Positive periodic solutions for an impulsive neutral delay model of single-species population growth on time scales

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Abstract: By using a fixed point theorem of strict-set-contraction, some criteria are established for the existence of positive periodic solutions for an impulsive neutral delay model of single-species population growth on time scales. Finally, an example is given to illustrate the main results.

Key–Words: Positive periodic solution; Neutral delay model; Impulse; Strict-set-contraction; Time scale.

1 Introduction

In 1993, Kuang [1] proposed an open problem (Open problem 9.2) to obtain sufficient conditions for the existence of positive periodic solutions to

$$\begin{aligned} x'(t) = x(t)\big[&a(t) - \beta(t)x(t) - b(t)x(t-\tau(t)) \\ &-c(t)x'(t-\tau(t))\big], \end{aligned} \tag{1}$$

where a, β, b, c, τ are nonnegative continuous periodic functions. Since then, different classes of neutral functional differential equations have been extensively studied, we refer the readers to [1-5] and the references therein.

However, in the natural world, there are many species whose developing processes are both continuous and discrete. Hence, using the only differential equation or difference equation can't accurately describe the law of their developments. Therefore, there is a need to establish correspondent dynamic models on new time scales.

The theory of calculus on time scales (see [6] and references cited therein) was initiated by Stefan Hilger [7] in order to unify continuous and discrete analysis, and it has a tremendous potential for applications and has recently received much attention since his foundational work (see, e.g., 8-14). Therefore, it is practicable to study that on time scales which can unify the continuous and discrete situations.

Motivated by above, the aim of this paper is to establish sufficient conditions for the existence of positive periodic solutions for a neutral delay model of single-species population growth on time scales. However, it is known that many real world phenomena often behave in a piecewise continuous frame interlaced with abrupt changes. Thus, the choice of system accompanied with impulsive conditions is much more appropriate.

Consider the following impulsive neutral delay model of single-species population growth on time scales

$$\begin{cases} x^{\Delta}(t) = x(t)\Big[r(t) - a(t)x(t) \\ \qquad - \sum\limits_{j=1}^{n} a_j(t)x(t-\tau_j(t)) \\ \qquad - \sum\limits_{j=1}^{n} b_j(t)\int_{-T_j}^{0} K_j(s)x(t+s)\Delta s \\ \qquad - \sum\limits_{j=1}^{n} c_j(t)x^{\Delta}(t-\sigma_j(t))\Big], \\ \qquad t \neq t_k,\, t\in\mathbb{T}, \\ x(t_k^+) = x(t_k^-) + I_k(x(t_k)), k = 1,2,\cdots, \end{cases} \tag{2}$$

where $\mathbb{T}$ is an ω-periodic time scale, and for each interval $\mathbb{I}$ of $\mathbb{R}$, we denote by $\mathbb{I}_{\mathbb{T}} = \mathbb{I}\cap\mathbb{T}$. $r, a, a_j, b_j, c_j \in C(\mathbb{T},\mathbb{R}^+)(j = 1,2,\dots,n)$ are ω-periodic functions, $T_j \in (0,\infty)_{\mathbb{T}}$, $K_j \in C([-T_j,0]_{\mathbb{T}},(0,\infty))$, $\int_{-T_j}^{0} K_j(s)\Delta s = 1\,(j = 1,2,\dots,n)$ and $\tau_j,\sigma_j \in C(\mathbb{T},\mathbb{T})\,(j = 1,2,\dots,n)$ are ω-periodic functions with respect to their first arguments, respectively. $x(t_k^+)$ and $x(t_k^-)$ represent the right and the left limit of $x(t_k)$ in the sense of time scales, in addition, if t_k is right-scattered, then $x(t_k^+) = x(t_k)$, whereas, if t_k is left-scattered, then $x(t_k^-) = x(t_k)$; $I_k \in C(\mathbb{R}^+,\mathbb{R}^+)$, $k\in\mathbb{Z}$. There exists a positive constant p such that $t_{k+p} = t_k+\omega, I_{k+p} = I_k, k\in\mathbb{Z}$, $[0,\omega)_{\mathbb{T}}\cap\{t_k, k\in\mathbb{Z}\} = \{t_1,t_2,\cdots,t_p\}$. For the ecological justification of (2), one can refer to [2-4].

2 Preliminaries

A time scale $\mathbb{T}$ is a nonempty closed subset of $\mathbb{R}$. The basic theories of calculus on time scales, one can see [6].

