Exponential stability of reaction-diffusion Cohen-Grossberg-type BAM neural networks with time delays

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Abstract: The global exponential stability is investigated for a class of reaction-diffusion Cohen-Grossberg-type BAM neural networks with time delays. By constructing suitable Lyapunov functional and using homeomorphism mapping, several sufficient conditions guaranteeing the existence, uniqueness and global exponential stability of reaction-diffusion Cohen-Grossberg-type BAM neural networks with time delays are given. Moreover, two illustrative examples are presented to illustrate the feasibility and effectiveness of our results.

Key–Words: Reaction-diffusion Cohen-Grossberg-type BAM neural networks; Time delays; Lyapunov functional; Equilibrium point; Exponential stability

1 Introduction

Bi-directional associative memory (BAM) neural networks, first introduced by Kosko[1], is a special class of recurrent neural networks that can store bipolar vector pairs. In recent decades, BAM neural networks has been successfully applied to pattern recognition and artificial intelligence due to its generalization of single-layer auto-associative Hebbian correlator to a two-layer pattern-matched hetero-associative circuits. In the designs and applications of networks, the stability of the designed neural network is one of the most important issues . There have been Many results concerning mainly on the existence and stability of the equilibrium point of BAM neural networks (see[2-10]).

As we know, Cohen-Grossberg neural network(CGNN), which includes a lot of famous neural networks such as Lotka-Volterra system, Hop field neural networks and cellular neural networks, and so on, has attracted considerable attention for its potential applications in classification, parallel computation, associative memory and great ability to solve difficult optimization problems since initially proposed and studied by Cohen and Grossberg in 1983 [11]. Based on BAM neural networks and Cohen-Grossberg neural networks, Cohen-Grossberg-type BAM neural networks (i.e., the BAM model that possesses Cohen-Grossberg dynamics) was naturally proposed and received overwhelming attention. Many researchers devoted to the dynamical analysis of Cohen-Grossberg-type BAM neural networks in recent years, especially the asymptotic and exponential stability (see[12-21]), which has the extremely close relation with the application of networks. For example, In [12], Cao and Song further investigated the global exponential stability for Cohen-Grossberg-type BAM neural networks with time-varying delays by using Lyapunov function, M-matrix theory and inequality technique. In [13], by constructing a suitable Lyapunov functional, the asymptotic stability was investigated for Cohen-Grossberg-type BAM neural network. In [14], the authors have proposed a new Cohen-Grossberg-type BAM neural network model with time delays, and some new sufficient conditions ensuring the existence and global asymptotical stability of equilibrium point for this model have been derived. The authors in [15-18] have investigated the periodicity of delayed Cohen-Grossberg-type BAM neural networks with variable coefficients. In [19-21], authors investigated the stability problem of Cohen-Grossberg-type BAM neural networks under the stochastic effects, impulsive effects and Markovian jumping effects , respectively.

However, In the factual operations, the diffusion phenomena could not be ignored in neural networks and electric circuits once electrons transport in a nonuniform electromagnetic field. So, it is essential to consider the state variables are varying with the time and space variables. On the other hand, due to the finite transmission speed of signals among neurons, time delays inevitably occur in artificial neural networks. Therefore, it is necessary to do some research on Cohen-Grossberg-type BAM neural networks with time delays and reaction-diffusion terms. In [22-27], authors have considered the stability of Cohen-Grossberg neural networks with reaction-diffusion terms neural networks , and with time-varying delays or continuous distribution delays. The study on the stability of BAM neural networks with delays and reaction-diffusion terms see ([28-32]). However, To the best of our knowledge, there have been very few results on analysis for Cohen-Grossberg-type BAM neural networks with time delays and reaction diffusion terms.

Motivated by the above discussions, a class of reaction-diffusion Cohen-Grossberg-type BAM neural networks with time delays is considered in this paper. We will derive some sufficient conditions of existence, uniqueness and exponential stability of equilibrium points for reaction-diffusion Cohen-Grossberg-type BAM neutral networks with time delays by constructing suitable Lyapunov functional and using homeomorphism mapping. The rest of this paper is organized as follows: In Section 2 the model formulation and some preliminaries are given. The main results are stated in Section 3. Finally, two illustrative examples are given to show the effectiveness of the proposed theory.

Consider the following reaction-diffusion Cohen-Grossberg-type BAM neural networks

$$\begin{cases} \frac{\partial u_i(t,x)}{\partial t} = \sum\limits_{k=1}^{l} \frac{\partial}{\partial x_k}(D_{ik}\frac{\partial u_i(t,x)}{\partial x_k}) \\ \qquad -a_i(u_i(t,x))[b_i(u_i(t,x)) \\ \qquad - \sum\limits_{j=1}^{m} a_{ij} f_j(v_j(t,x)) \\ \qquad - \sum\limits_{j=1}^{m} c_{ij} f_j(v_j(t-\tau_{ij},x)) - I_i], \\ \frac{\partial v_j(t,x)}{\partial t} = \sum\limits_{k=1}^{l} \frac{\partial}{\partial x_k}(E_{jk}\frac{\partial v_j(t,x)}{\partial x_k}) \\ \qquad -d_j(v_j(t,x))[e_j(v_j(t,x)) \\ \qquad - \sum\limits_{i=1}^{n} b_{ji} g_i(u_i(t,x)) \\ \qquad - \sum\limits_{i=1}^{n} h_{ji} g_i(u_i(t-\sigma_{ji},x)) - J_j], \end{cases} \tag{1}$$

for $i = 1,2,\cdots,n, j = 1,2,\cdots,m$, where $x = (x_1, x_2, \cdots, x_l)^T \in \Omega_i \subset R^l$ and Ω_i is a bounded compact set with smooth boundary $\partial\Omega_i$ and mes $\Omega_i > 0$ in space R^l; $u(t,x) = (u_1(t,x), u_2(t,x), \cdots, u_n(t,x))^T \in R^n, v(t,x) = (v_1(t,x), v_2(t,x), \cdots, v_m(t,x))^T \in R^m$, $u_i(t,x)$ and $v_j(t,x)$ are the state of the ith neurons from the neural field F_U and the jth neurons from the neural field F_V at time t and in space x , respectively; $D_{ik} > 0$ and $E_{jk} > 0$ correspond to the transmission reaction-diffusion operator along the ith neurons and the jth neurons, respectively; f_j and g_i denote the activation function of the jth neurons and the ith neurons at time t and in space x, respectively ; a_{ij} and c_{ij} weights the strength of the ith neuron on the jth neuron at the time t and $t-\tau_{ij}$, respectively; b_{ji} and h_{ji} weights the strength of the jth neuron on the ith neuron at the time t and $t-\sigma_{ji}$, respectively; $\tau_{ij} \geq 0$ and $\sigma_{ji} \geq 0$ are nonnegative constants, they correspond to finite speed of axonal signal transmission; I_i, J_j denote the external inputs on the ith neuron from F_U and the jth neuron from F_V , respectively; $a_i(u_i(t,x))$ and $d_j(v_j(t,x))$ represent amplification functions; $b_i(u_i(t,x))$ and $e_j(v_j(t,x))$ are appropriately behaved functions such that the solutions of model(1) remain bounded.

The boundary conditions and initial conditions of system (1) are given by

$$\begin{cases} \frac{\partial u_i(t,x)}{\partial n} = (\frac{\partial u_i(t,x)}{\partial x_1}, \frac{\partial u_i(t,x)}{\partial x_2}, \cdots, \frac{\partial u_i(t,x)}{\partial x_l})^T = 0, \\ \quad x \in \partial\Omega_i, \ \ i = 1,2,\cdots,n, \\ \frac{\partial v_j(t,x)}{\partial n} = (\frac{\partial v_j(t,x)}{\partial x_1}, \frac{\partial v_j(t,x)}{\partial x_2}, \cdots, \frac{\partial v_j(t,x)}{\partial x_l})^T = 0, \\ \quad x \in \partial\Omega_i, \ \ j = 1,2,\cdots,m. \end{cases} \tag{2}$$

and

$$\begin{cases} u_i(s,x) = \phi_{ui}(s,x), s \in [-\sigma, 0], \\ v_j(s,x) = \phi_{vj}(s,x), s \in [-\tau, 0], \end{cases} \tag{3}$$

for $x \in \Omega_i, i = 1,2,\cdots,n, j = 1,2,\cdots,m$, where $\sigma = \max\limits_{1\leq i\leq n, 1\leq j\leq m}\{\sigma_{ji}\}$, $\tau = \max\limits_{1\leq i\leq n, 1\leq j\leq m}\{\tau_{ij}\}$, $\phi_{ui}(s,x)$ and $\phi_{vj}(s,x)$ are bounded and continuous on $[-\delta, 0] \times \Omega_i$, $\delta = \max\{\sigma, \tau\}$.

2 Preliminaries

In order to establish the stability conditions for system (1), we first give some usual assumptions.

(H1): For each $i = 1,2,\ldots,n; j = 1,2,\ldots,m$, the amplification functions $a_i(z), d_j(z)$ are positive and continuously bounded, that is, there exist constants $0 < \underline{a}_i < \bar{a}_i$, and $0 < \underline{d}_j < \bar{d}_j$, such that $\underline{a}_i \leq a_i(z) \leq \bar{a}_i$, $\underline{d}_j \leq d_j(z) \leq \bar{d}_j$ for all $z \in R = (-\infty, +\infty)$.

(H2): $b_i(z)$ and $e_j(z)$ are locally Lipschitz continuous, and there exist $\beta_i > 0$ and $\gamma_j > 0$, $i = 1,2,\ldots,n; j = 1,2,\ldots,m$, such that $z[b_i(z+y) - b_i(y)] \geq \beta_i z^2$, $z[e_j(z+y) - e_j(y)] \geq \gamma_j z^2$, $z, y \in R$.

