{\partial \lambda_1} = \frac{4E(b-A)(1-c_n)}{[8E-2(b-A)(c_n-c_r-b)-4E\lambda_1-(b-A)^2\lambda_2]^2} > 0$$
$$\frac{\partial \gamma}{\partial \lambda_2} = \frac{(b-A)^3(1-c_n)}{[8E-2(b-A)(c_n-c_r-b)-4E\lambda_1-(b-A)^2\lambda_2]^2} > 0$$

2. Solve the first-order partial derivatives of γ' on

γ on λ_1 and λ_2.

$$\frac{\partial\gamma'}{\partial\lambda_1} = \frac{4E\alpha(\alpha-c_r)(b-A)}{[4E\alpha(2-\lambda_1)+(b-A)(2b-\lambda_2 b+\lambda_2 A)]^2} > 0$$
$$\frac{\partial\gamma'}{\partial\lambda_2} = \frac{(b-A)^3(\alpha-c_r)}{[4E\alpha(2-\lambda_1)+(b-A)(2b-\lambda_2 b+\lambda_2 A)]^2} > 0.$$

Therefore, γ and γ' are decreasing functions of λ_1 and λ_2, the recovery rate of waste products in the conditions of the two strategies increases with increasing the coefficient of cooperation.

Conclusion 4. Under the conditions of single-pricing strategy, the total profits of supply chain (π_D) increase with the level of cooperation.

Proof of conclusion 4:

Let $f(\lambda_1,\lambda_2) = 8E - 2(b-A)(c_n - c_r - b) - 4E\lambda_1 - (b-A)^2\lambda_2$, Then,

$$\pi_C = \frac{E(1-c_n)^2(2f-f(1,1))}{f^2}$$
$$\frac{\partial f}{\partial\lambda_1} = -4E < 0,$$
$$\frac{\partial f}{\partial\lambda_2} = -(b-A)^2 < 0$$

This is the decreasing function of λ_1 and λ_2 on $[0,1]$.

Solve the first-order partial derivatives of π_C on λ_1

$$\frac{\partial\pi_C}{\partial\lambda_1} = E(1-c_n)^2\times \left[\frac{8E+2}{f^2} + \frac{8E(f(\lambda_1,\lambda_2)-f(1,\lambda_2)-f(1,1)}{f^3}\right] > 0$$

Solve the first-order partial derivatives of π_C on λ_2

$$\frac{\partial\pi_C}{\partial\lambda_2} = E(1-c_n)^2\times \left[\frac{2(b-A)^2+2}{f^2} + \frac{2(b-A)^2(f(\lambda_1,\lambda_2)-f(1,\lambda_2)-f(1,1))}{f^3}\right] > 0$$

Hence, under the condition of single-pricing strategy, π_C is an increasing function of λ_1 and λ_2 , conclusion 4 is solved.

4 Numerical analysis

In this section we will use numerical example to verify the correctness of the above conclusions. First of all we inspect how cooperation level emerge effects on the prices, quantities and products return rate. Then we analyze the change of profits of each member with different level of cooperation. Take $c_n = 0.2, c_r = 0.06, b = 0.1, A = 0.05, E = 0.02, \alpha = 0.6$.

As is shown in figure2 and figure 4, whether single-pricing strategy condition or differentiated-pricing strategy condition, the sale price would go down with the raise of cooperation level. And the sale quantity would increase with the raise of cooperation level between manufacturer and dealer. Thus that proves the correctness of conclusion 1. This is

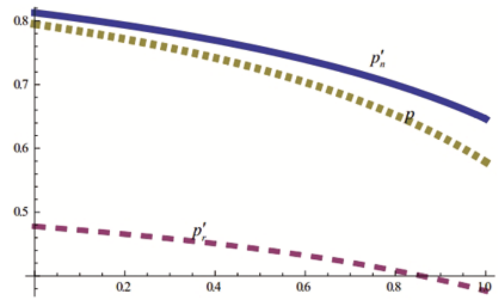

Figure 2: $\lambda_2 = 0$,the relation of sale price and λ_1

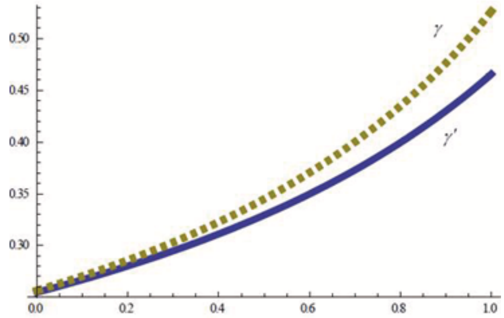

Figure 3: $\lambda_2 = 0$,the relation of recycle rate and λ_1

because with the increasing of supply chain cooperation, dealers would achieve more sales of products in favor of lower price. Although manufacturer's wholesale prices fall, the market demand become larger, the total profits grow higher. So we can see, as shown in figure 4, the change of total sales of products with the level of cooperation increasing, which shows higher level of the cooperation between manufacturers and dealers would bring more surplus.