Definition 1. *([15]) A time scale $\mathbb{T}$ is periodic if there exists $p > 0$ such that if $t \in \mathbb{T}$, then $t \pm p \in \mathbb{T}$. For $\mathbb{T} \neq \mathbb{R}$, the smallest positive p is called the period of the time scale.*

Definition 2. *([15]) Let $\mathbb{T} \neq \mathbb{R}$ be a periodic time scale with period p. A function $f : \mathbb{T} \to \mathbb{R}$ is periodic with period ω if there exists a natural number n such that $\omega = np$, $f(t+\omega) = f(t)$ for all $t \in \mathbb{T}$ and ω is the smallest number such that $f(t+\omega) = f(t)$.*

If $\mathbb{T} = \mathbb{R}$, f is ω-periodic if ω is the smallest positive number such that $f(t+\omega) = f(t)$ for all $t \in \mathbb{T}$.

Lemma 3. *([6]) If p be a regressive function on $\mathbb{T}$, then*
(i) $e_0(t,s) \equiv 1$ and $e_p(t,t) \equiv 1$;
(ii) $e_p(\sigma(t), s) = (I + \mu(t)p(t))e_p(t,s)$;
(iii) $e_p(t,s) = e_p^{-1}(s,t)$;
(iv) $e_p(t,s)e_p(s,r) = e_p(t,r)$.

Lemma 4. *([6]) Let $r : \mathbb{T} \to \mathbb{R}$ be right-dense continuous and regressive. The unique solution of the initial value problem*

$$y^{\Delta} = r(t)y + f(t), \;\; y(t_0) = y_0$$

is given by

$$y(t) = e_r(t,t_0)y_0 + \int_{t_0}^{t} e_r(t,\sigma(\tau))f(\tau)\Delta\tau.$$

Lemma 5. *The function $x(t)$ is an ω-periodic solution of (2), if and only if $x(t)$ is an ω-periodic solution of*

$$\begin{aligned} x(t) = & \int_{t}^{t+\omega} G(t,s)x(s)\Big[a(s)x(s) \\ & + \sum_{j=1}^{n} a_j(s)x(s-\tau_j(s)) \\ & + \sum_{j=1}^{n} b_j(s)\int_{-T_j}^{0} K_j(\theta)x(\theta+s)\,\Delta\theta \\ & + \sum_{j=1}^{n} c_j(s)x^{\Delta}(s-\sigma_j(s))\Big]\,\Delta s \\ & + \sum_{k:t_k\in[t,t+\omega)_{\mathbb{T}}} G(t,t_k)e_r(\sigma(t_k),t_k)I_k(x(t_k)), \end{aligned} \tag{3}$$

where

$$G(t,s) = \frac{e_r(t,\sigma(s))}{1-e_r(0,\omega)}. \tag{4}$$

Proof: If $x(t)$ is an ω-periodic solution of (2). For any $t \in \mathbb{T}$, there exists $k \in \mathbb{Z}$ such that t_k is the first impulsive point after t. By using Lemma 4, for $s \in [t,t_k]_{\mathbb{T}}$, we have

$$\begin{aligned} x(s) \;=\; & e_r(s,t)x(t) - \int_{t}^{s} e_r(s,\sigma(u))x(u) \\ & \times\Big[a(u)x(u) + \sum_{j=1}^{n} a_j(u)x(u-\tau_j(u)) \\ & + \sum_{j=1}^{n} b_j(u)\int_{-T_j}^{0} K_j(\theta)x(\theta+u)\,\Delta\theta \\ & + \sum_{j=1}^{n} c_j(u)x^{\Delta}(u-\sigma_j(u))\Big]\,\Delta u, \end{aligned}$$

then

$$\begin{aligned} x(t_k) \;=\; & e_r(t_k,t)x(t) - \int_{t}^{t_k} e_r(t_k,\sigma(u))x(u) \\ & \times\Big[a(u)x(u) + \sum_{j=1}^{n} a_j(u)x(u-\tau_j(u)) \\ & + \sum_{j=1}^{n} b_j(u)\int_{-T_j}^{0} K_j(\theta)x(\theta+u)\,\Delta\theta \\ & + \sum_{j=1}^{n} c_j(u)x^{\Delta}(u-\sigma_j(u))\Big]\,\Delta u. \end{aligned} \tag{5}$$