(H3): The activation functions f_j and $g_i(i = 1,2,\ldots,n; j = 1,2,\ldots,m)$ satisfy Lipschitz condition, that is, there exist constant $F_j > 0$ and $G_i > 0$, such that

$$|f_j(\xi_1) - f_j(\xi_2)| \leq F_j|\xi_1 - \xi_2|,$$
$$|g_i(\xi_1) - g_i(\xi_2)| \leq G_i|\xi_1 - \xi_2|,$$

for any $\xi_1, \xi_2 \in R$.

Let $a_i^* = \underline{a}_i/\bar{a}_i$, $d_j^* = \underline{d}_j/\bar{d}_j$, and

$$\begin{aligned}
&u(t,x) = (u_1(t,x), u_2(t,x), \cdots, u_n(t,x))^T,\\
&v(t,x) = (v_1(t,x), v_2(t,x), \cdots, v_m(t,x))^T,\\
&(u(t,x), v(t,x))^T\\
&= (u_1(t,x), \cdots, u_n(t,x), v_1(t,x), \cdots, v_m(t,x))^T\\
&u^* = (u_1^*, u_2^*, \cdots, u_n^*)^T, v^* = (v_1^*, v_2^*, \cdots, v_m^*)^T,\\
&(u^*, v^*)^T = (u_1^*, u_2^*, \cdots, u_n^*, v_1^*, v_2^*, \cdots, v_m^*)^T.
\end{aligned}$$

Definition 1 *The point $(u^*, v^*)^T$ is called a equilibrium point of system (1), if it satisfies the following equations*

$$\begin{cases} -b_i(u_i^*) + \sum\limits_{j=1}^{m} a_{ij} f_j(v_j^*) + \sum\limits_{j=1}^{m} c_{ij} f_j(v_j^*) + I_i = 0, \\ -e_j(v_j^*) + \sum\limits_{i=1}^{n} b_{ji} g_i(u_i^*) + \sum\limits_{i=1}^{n} h_{ji} g_i(u_i^*) + J_j = 0, \end{cases} \tag{4}$$

for $i = 1, 2, \cdots, n, j = 1, 2, \cdots, m$.

Definition 2 *Let $(u^*, v^*)^T$ be the equilibrium point of system (1), we define the norm*

$$\begin{aligned}
&\|u_i(t,x) - u_i^*\|_2^2 = \int_\Omega (u_i(t,x) - u_i^*)^2 dx,\\
&\|v_j(t,x) - v_j^*\|_2^2 = \int_\Omega (v_j(t,x) - v_j^*)^2 dx,\\
&\|\phi_u - u^*\|_2 = \sup_{-\sigma \le t \le 0} \sum_{i=1}^{n} \|\phi_{ui}(t,x) - u_i^*\|_2,\\
&\|\phi_v - v^*\|_2 = \sup_{-\tau \le t \le 0} \sum_{j=1}^{m} \|\phi_{vj}(t,x) - v_j^*\|_2,\\
&\|u\|^2 = \sum_{i=1}^{n} |u_i(t,x)|^2, \|v\|^2 = \sum_{j=1}^{m} |v_i(t,x)|^2,
\end{aligned}$$

where $\phi_u = (\phi_{u1}, \phi_{u2}, \cdots, \phi_{un})^T$ and $\phi_v = (\phi_{v1}, \phi_{v2}, \cdots, \phi_{vm})^T$ are initial values.

Definition 3 *The equilibrium point $(u^*, v^*)^T$ of system (1) is said to be globally exponentially stable, if there exist constants $\alpha > 0$ and $M \ge 1$ such that*

$$\begin{aligned}
&\sum_{i=1}^{n} \|u_i(t,x) - u_i^*\|_2 + \sum_{j=1}^{m} \|v_j(t,x) - v_j^*\|_2\\
&\le M e^{-\alpha t} [\|\phi_u - u^*\|_2 + \|\phi_v - v^*\|_2],
\end{aligned}$$

for all $t \ge 0$, where $(u(t,x), v(t,x))^T = (u_1(t,x), u_2(t,x), \cdots u_n(t,x), v_1(t,x), v_2(t,x), v_m(t,x))^T$ is any solution of system (1) with boundary conditions (2) and initial conditions(3).

Lemma 4 *[33] If $H(u) \in C^0$, and it satisfies the following conditions*

1) $H(u)$ is injective on R^n,

2) $\|H(u)\| \to +\infty$, as $\|u\| \to +\infty$,

then $H(u)$ is a homeomorphism of R^n.

Lemma 5 *Assume that*

$$-a_i^* \beta_i + \sum_{j=1}^{m} |b_{ji}| G_i + \sum_{j=1}^{m} |h_{ji}| G_i < 0,$$

$$-d_j^* \gamma_j + \sum_{i=1}^{n} |a_{ij}| F_j + \sum_{i=1}^{n} |c_{ij}| F_j < 0,$$

for $i = 1, 2, \cdots, n, j = 1, 2, \cdots, m$, then there exists $\alpha > 0$, such that

$$\frac{\alpha}{\bar{a}_i} - a_i^* \beta_i + \sum_{j=1}^{m} |b_{ji}| G_i + e^{\alpha\sigma} \sum_{j=1}^{m} |h_{ji}| G_i \le 0,$$

$$\frac{\alpha}{\bar{d}_j} - d_j^* \gamma_j + \sum_{i=1}^{n} |a_{ij}| F_j + e^{\alpha\tau} \sum_{i=1}^{n} |c_{ij}| F_j \le 0,$$

for $i = 1, 2, \ldots. n, j = 1, 2, \ldots. m$.

Proof. Let
$\varphi_i(\alpha) = \frac{\alpha}{\bar{a}_i} - a_i^* \beta_i + \sum\limits_{j=1}^{m} |b_{ji}| G_i + e^{\alpha\sigma} \sum\limits_{j=1}^{m} |h_{ji}| G_i$,
$i = 1, 2, \ldots, n$,
$\psi_j(\alpha) = \frac{\alpha}{\bar{d}_j} - d_j^* \gamma_j + \sum\limits_{i=1}^{n} |a_{ij}| F_j + e^{\alpha\tau} \sum\limits_{i=1}^{n} |c_{ij}| F_j$,
$j = 1, 2, \ldots, m$,
then they hold that
$\frac{d\varphi_i(\alpha)}{d\alpha} > 0, \quad \lim\limits_{\alpha \to +\infty} \varphi_i(\alpha) = +\infty, \quad \varphi_i(0) < 0$,
$i = 1, 2, \ldots, n$,
$\frac{d\psi_j(\alpha)}{d\alpha} > 0, \quad \lim\limits_{\alpha \to +\infty} \psi_j(\alpha) = +\infty, \quad \psi_j(0) < 0$,
$j = 1, 2, \ldots, m$.
Therefore, there exist constants α_i and $\alpha_j^* \in (0, +\infty)$, such that
$\varphi_i(\alpha_i) = 0, \quad i = 1, 2, \ldots, n, \quad \psi_j(\alpha_j^*) = 0$,
$j = 1, 2, \ldots, m$.
We choose
$\alpha = min\{\alpha_1, \alpha_2, \ldots, \alpha_n, \alpha_1^*, \alpha_2^*, \ldots, \alpha_m^*\}$, then $\alpha > 0$ and it satisfies that $\varphi_i(\alpha) \le 0, \quad i = 1, 2, \ldots, n, \quad \psi_j(\alpha) \le 0, j = 1, 2, \ldots, m$,
which means there exists constant $\alpha > 0$, such that
$\frac{\alpha}{\bar{a}_i} - a_i^* \beta_i + \sum\limits_{j=1}^{m} |b_{ji}| G_i + e^{\alpha\sigma} \sum\limits_{j=1}^{m} |h_{ji}| G_i \le 0$,
$i = 1, 2, \ldots, n$,
$\frac{\alpha}{\bar{d}_j} - d_j^* \gamma_j + \sum\limits_{i=1}^{n} |a_{ij}| F_j + e^{\alpha\tau} \sum\limits_{i=1}^{n} |c_{ij}| F_j \le 0$,
$j = 1, 2, \ldots, m$.
This completes the proof. □

Lemma 6 *Assume that*

$$-\underline{a}_i \beta_i + \sum_{j=1}^{m} \bar{d}_j |b_{ji}| G_i + \sum_{j=1}^{m} \bar{d}_j |h_{ji}| G_i < 0,$$

$$-\underline{d}_j\gamma_j+\sum_{i=1}^{n}\bar{a}_i|a_{ij}|F_j+\sum_{i=1}^{n}\bar{a}_i|c_{ij}|F_j<0,$$

for $i=1,2,\ldots.n, j=1,2,\ldots.m$, *then there exists* $\alpha>0$, *such that*

$$\alpha-\underline{a}_i\beta_i+\sum_{j=1}^{m}\bar{d}_j|b_{ji}|G_i+e^{\alpha\sigma}\sum_{j=1}^{m}\bar{d}_j|h_{ji}|G_i\le 0,$$

$$\alpha-\underline{d}_j\gamma_j+\sum_{i=1}^{n}\bar{a}_i|a_{ij}|F_j+e^{\alpha\tau}\sum_{i=1}^{n}\bar{a}_i|c_{ij}|F_j\le 0,$$

for $i=1,2,\ldots.n, j=1,2,\ldots.m$.

The proof is similar to that in proving Lemma 5, we omit it. □

3 Main results

In this section, we will derive some sufficient conditions to guarantee the existence, uniqueness and the exponential stability of the equilibrium point for system (1).