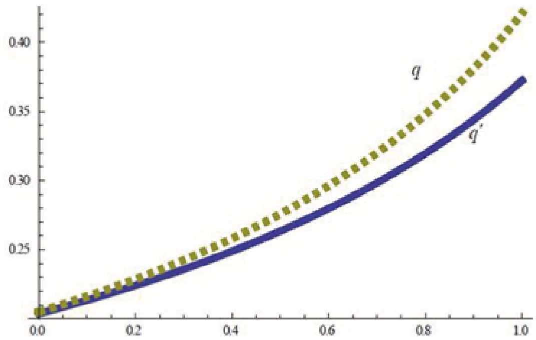

Figure 4: $\lambda_2 = 0$,the relation of sales quantity and λ_1

At the same time, figure 3 also shows the cooperation of manufacturer and dealer can do favor to the recovery of used-products. The higher the level of cooperation is, the higher the product return rate will be. That is because more products demand would provide larger amount of products that can be reused and the

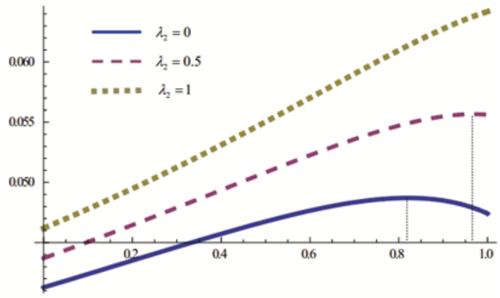

Figure 5: The relation of remanufactured sales quantity and λ_1

lower price would promote the demand of both new products and remanufactured products. So the recycler indirectly obtain benefits from cooperation relationship between manufacturer and dealer. From the figure, we know the raising of λ_1 would promote sale quantity but from figure 6 we can see the change when two types of cooperation level influence simultaneously. We can see that with the increasing of λ_2 the extrusion effect is receding.

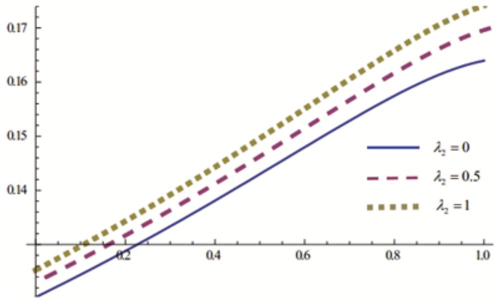

Figure 6: The relation of total profit and λ_1 under single strategy

The rising level of cooperation will promote the benefits of dealer and the third party recycler. At the same time, the cooperation would also promote the benefit of the whole supply chain. As is shown in figure 6, the change of total profit of supply chain under single-pricing strategy and differentiated-pricing strategy conditions.

5 The build of cooperation mechanism and profit distribution

In the process of cooperation, manufacturers will lose certain scale of profit with the reason that it has to drop down the wholesale price to keep the cooperation. Lower wholesale price can cause decreasing of its own profit, it is not to say that the manufacturer would sacrifice its own profit to rise the profit of the whole supply chain. Because there must be an mechanism to coordinate all the members. Supply chain channels are usually built cooperatively by manufacturers dealers and recyclers. In the real market, there are many relations that can be considered as cooperation, for example, franchise is a typical model of profits sharing between manufacturer and dealer. In this paper, the supply chain we consider is based on leading manufacturer game, the manufacturer is the leader not only in decision-making but also in channel building. Some literatures also point out that the cooperation level actually reflects the obligation and participation of manufacturer in channel construction. The profit increasing comes from collaboration which can be regarded as the foundation of cooperation. But because of the the limitation of building cooperation's cost, manufacturers may not have the ability to reach perfect cooperation $\lambda = 1$. However, in order to make its own profit more than that of the condition $\lambda = 0$, manufacturer will try hard to seek for the cooperation. And with considering the realistic factors the condition $0 < \lambda < 0$ is a more feasible scheme.

The above analysis has proved that the increase of cooperation level will cause the profit increasing of the system. Besides that, as the leader of the game, the manufacturer can generate the cooperation by reaching a contract which coordinate with other members. Three members of supply chain can consider redistribute the total profit under different level of cooperation .