Again using Lemma 4 and the equality (5), for $s \in (t_k,t_{k+1}]_{\mathbb{T}}$, then

$$\begin{aligned} x(s) \;=\; & e_r(s,t_k)x(t_k^+) - \int_{t_k}^{s} e_r(s,\sigma(u))x(u) \\ & \times\Big[a(u)x(u) + \sum_{j=1}^{n} a_j(u)x(u-\tau_j(u)) \\ & + \sum_{j=1}^{n} b_j(u)\int_{-T_j}^{0} K_j(\theta)x(\theta+u)\,\Delta\theta \\ & + \sum_{j=1}^{n} c_j(u)x^{\Delta}(u-\sigma_j(u))\Big]\,\Delta u \\ =\; & e_r(s,t_k)x(t_k) - \int_{t_k}^{s} e_r(s,\sigma(u))x(u) \\ & \times\Big[a(u)x(u) + \sum_{j=1}^{n} a_j(u)x(u-\tau_j(u)) \\ & + \sum_{j=1}^{n} b_j(u)\int_{-T_j}^{0} K_j(\theta)x(\theta+u)\,\Delta\theta \\ & + \sum_{j=1}^{n} c_j(u)x^{\Delta}(u-\sigma_j(u))\Big]\,\Delta u \end{aligned}$$

$$
\begin{aligned}
& +e_r(s,t_k)I_k(x(t_k)) \\
= {} & e_r(s,t)x(t) - \int_t^s e_r(s,\sigma(u))x(u) \\
& \times\Big[a(u)x(u) + \sum_{j=1}^n a_j(u)x(u-\tau_j(u)) \\
& + \sum_{j=1}^n b_j(u)\int_{-T_j}^0 K_j(\theta)x(\theta+u)\,\Delta\theta \\
& + \sum_{j=1}^n c_j(u)x^\Delta(u-\sigma_j(u))\Big]\,\Delta u \\
& +e_r(s,t_k)I_k(x(t_k)).
\end{aligned}
$$

Repeating the above process for $s \in [t, t+\omega]_{\mathbb{T}}$, we have

$$
\begin{aligned}
x(s) = {} & e_r(s,t)x(t) - \int_t^s e_r(s,\sigma(u))x(u) \\
& \times\Big[a(u)x(u) + \sum_{j=1}^n a_j(u)x(u-\tau_j(u)) \\
& + \sum_{j=1}^n b_j(u)\int_{-T_j}^0 K_j(\theta)x(\theta+u)\,\Delta\theta \\
& + \sum_{j=1}^n c_j(u)x^\Delta(u-\sigma_j(u))\Big]\,\Delta u \\
& + \sum_{k:t_k\in[t,s)_{\mathbb{T}}} e_r(s,t_k)I_k(x(t_k)).
\end{aligned}
$$

Let $s = t+\omega$ in the above equality, we have

$$
\begin{aligned}
x(t+\omega) = {} & e_r(t+\omega,t)x(t) \\
& - \int_t^{t+\omega} e_r(t+\omega,\sigma(u))x(u) \\
& \times\Big[a(u)x(u) + \sum_{j=1}^n a_j(u)x(u-\tau_j(u)) \\
& + \sum_{j=1}^n b_j(u)\int_{-T_j}^0 K_j(\theta)x(\theta+u)\,\Delta\theta \\
& + \sum_{j=1}^n c_j(u)x^\Delta(u-\sigma_j(u))\Big]\,\Delta u \\
& + \sum_{k:t_k\in[t,t+\omega)_{\mathbb{T}}} e_r(t+\omega,t_k)I_k(x(t_k)).
\end{aligned}
$$

Noticing that $x(t+\omega) = x(t)$ and $e_r(t,t+\omega) = e_r(0,\omega)$, we find that $x(t)$ satisfies (3).

Let $x(t)$ be an ω-periodic solution of (3). If $t \neq t_i, i \in \mathbb{Z}$, from (3) we get

$$
\begin{aligned}
& x^\Delta(t) \\
& = G(\sigma(t),t+\omega)x(t+\omega)\Big[a(t+\omega)x(t+\omega) \\
& \quad + \sum_{j=1}^n a_j(t+\omega)x(t+\omega-\tau_j(t+\omega)) \\
& \quad + \sum_{j=1}^n b_j(t+\omega)\int_{-T_j}^0 K_j(\theta)x(\theta+t+\omega)\,\Delta\theta \\
& \quad + \sum_{j=1}^n c_j(t+\omega)x^\Delta(t+\omega-\sigma_j(t+\omega))\Big] \\
& \quad -G(\sigma(t),t)x(t)\Big[a(t)x(t) \\
& \quad + \sum_{j=1}^n a_j(t)x(t-\tau_j(t)) \\
& \quad + \sum_{j=1}^n b_j(t)\int_{-T_j}^0 K_j(\theta)x(\theta+t)\,\Delta\theta \\
& \quad + \sum_{j=1}^n c_j(t)x^\Delta(t-\sigma_j(t))\Big] + r(t)x(t) \\
& = r(t)x(t) - x(t)\Big[a(t)x(t) \\
& \quad + \sum_{j=1}^n a_j(t)x(t-\tau_j(t)) \\
& \quad + \sum_{j=1}^n b_j(t)\int_{-T_j}^0 K_j(\theta)x(\theta+t)\,\Delta\theta \\
& \quad + \sum_{j=1}^n c_j(t)x^\Delta(t-\sigma_j(t))\Big].
\end{aligned}
$$