Theorem 7 *Under hypotheses* $(H1)-(H3)$, *then the system (1) has a unique equilibrium point if*

$$-a_i^*\beta_i+\frac{1}{2}\sum_{j=1}^{m}[|a_{ij}|+\|c_{ij}|]F_j+\sum_{j=1}^{m}[|b_{ji}|+|h_{ji}|]G_i<0, \tag{5}$$

$$-d_j^*\gamma_j+\sum_{i=1}^{n}[|a_{ij}|+|c_{ij}|]F_j+\frac{1}{2}\sum_{i=1}^{n}[|b_{ij}|+|h_{ji}|]G_i<0, \tag{6}$$

for $i=1,2,\ldots.n, j=1,2,\ldots.m$.

Proof. Let $H(u,v)=(H_1(u,v),H_2(u,v),\cdots,H_n(u,v), H_{n+1}(u,v),\cdots,H_{n+m}(u,v))^T$ where $H_i(u,v)=-b_i(u_i)+\sum\limits_{j=1}^{m}a_{ij}f_j(v_j)+\sum\limits_{j=1}^{m}c_{ij}f_j(v_j)+I_i, i=1,2\cdots,n$, and $H_{n+j}(u,v)=-e_j(v_j)+\sum\limits_{i=1}^{n}b_{ji}g_i(u_i)+\sum\limits_{i=1}^{n}h_{ji}g_i(u_i)+J_j, j=1,2\cdots,m$.

It is known that the solutions of $H(u,v)=0$ are the equilibriums of system (1). If the mapping $H(u,v)$ is a homeomorphism on R^{n+m}, then there exists a unique point (u^*,v^*), such that $H(u^*,v^*)=0$, i.e., system (1) has a unique equilibrium point (u^*,v^*). In the following, we shall prove that $H(u,v)$ is a homeomorphism.

Firstly, we prove that $H(u,v)$ is an injective mapping on R^{n+m}. In fact, if there exist $(u,v)=(u_1,u_2,\cdots,u_n,v_1,v_2,\cdots,v_m)^T$, $(\bar{u},\bar{v})=(\bar{u}_1,\bar{u}_2,\cdots,\bar{u}_n,\bar{v}_1,\bar{v}_2,\cdots,\bar{v}_m)^T\in R^{n+m}$, such that $H(u,v)=H(\bar{u},\bar{v})$ for $(u,v)\neq(\bar{u},\bar{v})$, then

$$-[b_i(u_i)-b_i(\bar{u}_i)]+\sum_{j=1}^{m}a_{ij}(f_j(v_j)-f_j(\bar{v}_j)) +\sum_{j=1}^{m}c_{ij}[f_j(v_j)-f_j(\bar{v}_j)]=0, i=1,2\cdots,n. \tag{7}$$

$$-[d_j(v_j)-d_j(\bar{v}_j)]+\sum_{i=1}^{n}b_{ji}(g_i(u_i)-g_i(\bar{u}_i)) +\sum_{i=1}^{n}h_{ji}[g_i(u_i)-g_i(\bar{u}_i)]=0, j=1,2\cdots,m. \tag{8}$$

By multiplying both sides of (7) by $(u_i-\bar{u}_i)$, we have

$$\begin{aligned}&-[b_i(u_i)-b_i(\bar{u}_i)](u_i-\bar{u}_i)\\&+(u_i-\bar{u}_i)\sum_{j=1}^{m}a_{ij}[f_j(v_j)-f_j(\bar{v}_j)]\\&+(u_i-\bar{u}_i)\sum_{j=1}^{m}c_{ij}[f_j(v_j)-f_j(\bar{v}_j)]=0,\end{aligned} \tag{9}$$

for $i=1,2\cdots,n$.
Which means

$$-a_i^*\beta_i(u_i-\bar{u}_i)^2+|u_i-\bar{u}_i|\sum_{j=1}^{m}|a_{ij}|.|f_j(v_j)-f_j(\bar{v}_j)| +|u_i-\bar{u}_i|\sum_{j=1}^{m}|c_{ij}|.|f_j(v_j)-f_j(\bar{v}_j)|\ge 0, \tag{10}$$

for $i=1,2\cdots,n$.
Since $|f_j(v_j)-f_j(\bar{v}_j)|\le F_j|v_j-\bar{v}_j|$, therefore from (10), we have

$$-a_i^*\beta_i(u_i-\bar{u}_i)^2+\sum_{j=1}^{m}|c_{ij}|F_j|u_i-\bar{u}_i|.|v_j-\bar{v}_j| +\sum_{j=1}^{m}|c_{ij}|F_j|u_i-\bar{u}_i|.|v_j-\bar{v}_j|\ge 0, \tag{11}$$

for $i=1,2\cdots,n$.
Applying the inequality: $a^2+b^2\ge 2|a|.|b|$ to (11), it follows that

$$-a_i^*\beta_i(u_i-\bar{u}_i)^2+\tfrac{1}{2}\sum_{j=1}^{m}[|a_{ij}| +|c_{ij}|]F_j[(u_i-\bar{u}_i)^2+(v_j-\bar{v}_j)^2]\ge 0, \tag{12}$$

for $i=1,2\cdots,n$.
Similarly, from (8), we derive

$$-d_j^*\gamma_j(v_j-\bar{v}_j)^2+\tfrac{1}{2}\sum_{i=1}^{n}[|b_{ji}| +|h_{ji}|]G_i[(u_i-\bar{u}_i)^2+(v_j-\bar{v}_j)^2]\ge 0, \tag{13}$$

for $j = 1, 2\cdots, m$.
Plus the left sides of (12)-(13), and merge the similar items, we can obtain

$$\sum_{i=1}^{n}\{-a_i^*\beta_i+\frac{1}{2}\sum_{j=1}^{m}[|a_{ij}|+|c_{ij}|]F_j$$
$$+\frac{1}{2}\sum_{j=1}^{m}[|b_{ji}|+|h_{ji}|]G_i\}(u_i-\bar{u}_i)^2$$
$$+\sum_{j=1}^{m}\{-d_j^*\gamma_j+\frac{1}{2}\sum_{i=1}^{n}[|a_{ij}|+|c_{ij}|]F_j$$
$$+\frac{1}{2}\sum_{i=1}^{n}[|b_{ji}|+|h_{ji}|]G_i\}(v_j-\bar{v}_j)^2\geq 0. \quad (14)$$

According to (5) and (6), from (14), it is easy to see that
$u_i = \bar{u}_i, v_j = \bar{v}_j$, for $i = 1, 2\cdots, n, j = 1, 2\cdots, m$, which contradict $(u, v) \neq (\bar{u}, \bar{v})$. So $H(u, v)$ is an injective mapping on R^{n+m}.
Secondly, we prove that $\|H(u, v)\| \to +\infty$ as $\|(u, v)\| \to +\infty$.
Let $\tilde{H}(u, v) = H(u, v) - H(0, 0)$
$= \Big(\tilde{H}_1(u, v), \tilde{H}_2(u, v), \cdots, \tilde{H}_n(u, v),$
$\tilde{H}_{n+1}(u, v), \tilde{H}_{n+2}(u, v), \cdots, \tilde{H}_{n+m}(u, v)\Big)^T$
where

$$\tilde{H}_i(u,v)=-[b_i(u_i)-b_i(0)]$$
$$+\sum_{j=1}^{m}a_{ij}[f_j(v_j)-f_j(0)]+\sum_{j=1}^{m}c_{ij}[f_j(v_j)-f_j(0)], \quad (15)$$
$$\tilde{H}_{n+j}(u,v)=-[e_j(v_j)-e_j(0)]$$
$$+\sum_{i=1}^{n}b_{ji}[g_i(u_i)-g_i(0)]+\sum_{i=1}^{n}h_{ji}[g_i(u_i)-g_i(0)], \quad (16)$$

for $i = 1, 2\cdots, n, j = 1, 2\cdots, m$.
By (15) and (16), we can find

$(u,v)^T\tilde{H}(u,v)=\sum_{i=1}^{n}u_i\tilde{H}_i(u,v)+\sum_{j=1}^{m}v_j\tilde{H}_{n+j}(u,v)$
$=\sum_{i=1}^{n}\{-[b_i(u_i)-b_i(0)]u_i+\sum_{j=1}^{m}a_{ij}u_i(f_j(v_j)-f_j(0))$
$+\sum_{j=1}^{m}c_{ij}u_i[f_j(v_j)-f_j(0)]\}$
$+\sum_{j=1}^{m}\{-[e_j(v_j)-e_j(0)]v_j+\sum_{i=1}^{n}b_{ji}v_j(g_i(u_i)-g_i(0))$
$+\sum_{i=1}^{n}h_{ji}v_j[g_i(u_i)-g_i(0)]\}$
$\leq\sum_{i=1}^{n}\{-a_i^*\beta_iu_i^2+\sum_{j=1}^{m}|a_{ij}|F_j|u_i|.|v_j|$
$+\sum_{j=1}^{m}|c_{ij}|F_j|u_i|.|v_j|\}+\sum_{j=1}^{m}\{-d_j^*\gamma_jv_j^2$
$+\sum_{i=1}^{n}|b_{ji}|G_i|u_i|.|v_j|+\sum_{i=1}^{n}|h_{ji}|G_i|u_i|.|v_j|\}$
$\leq\sum_{i=1}^{n}\{-a_i^*\beta_iu_i^2+\frac{1}{2}\sum_{j=1}^{m}[|a_{ij}|+|c_{ij}|]F_j(u_i^2+v_j^2)\}$
$+\sum_{j=1}^{m}\{-d_j^*\gamma_jv_j^2+\frac{1}{2}\sum_{i=1}^{n}[|b_{ji}|+|h_{ji}|]G_i(u_i^2+v_j^2)\}$
$=\sum_{i=1}^{n}\{-a_i^*\beta_i+\frac{1}{2}\sum_{j=1}^{m}[|a_{ij}|+|c_{ij}|]F_j$
$+\frac{1}{2}\sum_{j=1}^{m}[|b_{ji}|+|h_{ji}|]G_i\}u_i^2+\sum_{j=1}^{m}\{-d_j^*\gamma_j$
$+\frac{1}{2}\sum_{i=1}^{n}[|a_{ij}|+|c_{ij}|]F_j+\frac{1}{2}\sum_{i=1}^{n}[|b_{ji}|+|h_{ji}|]G_i\}v_j^2$
$\leq-\min_{1\leq i\leq n}\{a_i^*\beta_i-\frac{1}{2}\sum_{j=1}^{m}[|a_{ij}|+|c_{ij}]F_j-\frac{1}{2}\sum_{j=1}^{m}[|b_{ji}|$
$+|h_{ji}|]G_i\}\|u\|^2-\min_{1\leq j\leq m}\{d_j^*\gamma_j-\frac{1}{2}\sum_{i=1}^{n}[|a_{ij}|+|c_{ij}|]F_j$