Assume θ as allocation proportion, the value of θ based on the contribution and participation of members. With the contract, the manufacturer can share the increased income by the proportion θ_D and θ_R from dealer and recycler respectively. At first, the earnings of the three parties were π_M, π_D, π_R and the total profit of the supply chain is π_C . Considering the factors such as contract execution cost, manufacturers are almost impossible to complete wholesale prices of products sell to the dealers under the cooperation. But to achieve certain of cooperation, with the hypothesis, under single-pricing strategy condition, manufacturer sell the product to dealer with the price $1 - \frac{4E(1-c_n)}{8E-2(b-A)(c_n-c_r-b)-4E\lambda_1^*-(b-A)^2\lambda_2^*}$, then the total profit of three members is $\pi_M^{\lambda_1^*,\lambda_2^*}$, $\pi_D^{\lambda_1^*,\lambda_2^*}$, $\pi_R^{\lambda_1^*,\lambda_2^*}$, $\pi_C^{\lambda_1^*,\lambda_2^*}$and after the redistribution the final profit would be $\bar{\pi}_M^{\lambda_1^*,\lambda_2^*}$, $\bar{\pi}_D^{\lambda_1^*,\lambda_2^*}$, $\bar{\pi}_R^{\lambda_1^*,\lambda_2^*}$.

The redistribution model must meet:

$$\bar{\pi}_M^{\lambda_1^*,\lambda_2^*} = \pi_M^{\lambda_1^*,\lambda_2^*} + (\theta_D + \theta_R)(\pi_M^{\lambda_1^*,\lambda_2^*} - \pi_C) \geq \pi_M \quad (13)$$

$$\bar{\pi}_D^{\lambda_1^*,\lambda_2^*} = \pi_D^{\lambda_1^*,\lambda_2^*} - \theta_D(\pi_C^{\lambda_1^*,\lambda_2^*} - \pi_C) \geq \pi_D \quad (14)$$

$$\bar{\pi}_R^{\lambda_1^*,\lambda_2^*} = \pi_R^{\lambda_1^*,\lambda_2^*} - \theta_R(\pi_C^{\lambda_1^*,\lambda_2^*} - \pi_C) \geq \pi_R \quad (15)$$

where $\theta_i > 0, i = D, R$

The model illustrates a prerequisite of achieving cooperation that is the benefits of all supply chain participants can be improved. To maintain partnership, the members should establish the contract based on "powers and responsibilities of the peer". In practice, the value of θ not only depends on the contribution and influence of each member, but also depends on the bargaining power.

Solve the inequality (13) -(15):

$$\theta_D \leq \frac{\pi_D^{\lambda_1^*,\lambda_2^*} - \pi_D}{\pi_C^{\lambda_1^*,\lambda_2^*} - \pi_C}$$

$$\theta_R \leq \frac{\pi_R^{\lambda_1^*,\lambda_2^*} - \pi_R}{\pi_C^{\lambda_1^*,\lambda_2^*} - \pi_C}$$

$$\theta_D + \theta_R > \frac{\pi_M^{\lambda_1^*,\lambda_2^*} - \pi_M}{\pi_C^{\lambda_1^*,\lambda_2^*} - \pi_C}$$

Because the manufacturer is the Stackelberg leader in the supply chain, it has more bargaining power in the negotiation with dealer and third party. So in order to maximize its own profit, the manufacturer tends to share the proportion of $\frac{\pi_D^{\lambda_1^*,\lambda_2^*} - \pi_D}{\pi_C^{\lambda_1^*,\lambda_2^*} - \pi_C}$ and $\frac{\pi_R^{\lambda_1^*,\lambda_2^*} - \pi_R}{\pi_C^{\lambda_1^*,\lambda_2^*} - \pi_C}$ with dealer and recycler.

6 Conclusion

We analyze the closed-loop supply chain with one manufacturer, one dealer and one third-party recycler, and build a remanufacturing game model based on variable level of cooperation. We analyze the effects of the level of cooperation, manufacturing new products, manufactured products prices changes in yields and recoveries on the models. According to the analysis, the following conclusions are: the conditions that we do not consider cooperation costs, rising of the cooperate level can account for increasing in sales of both new products and remanufactured products, and used-products reusing. At the same time, as manufacturers have established cooperation with the two parties, dealers and recyclers will also indirectly get the profit from cooperation. Manufacturers, as the dominant party in supply chain, should take positive measures to contribute to maintain cooperation relationship with dealers, recyclers in building sales and recycling channels. Although manufacturer may find it is not necessarily to achieve complete cooperation, a certain level of cooperation can help improve the return of their own and the entire supply chain.

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