If $t = t_i, i \in \mathbb{Z}$, then by (3) we have

$$
\begin{aligned}
& x(t_i^+) - x(t_i^-) \\
& = \sum_{k:t_k\in[t_i^+,t_i^++\omega)_{\mathbb{T}}} G(t_i,t_k)e_r(\sigma(t_k),t_k)I_k(x(t_k)) \\
& \quad - \sum_{k:t_k\in[t_i^-,t_i^-+\omega)_{\mathbb{T}}} G(t_i,t_k)e_r(\sigma(t_k),t_k)I_k(x(t_k)) \\
& = G(t_i,t_i+\omega)e_r(\sigma(t_i+\omega),t_i+\omega)I_i(x(t_i+\omega)) \\
& \quad -G(t_i,t_i)e_r(\sigma(t_i),t_i)I_i(x(t_i)) \\
& = I_i(x(t_i)).
\end{aligned}
$$

So, $x(t)$ is also an ω-periodic solution of (2). This completes the proof.

In order to obtain the existence of a periodic solution of system (2), we first make the following preparations:

Let E be a Banach space and K be a cone in E. The semi-order induced by the cone K is denoted by "$\leq$", that is, $x \leq y$ if and only if $y - x \in K$. In addition, for a bounded subset $A \subset E$, let $\alpha_E(A)$ denote the (Kuratowski) measure of non-compactness defined by

$$\alpha_E(A) = \inf\big\{\delta > 0 : \text{there is a finite number of subsets } A_i \subset A \text{ such that } A = \bigcup_i A_i, \ \operatorname{diam}(A_i) \leq \delta\big\},$$

where $\operatorname{diam}(A_i)$ denotes the diameter of the set A_i.

Let E, F be two Banach spaces and $D \subset E$, a continuous and bounded map $\Phi : \bar{\Omega} \to F$ is called k-set contractive if for any bounded set $S \subset D$ we have

$$\alpha_F(\Phi(S)) \leq k\alpha_E(S).$$

Φ is called strict-set-contractive if it is k-set-contractive for some $0 \leq k < 1$.

The following lemma comes from [16] which is useful for the proof of our main results.

Lemma 6. *([16]) Let K be a cone of the real Banach space X and $K_{r,R} = \{x \in K | r \leq x \leq R\}$ with $R > r > 0$. Suppose that $\Phi : K_{r,R} \to K$ is strict-set-contractive such that one of the following two conditions is satisfied:*

(i) $\Phi x \nleq x$, $\forall x \in K$, $||x|| = r$ *and* $\Phi x \ngeq x$, $\forall x \in K$, $||x|| = R$.

(ii) $\Phi x \ngeq x$, $\forall x \in K$, $||x|| = r$ *and* $\Phi x \nleq x$, $\forall x \in K$, $||x|| = R$.

Then Φ has at least one fixed point in $K_{r,R}$.

In order to apply Lemma 6 to system (2), we set

$$C^0_\omega = \{x : x \in C^0(\mathbb{T}, \mathbb{R}), x(t + \omega) = x(t)\}$$

with the norm defined by $|x|_0 = \max_{t\in[0,\omega]_{\mathbb{T}}}\{|x(t)|\}$, and

$$C^1_\omega = \{x : x \in C^1(\mathbb{T}, \mathbb{R}), x(t + \omega) = x(t)\}$$

with the norm defined by $|x|_1 = \max\{|x|_0, |x^\Delta|_0\}$. Then C^0_ω and C^1_ω are all Banach spaces.

Since $\mathbb{T}$ is ω-periodic, $\mu(t)$ is an ω-periodic function, then $\sigma(t+\omega) = \sigma(t)+\omega$. From (4), it is easy to see that $G(t + \omega, s + \omega) = G(t, s)$ and

$$A_0 = \frac{\Upsilon}{1 - \Upsilon} \leq G(t, s) \leq \frac{1}{1 - \Upsilon} = B_0, s \in [t, t+\omega]_{\mathbb{T}},$$

where $\Upsilon = e_r(0, \omega) < 1$.