$$-\frac{1}{2}\sum_{i=1}^{n}[|b_{ji}|+|h_{ji}|]G_i\}\|v\|^2. \quad (17)$$

Using the Schwartz inequality

$$-X^TY\leq|X^TY|\leq\|X\|\cdot\|Y\|, \quad (18)$$

where $\|X\|, \|Y\|$ are the norms of vectors X and Y, respectively. From (17), we get
$\|(u,v)\|\cdot\|\tilde{H}(u,v)\|\geq\min_{1\leq i\leq n}\{a_i^*\beta_i-\frac{1}{2}\sum_{j=1}^{m}[|a_{ij}|+|c_{ij}]F_j-\frac{1}{2}\sum_{j=1}^{m}[|b_{ji}|+|h_{ji}|]G_i\}\|u\|^2+\min_{1\leq j\leq m}\{d_j^*\gamma_j-\frac{1}{2}\sum_{i=1}^{n}[|a_{ij}|+|c_{ij}|]F_j-\frac{1}{2}\sum_{i=1}^{n}[|b_{ji}|+|h_{ji}|]G_i\}\|v\|^2$
$\geq M(\|u\|^2+\|v\|^2)=M\|(u,v)\|^2,$
where
$M=\min\{\min_{1\leq i\leq n}\{a_i^*\beta_i-\frac{1}{2}\sum_{j=1}^{m}[|a_{ij}|+|c_{ij}|]F_j-\frac{1}{2}\sum_{j=1}^{m}[|b_{ji}|+|h_{ji}|]G_i\},\min_{1\leq j\leq m}\{d_j^*\gamma_j-\frac{1}{2}\sum_{i=1}^{n}[|a_{ij}|+|c_{ij}|]F_j-\frac{1}{2}\sum_{i=1}^{n}[|b_{ji}|+|h_{ji}|]G_i\}\}.$

When $\|(u, v)\| \neq 0$, we have $\|\tilde{H}(u, v)\| \geq M\|(u, v)\|$. Therefore $\|\tilde{H}(u, v)\| \to +\infty$ as $\|(u, v)\| \to +\infty$, which implies that $\|H(u, v)\| \to +\infty$ as $\|(u, v)\| \to +\infty$. From Lemma 4, we know that $H(u, v)$ is a homeomorphism on R^{n+m}. Thus , system (1) has a unique equilibrium point. This completes the proof. □

Theorem 8 *Under hypotheses $(H1)-(H3)$, then the unique equilibrium point of system (1) is globally exponentially stable if (5) and (6) in Theorem 7 hold.*

Proof. By using Theorem 7, system (1) has a unique equilibrium point. In the following we will prove the unique equilibrium point $(u^*, v^*) = (u_1^*, u_2^*, \cdots, u_n^*, v_1^*, v_2^*, \cdots, v_m^*)^T$ is globally exponentially stable.

Let

$$\begin{aligned}
&y_i(t,x) = u_i(t,x) - u_i^*,\\
&z_j(t,x) = v_j(t,x) - v_j^*,\\
&\bar{a}_i(y_i(t,x)) = a_i(y_i(t,x) + u_i^*),\\
&\bar{d}_j(z_j(t,x)) = d_j(z_j(t,x) + v_j^*),\\
&\bar{b}_i(y_i(t,x)) = b_i(y_i(t,x) + u_i^*) - b_i(u_i^*),\\
&\bar{e}_j(z_j(t,x)) = e_j(z_j(t,x) + v_j^*) - e_j(v_j^*),\\
&\bar{f}_j(z_j(t,x)) = f_j(z_j(t,x) + v_j^*) - f_j(v_j^*),\\
&\bar{g}_i(y_i(t,x)) = g_i(y_i(t,x) + u_i^*) - g_i(u_i^*),
\end{aligned}$$

for $i = 1, 2, \ldots, n, j = 1, 2, \ldots, m.$
From (1),(4), we derive

$$\frac{\partial y_i(t,x)}{\partial t} = \sum_{k=1}^{l} \frac{\partial}{\partial x_k}(D_{ik}\frac{\partial y_i(t,x)}{\partial x_k})$$
$$- \bar{a}_i(y_i(t,x))[\bar{b}_i(y_i(t,x)) - \sum_{j=1}^{m} a_{ij}\bar{f}_j(z_j(t,x))$$
$$- \sum_{j=1}^{m} c_{ij}\bar{f}_j(z_j(t-\tau_{ij},x))], \quad x \in \Omega_i \tag{19}$$

$$\frac{\partial z_j(t,x)}{\partial t} = \sum_{k=1}^{l} \frac{\partial}{\partial x_k}(E_{jk}\frac{\partial z_j(t,x)}{\partial x_k})$$
$$- \bar{d}_j(z_j(t,x))[\bar{e}_j(z_j(t,x)) - \sum_{i=1}^{n} b_{ij}\bar{g}_i(y_i(t,x))$$
$$- \sum_{i=1}^{n} h_{ji}\bar{g}_i(y_i(t-\sigma_{ji},x))], \quad x \in \Omega_i, \tag{20}$$

for $i = 1, 2, \cdots, n, j = 1, 2, \cdots, m.$
Multiply both sides of (19) by $y_i(t,x)$ and integrate with respect to x, we get

$$\frac{1}{2}\frac{d}{dt}\int_{\Omega_i} y_i(t,x)^2 dx = \sum_{k=1}^{l}\int_{\Omega_i} y_i \frac{\partial}{\partial x_k}(D_{ik}\frac{\partial y_i(t,x)}{\partial x_k})dx$$
$$- \int_{\Omega_i} \bar{a}_i(y_i(t,x))[y_i\bar{b}_i(y_i(t,x)) - \sum_{j=1}^{m} a_{ij}y_i\bar{f}_j(z_j(t,x)$$
$$- \sum_{j=1}^{m} c_{ij}y_i\bar{f}_j(z_j(t-\tau_{ij},x))]dx, \tag{21}$$

for $i = 1, 2, \cdots, n, j = 1, 2, \cdots, m.$
It follows from the boundary condition that

$$\sum_{k=1}^{l}\int_{\Omega_i} y_i \frac{\partial}{\partial x_k}(D_{ik}\frac{\partial y_i(t,x)}{\partial x_k})dx$$
$$= \int_{\Omega_i} y_i \nabla(D_{ik}\frac{\partial y_i(t,x)}{\partial x_k})_{k=1}^{l} dx$$
$$= \int_{\Omega_i} \nabla.y_i(D_{ik}\frac{\partial y_i(t,x)}{\partial x_k})_{k=1}^{l} dx$$
$$- \int_{\Omega_i} (D_{ik}\frac{\partial y_i(t,x)}{\partial x_k})_{k=1}^{l}.\nabla y_i dx$$
$$= \int_{\partial\Omega_i} (y_i D_{ik}\frac{\partial y_i(t,x)}{\partial x_k})_{k=1}^{l} ds$$
$$- \sum_{k=1}^{l}\int_{\Omega_i} D_{ik}(\frac{\partial y_i(t,x)}{\partial x_k})^2 dx$$
$$= - \sum_{k=1}^{l}\int_{\Omega_i} D_{ik}(\frac{\partial y_i(t,x)}{\partial x_k})^2 dx,$$

where $(D_{ik}\frac{\partial y_i(t,x)}{\partial x_k})_{k=1}^{l}$
$= (D_{i1}\frac{\partial y_i(t,x)}{\partial x_1}, D_{i2}\frac{\partial y_i(t,x)}{\partial x_2}, \ldots, D_{il}\frac{\partial y_i(t,x)}{\partial x_l})^T.$
By (5),(6) and Hölder inequality, from (21) we obtain

$$\frac{d\|y_i\|_2^2}{dt} \le -2\sum_{k=1}^{l}\int_{\Omega_i} D_{ik}(\frac{\partial y_i(t,x)}{\partial x_k})^2 dx - 2\underline{a}_i\beta_i\|y_i\|_2^2$$
$$+ 2\bar{a}_i\sum_{j=1}^{m}|a_{ij}|\int_{\Omega_i}|y_i|.|\bar{f}_j(z_j(t,x))|dx$$
$$+ 2\bar{a}_i\sum_{j=1}^{m}|c_{ij}|\int_{\Omega_i}|y_i|.|\bar{f}_j(z_j(t-\tau_{ij},x))|dx$$
$$\le -2\underline{a}_i\|y_i(t)\|_2^2 + 2\bar{a}_i\sum_{j=1}^{m}|a_{ij}|F_j\|y_i(t)\|_2.\|z_j(t)\|_2$$
$$+ 2\bar{a}_i\sum_{j=1}^{m}|c_{ij}|F_j\|y_i(t)\|_2.\|z_j(t-\tau_{ij})\|_2, \text{ that is}$$
$$\frac{d\|y_i(t)\|_2}{dt} \le \bar{a}_i[-a_i^*\beta_i\|y_i(t)\|_2$$
$$+ \sum_{j=1}^{m}|a_{ij}|F_j\|z_j(t)\|_2 + \sum_{j=1}^{m}|c_{ij}|F_j\|z_j(t-\tau_{ij})\|_2], \tag{22}$$