For convenience, we introduce the following notations

$$\begin{aligned}
A_1 &= \min_{t,t_k\in[0,\omega]_{\mathbb{T}}}\{G(t, t_k)e_r(\sigma(t_k), t_k)\},\\
B_1 &= \max_{t,t_k\in[0,\omega]_{\mathbb{T}}}\{G(t, t_k)e_r(\sigma(t_k), t_k)\},\\
A &= \min\{A_0, A_1\},\ B = \max\{B_0, B_1\},\\
\Theta &= \frac{A}{B}\ (0 < \Theta < 1),\\
\Gamma &= \int_0^\omega \Big[\Theta a(s) + \sum_{j=1}^n \Theta a_j(s) + \sum_{j=1}^n \Theta b_j(s) - \sum_{j=1}^n c_j(s)\Big]\Delta s,\\
\Pi &= \int_0^\omega \Big[a(s) + \sum_{j=1}^n a_j(s) + \sum_{j=1}^n b_j(s) + \sum_{j=1}^n c_j(s)\Big]\Delta s,\\
f^M &= \max_{t\in[0,\omega]_{\mathbb{T}}}\{f(t)\},\ f^m = \min_{t\in[0,\omega]_{\mathbb{T}}}\{f(t)\},
\end{aligned}$$

where f is a continuous ω-periodic function.

Throughout this paper, we assume that

(H_1) $\Theta a(t) + \sum_{j=1}^n \Theta a_j(t) + \sum_{j=1}^n \Theta b_j(t) - \sum_{j=1}^n c_j(t) \geq 0.$

(H_2) $(1 + r^m)A_0\Theta\Gamma \geq \max_{t\in[0,\omega]_{\mathbb{T}}}\Big\{a(t) + \sum_{j=1}^n a_{ij}(t) + \sum_{j=1}^n b_{ij}(t) + \sum_{j=1}^n c_{ij}(t)\Big\}.$

(H_3) $\frac{\Pi(r^M-1)B_0}{\Theta} \leq \min_{t\in[0,\omega]_{\mathbb{T}}}\Big\{\Theta a(t) + \sum_{j=1}^n \Theta a_{ij}(t) + \sum_{j=1}^n \Theta b_{ij}(t) - \sum_{j=1}^n c_{ij}(t)\Big\}.$

(H_4) $\max\big\{\sum_{k=1}^p I_k(v)\big\} \leq M|v|_0$, where $M > 0$ is a sufficient large number.

Define the cone K in C^1_ω by

$$K = \{x :\in C^1_\omega, x(t) \geq \Theta|x|_1, t \in [0, \omega]_{\mathbb{T}}\}. \tag{6}$$

Let Φ be a map defined by

$$\begin{aligned}(\Phi x)(t) &= \int_t^{t+\omega} G(t,s)x(s)\Big[a(s)x(s)\\ &\quad+\sum_{j=1}^n a_j(s)x(s-\tau_j(s))\\ &\quad+\sum_{j=1}^n b_j(s)\int_{-T_j}^0 K_j(\theta)x(\theta+s)\,\Delta\theta\\ &\quad+\sum_{j=1}^n c_j(s)x^\Delta(s-\sigma_j(s))\Big]\,\Delta s\\ &\quad+\sum_{k:t_k\in[t,t+\omega)_{\mathbb{T}}} G(t,t_k)e_r(\sigma(t_k),t_k)I_k(x(t_k)),\end{aligned} \tag{7}$$

where $x \in K, t \in \mathbb{R}$, and $G(t,s)$ is given by (4).

In the following, we will give some lemmas concerning K and Φ defined by (6) and (7), respectively.

Lemma 7. *Assume that $(H_1)-(H_2)$ hold.*

(i) *If $r^M \le 1$, then $\Phi : K \to K$ is well defined.*

(ii) *If (H_3) holds and $r^M > 1$, then $\Phi : K \to K$ is well defined.*

Proof: For any $x \in K$, it is clear that $\Phi x \in C^1(\mathbb{T},\mathbb{R})$. In view of (7), for $t \in \mathbb{T}$, we obtain