for $i = 1, 2, \cdots, n.$
Multiply both sides of (20) by $z_j(t,x)$, similarly, we also get

$$\frac{d\|z_j(t)\|_2}{dt} \le \bar{d}_j[-d_j^*\gamma_j\|z_j(t)\|_2$$
$$+ \sum_{i=1}^{n}|b_{ji}|G_i\|y_i(t)\|_2 + \sum_{i=1}^{n}|h_{ji}|G_i\|y_i(t-\sigma_{ji})\|_2], \tag{23}$$

for $j = 1, 2, \cdots, m.$
We consider the following Lyapunov functional

$$V(t) = \sum_{i=1}^{n}\frac{1}{\bar{a}_i}e^{\alpha t}\|y_i\|_2 + \sum_{j=1}^{m}\frac{1}{\bar{d}_j}e^{\alpha t}\|z_j\|_2$$
$$+ \sum_{i=1}^{n}\sum_{j=1}^{m}|c_{ij}|F_j\int_{t-\tau_{ij}}^{t} e^{\alpha(s+\tau_{ij})}\|z_j(s)\|_2 ds$$
$$+ \sum_{j=1}^{m}\sum_{i=1}^{n}|h_{ji}|G_i\int_{t-\sigma_{ji}}^{t} e^{\alpha(s+\sigma_{ji})}\|y_i(s)\|_2 ds, \tag{24}$$

where α is given by Lemma 5.

Calculate the rate of change of $V(t)$ along (19)-(20), we derive

$$D^+V(t) \le e^{\alpha t}\sum_{i=1}^{n}\{\tfrac{\alpha}{\bar{a}_i}\|y_i(t)\|_2 - a_i^*\beta_i\|y_i(t)\|_2$$
$$+\sum_{j=1}^{m}|a_{ij}|F_j\|z_j(t)\|_2 + \sum_{j=1}^{m}|c_{ij}|F_j e^{\alpha\tau_{ij}}\|z_j(t)\|_2\}$$
$$+e^{\alpha t}\sum_{j=1}^{m}\{\tfrac{\alpha}{\bar{d}_j}\|z_j(t)|_2 - d_j^*\gamma_j\|z_j(t)\|_2$$
$$+\sum_{i=1}^{n}|b_{ji}|G_i\|y_i(t)\|_2 + \sum_{i=1}^{n}|h_{ji}|.G_i e^{\alpha\sigma_{ji}}\|y_i(t)\|_2\}$$
$$\le e^{\alpha t}\sum_{i=1}^{n}\{\tfrac{\alpha}{\bar{a}_i} - a_i^*\beta_i + \sum_{j=1}^{m}|b_{ji}|G_i$$
$$+e^{\alpha\sigma}\sum_{j=1}^{m}|h_{ji}|G_i\}\|y_i\|_2 + e^{\alpha t}\sum_{j=1}^{m}\{\tfrac{\alpha}{\bar{d}_j}$$
$$-d_j^*\gamma_j + \sum_{i=1}^{n}|a_{ij}|F_j + e^{\alpha\tau}\sum_{i=1}^{n}|c_{ij}|F_j\}\|z_j|_2. \quad (25)$$

By (5)-(6) and Lemma 5, we have

$$\tfrac{\alpha}{\bar{a}_i} - a_i^*\beta_i + \sum_{j=1}^{m}|b_{ji}|G_i + e^{\alpha\sigma}\sum_{j=1}^{m}|h_{ji}|G_i \le 0,$$
$$\tfrac{\alpha}{\bar{d}_j} - d_j^*\gamma_j + \sum_{i=1}^{n}|a_{ij}|F_j + e^{\alpha\tau}\sum_{i=1}^{n}|c_{ij}|F_j \le 0,$$

for $i = 1, 2, \ldots .n, j = 1, 2, \ldots .m.$
From (25), we can find $D^+V(t) \le 0$, and so $V(t) \le V(0)$, for all $t \ge 0$. From (24), we have

$$V(t) \ge \sum_{i=1}^{n}\frac{1}{\bar{a}_i}e^{\alpha t}\|y_i\|_2 + \sum_{j=1}^{m}\frac{1}{\bar{d}_j}e^{\alpha t}\|z_j\|_2, \quad t \ge 0. \quad (26)$$

$$V(0) = \sum_{i=1}^{n}\tfrac{1}{\bar{a}_i}\|y_i(0)\|_2 + \sum_{j=1}^{m}\tfrac{1}{\bar{d}_j}\|z_j(0)\|_2$$
$$+\sum_{i=1}^{n}\sum_{j=1}^{m}|c_{ij}|F_j\int_{-\tau_{ij}}^{0}e^{\alpha(s+\tau_{ij})}\|z_j(s)\|_2 ds$$
$$+\sum_{j=1}^{m}\sum_{i=1}^{n}|h_{ji}|G_i\int_{-\sigma_{ji}}^{0}e^{\alpha(s+\sigma_{ji})}\|y_i(s)\|_2 ds,$$
$$\le \sum_{i=1}^{n}\tfrac{1}{\bar{a}_i}\|\phi_{u_i}(0,x) - u_i^*\|_2$$
$$+\sum_{i=1}^{n}\int_{-\tau}^{0}e^{\alpha(s+\tau)}\sum_{j=1}^{m}|c_{ij}|F_j\|\phi_{v_j}(s,x) - v_j^*\|_2 ds$$
$$+\sum_{j=1}^{m}\tfrac{1}{\bar{d}_j}\|\phi_{v_j}(0,x) - v_j^*\|_2$$
$$+\sum_{j=1}^{m}\int_{-\sigma}^{0}e^{\alpha(s+\sigma)}\sum_{i=1}^{n}|h_{ji}|G_i\|\phi_{u_i}(s,x) - u_i^*\|_2 ds,$$
$$\le \sup_{-\sigma\le s\le 0}\sum_{i=1}^{n}[\tfrac{1}{\bar{a}_i} + \sum_{j=1}^{m}(\max_{1\le i\le n}|h_{ji}|G_i)$$
$$\cdot\int_{-\sigma}^{0}e^{\alpha(s+\sigma)}ds]\|\phi_{u_i}(s,x) - u_i^*\|_2$$
$$+\sup_{-\tau\le s\le 0}\sum_{j=1}^{m}[\tfrac{1}{\bar{d}_j} + \sum_{i=1}^{n}(\max_{1\le j\le m}|c_{ij}|F_j)$$
$$\cdot\int_{-\tau}^{0}e^{\alpha(s+\tau)}ds]\|\phi_{v_j}(s,x) - v_j^*\|_2$$
$$\le \sup_{-\sigma\le s\le 0}\sum_{i=1}^{n}[\tfrac{1}{\bar{a}_i} + \sum_{j=1}^{m}(\max_{1\le i\le n}|h_{ji}|G_i)\tfrac{e^{\alpha\sigma}}{\alpha}]$$
$$\cdot\|\phi_{u_i}(s,x) - u_i^*\|_2$$
$$+\sup_{-\tau\le s\le 0}\sum_{j=1}^{m}[\tfrac{1}{\bar{d}_j} + \sum_{i=1}^{n}(\max_{1\le j\le m}|c_{ij}|F_j)\tfrac{e^{\alpha\tau}}{\alpha}]$$
$$\cdot\|\phi_{v_j}(s,x) - v_j^*\|_2$$
$$\le \bar{M}[\sup_{-\sigma\le s\le 0}\sum_{i=1}^{n}\|\phi_{u_i}(s,x) - u_i^*\|_2$$
$$+\sup_{-\tau\le s\le 0}\sum_{j=1}^{m}\|\phi_{v_j}(s,x) - v_j^*\|_2]$$
$$= \bar{M}[\|\phi_u - u^*\|_2 + \|\phi_v - v^*\|_2], \quad t > 0, \quad (27)$$

where

$$\bar{M} = \max\{\max_{1\le i\le n}(\tfrac{1}{\bar{a}_i}) + \sum_{j=1}^{m}(\max_{1\le i\le n}|h_{ji}|G_i)\tfrac{e^{\alpha\sigma}}{\alpha}, \max_{1\le j\le m}(\tfrac{1}{\bar{d}_j}) + \sum_{i=1}^{n}(\max_{1\le j\le m}|c_{ij}|F_j)\tfrac{e^{\alpha\tau}}{\alpha}\}.$$

Since $V(0) \ge V(t)$, we can obtain

$$\sum_{i=1}^{n}\tfrac{1}{\bar{a}_i}\|y_i\|_2 + \sum_{j=1}^{m}\tfrac{1}{\bar{d}_j}\|z_j\|_2 \le \bar{M}e^{-\alpha t}[\|\phi_u - u^*\|_2 + \|\phi_v - v^*\|_2], \quad t > 0,$$

i.e.,

$$\sum_{i=1}^{n}\|u_i - u_i^*\|_2 + \sum_{j=1}^{m}\|v_j - v_j^*\|_2 \le Me^{-\alpha t}[\|\phi_u - u^*\|_2 + \|\phi_v - v^*\|_2], \quad t > 0,$$

where

$$M = p\bar{M} > 1, p = \max\{\max_{1\le i\le n}(\bar{a}_i), \max_{1\le j\le m}(\bar{d}_j)\}$$

By Definition 3, the equilibrium point (u^*, v^*) of system (1) is globally exponentially stable. □