$$\begin{aligned}(\Phi x)(t+\omega) &= \int_{t+\omega}^{t+2\omega} G(t+\omega,s)x(s)\\ &\quad\times\Big[a(s)x(s)+\sum_{j=1}^n a_j(s)x(s-\tau_j(s))\\ &\quad+\sum_{j=1}^n b_j(s)\int_{-T_j}^0 K_j(\theta)x(\theta+s)\,\Delta\theta\\ &\quad+\sum_{j=1}^n c_j(s)x^\Delta(s-\sigma_j(s))\Big]\,\Delta s\\ &\quad+\sum_{k:t_k\in[t+\omega,t+2\omega)_{\mathbb{T}}} G(t+\omega,t_k)e_r(\sigma(t_k),t_k)\\ &\quad\times I_k(x(t_k))\\ &= \int_t^{t+\omega} G(t+\omega,u+\omega)x(u+\omega)\\ &\quad\times\Big[a(u+\omega)x(u+\omega)\\ &\quad+\sum_{j=1}^n a_j(u+\omega)x(u+\omega-\tau_j(u+\omega))\\ &\quad+\sum_{j=1}^n b_j(u+\omega)\int_{-T_j}^0 K_j(\theta)x(\theta+u+\omega)\,\Delta\theta\\ &\quad+\sum_{j=1}^n c_j(u+\omega)x^\Delta(u+\omega-\sigma_j(u+\omega))\Big]\,\Delta u\\ &\quad+\sum_{k:t_k\in[t,t+\omega)_{\mathbb{T}}} G(t+\omega,t_k+\omega)\\ &\quad\times e_r(\sigma(t_k+\omega),t_k+\omega)I_k(x(t_k+\omega))\\ &= \int_t^{t+\omega} G(t,u)x(u)\Big[a(u)x(u)\\ &\quad+\sum_{j=1}^n a_j(u)x(u-\tau_j(u))\\ &\quad+\sum_{j=1}^n b_j(u)\int_{-T_j}^0 K_j(\theta)x(\theta+u)\,\Delta\theta\\ &\quad+\sum_{j=1}^n c_j(u)x^\Delta(u-\sigma_j(u))\Big]\,\Delta u\\ &\quad+\sum_{k:t_k\in[t,t+\omega)_{\mathbb{T}}} G(t,t_k)e_r(\sigma(t_k),t_k)I_k(x(t_k))\\ &= (\Phi x)(t).\end{aligned}$$

That is, $(\Phi x)(t+\omega) = (\Phi x)(t), t \in \mathbb{R}$. So $\Phi x \in C^1_\omega$. In view of (H_1), for $x \in K, t \in [0,\omega]_{\mathbb{T}}$, we have

$$\begin{aligned}&a(t)x(t)+\sum_{j=1}^n a_j(t)x(t-\tau_j(t))\\ &\quad+\sum_{j=1}^n b_j(t)\int_{-T_j}^0 K_j(\theta)x(\theta+t)\,\Delta\theta\\ &\quad+\sum_{j=1}^n c_j(t)x^\Delta(t-\sigma_j(t))\\ &\ge a(t)x(t)+\sum_{j=1}^n a_j(t)x(t-\tau_j(t))\\ &\quad+\sum_{j=1}^n b_j(t)\int_{-T_j}^0 K_j(\theta)x(\theta+t)\,\Delta\theta\\ &\quad-\sum_{j=1}^n c_j(t)|x^\Delta(t-\sigma_j(t))|\\ &\ge \Theta a(t)|x^\Delta|_1+\sum_{j=1}^n \Theta a_j(t)|x^\Delta|_1\\ &\quad+\sum_{j=1}^n \Theta b_j(t)|x^\Delta|_1-\sum_{j=1}^n c_j(t)|x^\Delta|_1\\ &= \Big[\Theta a(t)+\sum_{j=1}^n \Theta a_j(t)+\sum_{j=1}^n \Theta b_j(t)\\ &\quad-\sum_{j=1}^n c_j(t)\Big]|x^\Delta|_1 \ge 0.\end{aligned} \tag{8}$$

Therefore, for $x \in K, t \in [0,\omega]_{\mathbb{T}}$, we can get

$$\begin{aligned}|\Phi x|_0 &\leq B\int_0^\omega x(s)\Big[a(s)x(s) \\ &+\sum_{j=1}^n a_j(s)x(s-\tau_j(s)) \\ &+\sum_{j=1}^n b_j(s)\int_{-T_j}^0 K_j(\theta)x(\theta+s)\,\Delta\theta \\ &+\sum_{j=1}^n c_j(s)x^\Delta(s-\sigma_j(s))\Big]\,\Delta s \\ &+B\sum_{k=1}^p I_k(x(t_k))\end{aligned}$$

and

$$\begin{aligned}(\Phi x)(t) &\geq A\int_t^{t+\omega} x(s)\Big[a(s)x(s) \\ &+\sum_{j=1}^n a_j(s)x(s-\tau_j(s)) \\ &+\sum_{j=1}^n b_j(s)\int_{-T_j}^0 K_j(\theta)x(\theta+s)\,\Delta\theta \\ &+\sum_{j=1}^n c_j(s)x^\Delta(s-\sigma_j(s))\Big]\,\Delta s \\ &+A\sum_{k=1}^p I_k(x(t_k)) \\ &\geq \Theta|\Phi x|_0. \qquad (9)\end{aligned}$$

Now, we show that

$$(\Phi x)^\Delta(t) \geq \Theta|(\Phi x)^\Delta|_0, \quad t\in[0,\omega]_{\mathbb{T}}.$$