Theorem 9 *Under hypotheses $(H1) - (H3)$, then the unique equilibrium point of system (1) is globally exponentially stable if*

$$-\underline{a}_i\beta_i + \tfrac{1}{2}\bar{a}_i\sum_{j=1}^{m}[|a_{ij}| + |c_{ij}|]F_j + \sum_{j=1}^{m}[|b_{ji}| + |h_{ji}|]G_i(\bar{a}_i + \bar{d}_j) < 0, \quad (28)$$

$$-\underline{d}_j\gamma_j + \sum_{i=1}^{n}[|a_{ij}| + |c_{ij}|]F_j(\bar{d}_j + \bar{a}_i) + \tfrac{1}{2}\bar{d}_j\sum_{i=1}^{n}[|b_{ji}| + |h_{ji}|]G_i < 0, \quad (29)$$

for $i = 1, 2, \ldots .n, j = 1, 2, \ldots .m.$

Proof. From (28)-(29), by simple calculations, we can find that the conditions (5)-(6) of theorem 7 are satisfied. By using Theorem 7, system (1) has a unique equilibrium point. In the following we will prove the unique equilibrium point $(u^*, v^*) = (u_1^*, u_2^*, \cdots, u_n^*, v_1^*, v_2^*, \cdots, v_m^*)^T$ is globally exponentially stable.
From (22)-(23), we can obtain

$$\|y_i(t)\|_2 \le \bar{a}_i\{\tfrac{1}{\bar{a}_i}e^{-\underline{a}_i\beta_i t}\|y_i(0)\|_2 + \sum_{j=1}^m F_j \cdot \int_0^t e^{\underline{a}_i\beta_i(s-t)}[|a_{ij}\|z_j(s)\|_2 + |c_{ij}|\|z_j(s-\tau_{ij})\|_2]ds\}, \tag{30}$$

for $i = 1, 2, \cdots, n$, and

$$\|z_j(t)\|_2 \le \bar{d}_j\{\tfrac{1}{\bar{d}_j}e^{-\underline{d}_j\gamma_j t}\|z_j(0)\|_2 + \sum_{i=1}^n G_i \cdot \int_0^t e^{\underline{d}_j\gamma_j(s-t)}[|b_{ji}|\|y_i(s)\|_2 + |h_{ji}|\|y_i(s-\sigma_{ji})\|_2]ds\}, \tag{31}$$

for $j = 1, 2, \cdots, m$. Let

$$\begin{aligned} Q(t) &= e^{\alpha t}\sum_{i=1}^n \bar{a}_i\{\tfrac{1}{\bar{a}_i}e^{-\underline{a}_i\beta_i t}\|y_i(0)\|_2 \\ &+ \sum_{j=1}^m F_j\int_0^t e^{\underline{a}_i\beta_i(s-t)}[|a_{ij}\|z_j(s)\|_2 \\ &+ |c_{ij}|\|z_j(s-\tau_{ij})\|_2]ds\} \\ &+ e^{\alpha t}\sum_{j=1}^m \bar{d}_j\{\tfrac{1}{\bar{d}_j}e^{-\underline{d}_j\gamma_j t}\|z_j(0)\|_2 \\ &+ \sum_{i=1}^n G_i\int_0^t e^{\underline{d}_j\gamma_j(s-t)}[|b_{ji}|\|y_i(s)\|_2 \\ &+ |h_{ji}|\|y_i(s-\sigma_{ji})\|_2]ds\}, \end{aligned} \tag{32}$$

where α is given by Lemma 6.

From (30)-(31), we have

$$e^{\alpha t}[\sum_{i=1}^n \|y_i(t)\|_2 + \sum_{j=1}^m \|z_j(t)\|_2] \le Q(t). \tag{33}$$

By calculation, we have

$$\begin{aligned} D^+Q(t) &\le e^{\alpha t}\sum_{i=1}^n[\alpha - \underline{a}_i\beta_i + \sum_{j=1}^m \bar{d}_j G_i|b_{ji}|]\|y_i(t))\|_2 \\ &+ e^{\alpha t}\sum_{j=1}^m[\alpha - \underline{d}_j\gamma_j + \sum_{i=1}^n \bar{a}_i F_j|a_{ij}|]\|z_j(t))\|_2 \\ &+ e^{\alpha t}\sum_{i=1}^n \bar{a}_i\sum_{j=1}^m F_j|c_{ij}|\|z_j(t-\tau_{ij}))\|_2 \\ &+ e^{\alpha t}\sum_{j=1}^m \bar{d}_j\sum_{i=1}^n G_i|h_{ji}|\|y_i(t-\sigma_{ji}))\|_2. \end{aligned} \tag{34}$$

We consider the following Lyapunov functional

$$\begin{aligned} V(t) &= Q(t) \\ &+ \sum_{i=1}^n \bar{a}_i\sum_{j=1}^m |c_{ij}|F_j\int_{t-\tau_{ij}}^t e^{\alpha(s+\tau_{ij})}\|z_j(s)\|_2 ds \\ &+ \sum_{j=1}^m \bar{d}_j\sum_{i=1}^n |h_{ji}|G_i\int_{t-\sigma_{ji}}^t e^{\alpha(s+\sigma_{ji})}\|y_i(s)\|_2 ds, \end{aligned} \tag{35}$$

Calculate the rate of change of $V(t)$ we derive

$$\begin{aligned} D^+V(t) &= D^+Q(t) \\ &+ e^{\alpha t}\sum_{i=1}^n \bar{a}_i\sum_{j=1}^m |c_{ij}|F_j[e^{\alpha\tau_{ij}}\|z_j(t)\|_2 - \|z_j(t-\tau_{ij})\|_2] \\ &+ e^{\alpha t}\sum_{j=1}^m \bar{d}_j\sum_{i=1}^n |h_{ji}|G_i[e^{\alpha\sigma_{ji}}\|y_i(t)\|_2 - \|y_i(t-\sigma_{ji})\|_2] \\ &\le e^{\alpha t}\sum_{i=1}^n[\alpha - \underline{a}_i\beta_i + \sum_{j=1}^m \bar{d}_j G_i|b_{ji}|]\|y_i(t)\|_2 \\ &+ e^{\alpha t}\sum_{j=1}^m[\alpha - \underline{d}_j\gamma_j + \sum_{i=1}^n \bar{a}_i F_j|a_{ij}|]\|z_j(t)\|_2 \\ &+ e^{\alpha t}\sum_{i=1}^n \bar{a}_i\sum_{j=1}^m F_j|c_{ij}|\|z_j(t-\tau_{ij})\|_2 \\ &+ e^{\alpha t}\sum_{j=1}^m \bar{d}_j\sum_{i=1}^n G_i|h_{ji}|\|y_i(t-\sigma_{ji})\|_2. \\ &+ e^{\alpha t}\sum_{i=1}^n \bar{a}_i\sum_{j=1}^m |c_{ij}|F_j[e^{\alpha\tau}\|z_j(t)\|_2 - \|z_j(t-\tau_{ij})\|_2] \\ &+ e^{\alpha t}\sum_{j=1}^m \bar{d}_j\sum_{i=1}^n |h_{ji}|G_i[e^{\alpha\sigma}\|y_i(t)\|_2 - \|y_i(t-\sigma_{ji})\|_2] \\ &= e^{\alpha t}\sum_{i=1}^n[\alpha - \underline{a}_i\beta_i + \sum_{j=1}^m \bar{d}_j|b_{ji}|G_i \\ &+ e^{\alpha\sigma}\sum_{j=1}^m \bar{d}_j|h_{ji}|G_i\}\|y_i\|_2 \\ &+ e^{\alpha t}\sum_{j=1}^m[\alpha - \underline{d}_j\gamma_j \\ &+ \sum_{i=1}^n \bar{a}_i|a_{ij}|F_j + e^{\alpha\tau}\sum_{i=1}^n \bar{a}_i|c_{ij}|F_j\}\|z_j|_2. \end{aligned} \tag{36}$$

By (28)-(29) and Lemma 6, we have

$$\alpha - \underline{a}_i\beta_i + \sum_{j=1}^m \bar{d}_j|b_{ji}|G_i + e^{\alpha\sigma}\sum_{j=1}^m \bar{d}_j|h_{ji}|G_i \le 0,$$

$$\alpha - \underline{d}_j\gamma_j + \sum_{i=1}^n \bar{a}_i|a_{ij}|F_j + e^{\alpha\tau}\sum_{i=1}^n \bar{a}_i|c_{ij}|F_j \le 0,$$

for $i = 1, 2, \ldots . n, j = 1, 2, \ldots . m$.