In fact, from (7) we have

$$\begin{aligned}(\Phi x)^\Delta(t) &= r(t)(\Phi x)(t) - x(t)\Big[a(t)x(t) \\ &+\sum_{j=1}^n a_j(t)x(t-\tau_j(t)) \\ &+\sum_{j=1}^n b_j(t)\int_{-T_j}^0 K_j(\theta)x(\theta+t)\,\Delta\theta \\ &+\sum_{j=1}^n c_j(t)x^\Delta(t-\sigma_j(t))\Big]. \qquad (10)\end{aligned}$$

It follows from (8) and (10) that if $(\Phi x)^\Delta(t) \geq 0$, then

$$\begin{aligned}(\Phi x)^\Delta(t) &\leq r(t)(\Phi x)(t) \leq r^M(\Phi x)(t) \\ &\leq (\Phi x)(t). \qquad (11)\end{aligned}$$

On the other hand, from (9), (10) and (H_2), if $(\Phi x)^\Delta(t) < 0$, then

$$\begin{aligned}&-(\Phi x)^\Delta(t) \\ &= x(t)\Big[a(t)x(t)+\sum_{j=1}^n a_j(t)x(t-\tau_j(t)) \\ &+\sum_{j=1}^n b_j(t)\int_{-T_j}^0 K_j(\theta)x(\theta+t)\,\Delta\theta \\ &+\sum_{j=1}^n c_j(t)x^\Delta(t-\sigma_j(t))\Big] - r(t)(\Phi x)(t) \\ &\leq |x|_1^2\Big[a(t)+\sum_{j=1}^n a_j(t)+\sum_{j=1}^n b_j(t)+\sum_{j=1}^n c_j(t)\Big] \\ &-r^m(\Phi x)(t) \\ &\leq (1+r^m)A_0\Theta|x|_1^2\int_0^\omega\Big[\Theta a(s)+\sum_{j=1}^n\Theta a_j(s) \\ &+\sum_{j=1}^n\Theta b_j(s)-\sum_{j=1}^n c_j(s)\Big]\,\Delta s - r^m(\Phi x)(t) \\ &= (1+r^m)\int_0^\omega A_0\Theta|x|_1\Big[\Theta|x|_1 a(s) \\ &+\sum_{j=1}^n\Theta|x|_1 a_j(s)+\sum_{j=1}^n\Theta|x|_1 b_j(s) \\ &-\sum_{j=1}^n |x|_1 c_j(s)\Big]\,\Delta s - r^m(\Phi x)(t) \\ &\leq (1+r^m)\int_t^{t+\omega} G(t,s)x(s)\Big[a(s)x(s) \\ &+\sum_{j=1}^n a_j(s)x(s-\tau_j(s)) \\ &+\sum_{j=1}^n b_j(s)\int_{-T_j}^0 K_j(\theta)x(\theta+s)\,\Delta\theta \\ &-\sum_{j=1}^n c_j(s)|x^\Delta(s-\sigma_j(s))|\Big]\,\Delta s - r^m(\Phi x)(t) \\ &\leq (1+r^m)\int_t^{t+\omega} G(t,s)x(s)\Big[a(s)x(s) \\ &+\sum_{j=1}^n a_j(s)x(s-\tau_j(s)) \\ &+\sum_{j=1}^n b_j(s)\int_{-T_j}^0 K_j(\theta)x(\theta+s)\,\Delta\theta \\ &+\sum_{j=1}^n c_j(s)x^\Delta(s-\sigma_j(s))\Big]\,\Delta s - r^m(\Phi x)(t)\end{aligned}$$

$$
\begin{aligned}
&= (1+r^m)(\Phi x)(t) - r^m(\Phi x)(t) \\
&= (\Phi x)(t). \qquad (12)
\end{aligned}
$$

It follows from (11) and (12) that $|(\Phi x)^\Delta|_0 \le |\Phi x|_0$. So $|\Phi x|_1 = |\Phi x|_0$. By (9) we have $(\Phi x)(t) \ge \Theta|\Phi x|_1$. Hence, $\Phi x \in K$. The proof of (i) is completed.