From (36), we can find $D^+V(t) \le 0$, and so $V(t) \le V(0)$, for all $t \ge 0$. From (35) and (32), we have

$$
\begin{aligned}
&V(0) = Q(0)\\
&+\sum_{i=1}^{n}\bar{a}_i\sum_{j=1}^{m}|c_{ij}|F_j\int_{-\tau_{ij}}^{0}e^{\alpha(s+\tau_{ij})}\|z_j(s)\|_2ds\\
&+\sum_{j=1}^{m}\bar{d}_j\sum_{i=1}^{n}|h_{ji}|G_i\int_{-\sigma_{ji}}^{0}e^{\alpha(s+\sigma_{ji})}\|y_i(s)\|_2ds\\
&\leq\sum_{i=1}^{n}\|y_i(0)\|_2+\sum_{j=1}^{m}\|z_j(0)\|_2\\
&+\sum_{i=1}^{n}\bar{a}_i\sum_{j=1}^{m}|c_{ij}|F_j\int_{-\tau}^{0}e^{\alpha(s+\tau)}\|z_j(s)\|_2ds\\
&+\sum_{j=1}^{m}\bar{d}_j\sum_{i=1}^{n}|h_{ji}|G_i\int_{-\sigma}^{0}e^{\alpha(s+\sigma)}\|y_i(s)\|_2ds\\
&\leq\sum_{i=1}^{n}\|\phi_{u_i}(0,x)-u_i^*\|_2+\sum_{j=1}^{m}\|\phi_{v_j}(0,x)-v_j^*\|_2\\
&+\sum_{i=1}^{n}\bar{a}_i\max_{1\leq j\leq m}(F_j|c_{ij}|)\int_{-\tau}^{0}e^{\alpha(s+\tau)}\sum_{j=1}^{m}\|z_j(s)\|_2ds\\
&+\sum_{j=1}^{m}\bar{d}_j\max_{1\leq i\leq n}(G_i|h_{ji}|)\int_{-\sigma}^{0}e^{\alpha(s+\sigma)}\sum_{i=1}^{n}\|y_i(s)\|_2ds\\
&\leq\sup_{-\sigma\leq s\leq 0}\sum_{i=1}^{n}[1+\tfrac{e^{\alpha\sigma}}{\alpha}\sum_{j=1}^{m}\max_{1\leq i\leq n}(G_i|h_{ji}|)]\\
&\cdot\|\phi_{u_i}(s,x)-u_i^*\|_2\\
&+\sup_{-\tau\leq s\leq 0}\sum_{i=j}^{m}[1+\tfrac{e^{\alpha\tau}}{\alpha}\sum_{i=1}^{n}\max_{1\leq j\leq m}(F_j|c_{ij}|)]\\
&\cdot\|\phi_{v_j}(s,x)-v_j^*\|_2\\
&=[1+\tfrac{e^{\alpha\sigma}}{\alpha}\sum_{j=1}^{m}\max_{1\leq i\leq n}(G_i|h_{ji}|)]\|\phi_u-u^*\|_2\\
&+[1+\tfrac{e^{\alpha\tau}}{\alpha}\sum_{i=1}^{n}\max_{1\leq j\leq m}(F_j|c_{ij}|)]\|\phi_v-v^*\|_2\\
&=\bar{M}[\|\phi_u-u^*\|_2+\|\phi_v-v^*\|_2],\quad t>0,
\end{aligned}\tag{37}
$$

where $\bar{M} = \max\{1+\frac{e^{\alpha\sigma}}{\alpha}\sum_{j=1}^{m}\max_{1\leq i\leq n}(G_i|h_{ji}|), 1+\frac{e^{\alpha\tau}}{\alpha}\sum_{i=1}^{n}\max_{1\leq j\leq m}(F_j|c_{ij}|)\}>1$.

Since $Q(t)\leq V(t)\leq V(0)$, from (33) and (37), we have

$$
\begin{aligned}
&\sum_{i=1}^{n}\|y_i\|_2+\sum_{j=1}^{m}\|z_j\|_2\\
&\leq\bar{M}e^{-\alpha t}[\|\phi_u-u^*\|_2+\|\phi_v-v^*\|_2],\quad t>0,
\end{aligned}
$$

i.e.,

$$
\begin{aligned}
&\sum_{i=1}^{n}\|u_i-u_i^*\|_2+\sum_{j=1}^{m}\|v_j-v_j^*\|_2\\
&\leq\bar{M}e^{-\alpha t}[\|\phi_u-u^*\|_2+\|\phi_v-v^*\|_2],\quad t>0.
\end{aligned}
$$

By Definition 3, the equilibrium point (u^*, v^*) of system (1) is globally exponentially stable. □

Remark 1. When $a_i(u_i(t,x)) = d_j(v_j(t,x)) \equiv 1$, $a_{ij} = b_{ji} \equiv 0$, $b_i(u_i(t,x)) = b_i\cdot u_i(t,x)$, $e_j(v_j(t,x)) = e_j\cdot v_j(t,x)$ in (1), where b_i and e_j are positive constants, $i = 1,2,\cdots n, j = 1,2,\cdots,m$, then system (1) is the system (1) in [34]. From this point, our model is more general. Moreover, by comparing our paper with reference[34], we find that the construction of Lyapunov functional and analysis techniques are different. Therefore, the given algebra criteria guaranteeing the globally exponential stability of the equilibrium point are different in this paper and paper [34], and they will bring different advantages for those who design and verify these neural networks.

Remark 2. Theorem 7-8 and Theorem 9 are developed under different assumptions and use of various lemmas. They provide different sufficient conditions ensuring the equilibrium point of system (1) to be unique and exponentially stable. Therefore, we can select suitable theorems for reaction-diffusion Cohen-Grossberg-type BAM neural networks with time delays to determine its exponential stability.

4 Examples

This Section, we give two examples for showing our results.

Example 1. Consider the following reaction-diffusion Cohen-Grossberg-type BAM neural networks $(n = m = l = 2)$

$$
\left\{
\begin{aligned}
&\frac{\partial u_i(t,x)}{\partial t}=\sum_{k=1}^{2}\frac{\partial}{\partial x_k}(D_{ik}\frac{\partial u_i(t,x)}{\partial x_k})\\
&-a_i(u_i(t,x))[b_i(u_i(t,x))-\sum_{j=1}^{2}a_{ij}f_j(v_j(t,x))\\
&-\sum_{j=1}^{2}c_{ij}f_j(v_j(t-\tau_{ij},x))-I_i],\\
&\frac{\partial v_j(t,x)}{\partial t}=\sum_{k=1}^{2}\frac{\partial}{\partial x_k}(E_{jk}\frac{\partial v_j(t,x)}{\partial x_k})\\
&-d_j(v_j(t,x))[e_j(v_j(t,x))-\sum_{i=1}^{2}b_{ji}g_i(u_i(t,x))\\
&-\sum_{i=1}^{2}h_{ji}g_i(u_i(t-\sigma_{ji},x))-J_j],
\end{aligned}
\right.\tag{38}
$$

for $i = 1,2, j = 1,2$, where $f_i(r) = 2\sin r$, $g_i(r) = \cos 2r$, $a_i(r) = 2+\sin r$, $d_i(r) = 2+\cos r$, $b_i(r) = 12r$, $e_i(r) = 18r$, $i = 1,2$.

Since $r_1, r_2\in R$,

$$
\begin{aligned}
&|f_i(r_1)-f_i(r_2)|\leq 2|r_1-r_2|,\\
&|g_i(r_1)-g_i(r_2)|\leq 2|r_1-r_2|,\\
&1\leq a_i(r)\leq 3,\quad 1\leq d_i(r)\leq 3,\\
&b_i(r_1)-b_i(r_2)=12(r_1-r_2),\\
&e_i(r_1)-e_i(r_2)=18(r_1-r_2),\quad i=1,2.
\end{aligned}
$$

We select $F_i = G_i = 2$, $\beta_i = 12$,$\gamma_i = 18$, $\bar{a}_i = 3$, $\underline{a}_i = 1$, $\bar{d}_i = 3$, $\underline{d}_i = 1$, $a_i^* = \frac{1}{3}$, $d_i^* = \frac{1}{3}$, $i = 1, 2$.
Let $a_{11} = \frac{1}{4}$, $a_{12} = -\frac{1}{2}$, $a_{21} = \frac{1}{2}$, $a_{22} = -\frac{1}{2}$, $b_{11} = \frac{1}{2}$, $b_{12} = -\frac{1}{2}$, $b_{21} = \frac{1}{4}$, $b_{22} = \frac{1}{8}$, $c_{11} = \frac{1}{4}$, $c_{12} = -\frac{1}{4}$, $c_{21} = \frac{1}{2}$, $c_{22} = \frac{1}{2}$, $h_{11} = -\frac{1}{4}$, $h_{12} = \frac{1}{8}$, $h_{21} = \frac{1}{4}$, $h_{22} = \frac{1}{8}$, $I_1 = 3\pi - 1$, $I_2 = 2\pi - 2$, $J_1 = 9\pi - \frac{3}{16}$, $J_2 = 18\pi - \frac{1}{8}$.
From (38), we get the equation of the equilibriums

$$\begin{cases} 12u_1 - sin(v_1) + \frac{3}{2}sin(v_2) - 3\pi + 1 = 0, \\ 12u_2 - 2sin(v_1) - 2\pi + 2 = 0, \\ 18v_1 - \frac{1}{4}cos(2u_1) + \frac{3}{8}cos(2u_2) - 9\pi + \frac{3}{16} = 0, \\ 18v_2 - \frac{1}{2}cos(2u_1) - \frac{1}{4}cos(2u_2) - 18\pi + \frac{1}{8} = 0. \end{cases} \tag{39}$$

By calculation, we can solve the unique equilibrium point

$$(u_1^*, u_2^*, v_1^*, v_2^*) = (\frac{\pi}{4}, \frac{\pi}{6}, \frac{\pi}{2}, \pi).$$

On the other hand, we have the following results by simple calculation

$$-a_i^*\beta_i+\frac{1}{2}\sum_{j=1}^{2}[|a_{ij}|+\||c_{ij}|]F_j+\sum_{j=1}^{2}[|b_{ji}|+|h_{ji}|]G_i < 0,$$

$$-d_j^*\gamma_j+\sum_{i=1}^{2}[|a_{ij}|+|c_{ij}|]F_j+\frac{1}{2}\sum_{i=1}^{2}[|b_{ij}|+|h_{ji}|]G_i < 0,$$

for $i = 1, 2, j = 1, 2$.

It follows from Theorem 7-8 that this system has one unique equilibrium point, which is globally exponentially stable.

Example 2. For the neural networks described by system (38), let $a_i(r) = 3 + \sin r, d_i(r) = 2 + \cos r, i = 1, 2$, then have $2 \le a_i(r) \le 4, 1 \le d_i(r) \le 3, i = 1, 2$, we select $\underline{a}_i = 2$, $\bar{a}_i = 4$, $\underline{d}_i = 1$, $\bar{d}_i = 3, i = 1, 2$.

Take

$a_{11} = \frac{1}{4}$, $a_{12} = -\frac{1}{2}$, $a_{21} = \frac{1}{8}$, $a_{22} = -\frac{1}{8}$,
$b_{11} = \frac{1}{2}$, $b_{12} = \frac{1}{8}$, $b_{21} = \frac{1}{5}$, $b_{22} = \frac{1}{8}$,
$c_{11} = \frac{1}{4}$, $c_{12} = -\frac{1}{4}$, $c_{21} = \frac{1}{8}$, $c_{22} = \frac{1}{8}$,
$h_{11} = -\frac{1}{2}$, $h_{12} = \frac{1}{8}$, $h_{21} = \frac{1}{5}$, $h_{22} = \frac{1}{8}$,
$I_1 = 6\pi + 1$, $I_2 = 12\pi - \frac{1}{4}$,
$J_1 = 3\pi - \frac{1}{4}$, $J_2 = 9\pi + \frac{3}{20}$.

The other parameters are the same as that in Example 1.

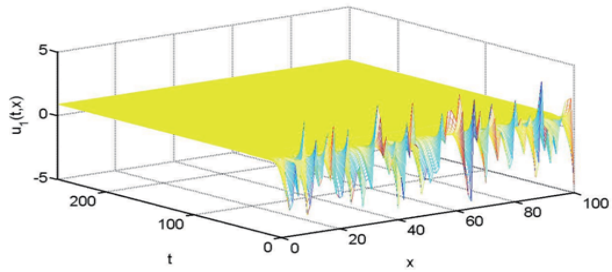

Figure 1: Three-dimensional view of (t, x, u_1) in Example 1

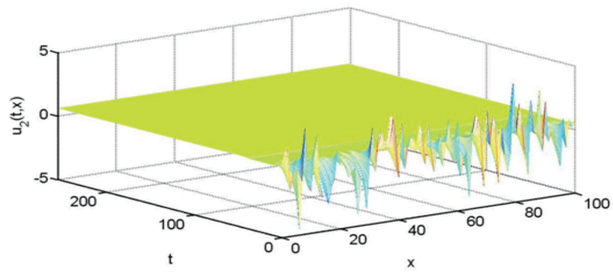

Figure 2: Three-dimensional view of (t, x, u_2) in Example 1

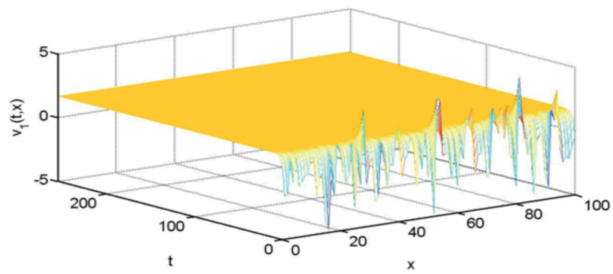

Figure 3: Three-dimensional view of (t, x, v_1) in Example 1

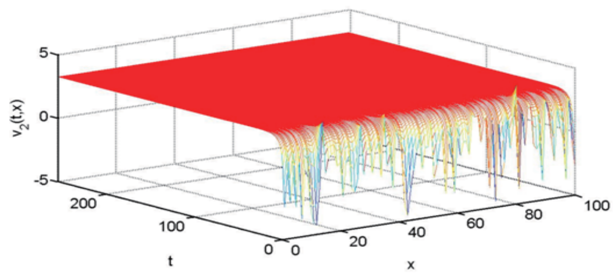

Figure 4: Three-dimensional view of (t, x, v_2) in Example 1

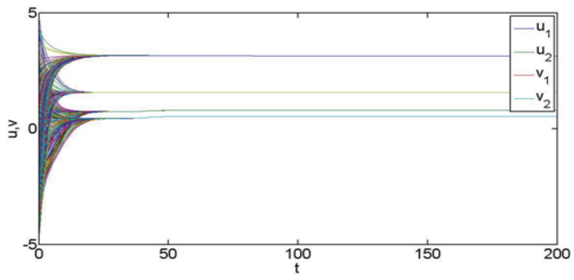

Figure 5: Transient response of state variable $u_i(t,x), v_i(t,x)$ in Example 1

From (38), we get the equation of the equilibriums

$$\begin{cases} 12u_1 - \sin(v_1) + \frac{3}{2}\sin(v_2) - 6\pi - 1 = 0, \\ 12u_2 - \frac{1}{2}\sin(v_1) - 12\pi + \frac{1}{4} = 0, \\ 18v_1 - \frac{1}{4}\cos(2u_2) - 3\pi + \frac{1}{4} = 0, \\ 18v_2 - \frac{2}{5}\cos(2u_1) - \frac{1}{4}\cos(2u_2) - 9\pi - \frac{3}{20} = 0. \end{cases} \tag{40}$$

By calculation, we can solve the unique equilibrium point

$$(u_1^*, u_2^*, v_1^*, v_2^*) = (\frac{\pi}{2}, \pi, \frac{\pi}{6}, \frac{\pi}{2}).$$

On the other hand, we have the following results by simple calculation

$$\begin{aligned} &-\underline{a}_i\beta_i + \tfrac{1}{2}\bar{a}_i\sum_{j=1}^{2}[|a_{ij}| + |c_{ij}|]F_j \\ &+ \sum_{j=1}^{2}[|b_{ji}| + |h_{ji}|]G_i(\bar{a}_i + \bar{d}_j) < 0, \\ &-\underline{d}_j\gamma_j + \sum_{i=1}^{2}[|a_{ij}| + |c_{ij}|]F_j(\bar{d}_j + \bar{a}_i) \\ &+ \tfrac{1}{2}\bar{d}_j\sum_{i=1}^{2}[|b_{ji}| + |b_{ji}|]G_i < 0, \end{aligned}$$

for $i, j = 1, 2$.

It follows from Theorem 9 that this system has one unique equilibrium point, which is globally exponentially stable.

Remark 3. By simple calculations, we can find that the conditions (28)-(29) of Theorem 9 aren't satisfied for the system in Example 1, while for the system in Example 4.2, conditions (5)-(6) of Theorem 8 aren't satisfied. Therefore, Theorem 3.2 are suitable for the exponential stability of the system in Example 1, but Theorem 9 isn't; and Theorem 9 is suitable for the exponential stability of the system in Example 2, but Theorem 8 is not. The above two examples show that all the Theorems 8-9 in this paper have advantages in different problems and applications.

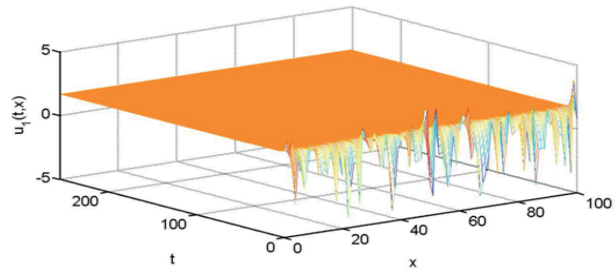

Figure 6: Three-dimensional view of (t, x, u_1) in Example 2

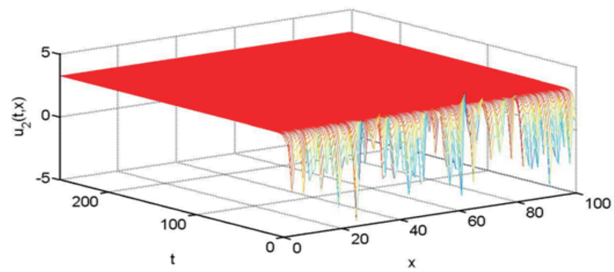

Figure 7: Three-dimensional view of (t, x, u_2) in Example 2

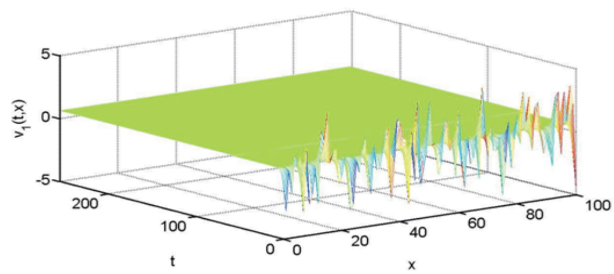

Figure 8: Three-dimensional view of (t, x, v_1) in Example 2

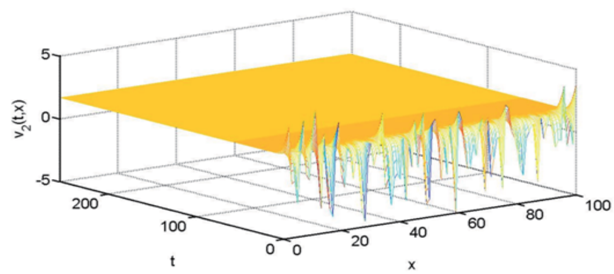

Figure 9: Three-dimensional view of (t, x, v_2) in Example 2

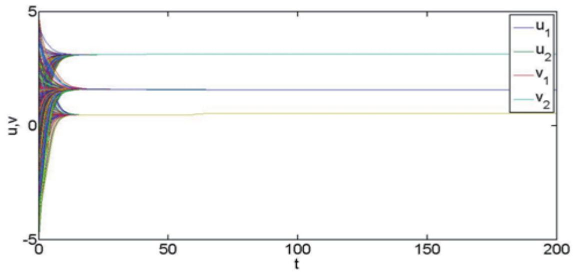

Figure 10: Transient response of state variable $u_i(t,x), v_i(t,x)$ in Example 2

5 Conclusions

Under different assumption conditions, three theorems are given to ensure the existence, uniqueness and the exponential stability of the equilibrium point for a reaction-diffusion Cohen-Grossberg-type BAM neural networks with time delays by constructing a suitable Lyapunov functional, utilizing some analytical techniques. Two examples are given to show the effectiveness of the results. The given algebra conditions are easily verifiable and useful in theories and applications.

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