(ii) In view of the proof of (i), we only need to prove that $(\Phi x)^\Delta(t) \ge 0$ implies

$$(\Phi x)^\Delta(t) \le (\Phi x)(t).$$

From (8), (10), (H_1) and (H_3), we obtain

$$
\begin{aligned}
& (\Phi_i x)^\Delta(t) \\
\le\ & r(t)(\Phi x)(t) - \Theta|x|_1 \Big[a(t)x(t) \\
& + \sum_{j=1}^n a_j(t)x(t-\tau_j(t)) \\
& + \sum_{j=1}^n b_j(t)\int_{-T_j}^0 K_j(\theta)x(\theta+t)\,\Delta\theta \\
& - \sum_{j=1}^n c_j(t)|x^\Delta(t-\sigma_j(t))|\Big] \\
\le\ & r(t)(\Phi x)(t) - \Theta|x|_1^2\Big[a(t) + \sum_{j=1}^n a_j(t) \\
& + \sum_{j=1}^n b_j(t) - \sum_{j=1}^n c_j(t)\Big] \\
\le\ & r^M(\Phi x)(t) - \Theta|x|_1^2 \frac{r^M-1}{\Theta}B_0 \\
& \times \int_0^\omega \Big[a(s) + \sum_{j=1}^n a_j(s) + \sum_{j=1}^n b_j(s) \\
& + \sum_{j=1}^n c_j(s)\Big]\,\Delta s \\
\le\ & r^M(\Phi x)(t) - (r^M-1)\int_t^{t+\omega} B_0|x|_1 \\
& \times\Big[a(s)|x|_1 + \sum_{j=1}^n a_j(s)|x|_1 \\
& + \sum_{j=1}^n b_j(s)|x|_1 + \sum_{j=1}^n c_j(s)|x|_1\Big]\,\Delta s \\
\le\ & r^M(\Phi x)(t) - (r^M-1)\int_t^{t+\omega} G(t,s)x(s) \\
& \times\Big[a(s)x(s) + \sum_{j=1}^n a_j(s)x(s-\tau_j(s)) \\
& + \sum_{j=1}^n b_j(s)\int_{-T_k}^0 K_j(\theta)x(\theta+s)\,\Delta\theta \\
& + \sum_{j=1}^n c_j(s)|x^\Delta(s-\sigma_j(s))|\Big]\,\Delta s \\
\le\ & r^M(\Phi x)(t) - (r^M-1)\int_t^{t+\omega} G(t,s)x(s) \\
& \times\Big[a(s)x(s) + \sum_{j=1}^n a_j(s)x(s-\tau_j(s)) \\
& + \sum_{j=1}^n b_j(s)\int_{-T_k}^0 K_j(\theta)x(\theta+s)\,\Delta\theta \\
& + \sum_{j=1}^n c_j(s)x^\Delta(s-\sigma_j(s))\Big]\,\Delta s \\
=\ & r^M(\Phi x)(t) - (r^M-1)(\Phi x)(t) \\
=\ & (\Phi x)(t).
\end{aligned}
$$

The proof of (ii) is completed. □

Lemma 8. *Assume that $(H_1)-(H_2),(H_4)$ hold and $R\sum_{j=1}^n c_j^M < 1$.*

(i) If $r^M \le 1$, then $\Phi : K\bigcap\bar{\Omega}_R \to K$ is strict-set-contractive;

(ii) If (H_3) holds and $r^M > 1$, then $\Phi : K\bigcap\bar{\Omega}_R \to K$ is strict-set-contractive;

where $\Omega_R = \{x \in C_\omega^1 : |x|_1 < R\}$.

Proof: We only need to prove (i), since the proof of (ii) is similar. It is easy to see that Φ is continuous and bounded. Now we prove that $\alpha_{C_\omega^1}(\Phi(S)) \le \left(R\sum_{j=1}^n c_j^M\right)\alpha_{C_\omega^1}(S)$ for any bounded set $S \subset \bar{\Omega}_R$. Let $\eta = \alpha_{C_\omega^1}(S)$. Then, for any positive number $\varepsilon < \left(R\sum_{j=1}^n c_j^M\right)\eta$, there is a finite family of subsets $\{S_i\}$ satisfying $S = \bigcup_i S_i$ with $\mathrm{diam}(S_i) \le \eta+\varepsilon$. Therefore

$$|x-y|_1 \le \eta+\varepsilon \quad \text{for any } x, y \in S_i. \qquad (13)$$

As S and S_i are precompact in C_ω^0, it follows that there is a finite family of subsets $\{S_{ij}\}$ of S_i such that $S_i = \bigcup_j S_{ij}$ and

$$|x-y|_0 \le \varepsilon \quad \text{for any } x, y \in S_{ij}. \qquad (14)$$

In addition, for any $x \in S$ and $t \in [0,\omega]_{\mathbb{T}}$, we have

$$|(\Phi x)(t)| \;=\; \int_t^{t+\omega} G(t,s)x(s)\Big[a(s)x(s)$$

$$
\begin{aligned}
&+\sum_{j=1}^{n} a_j(s)x(s-\tau_j(s)) \\
&+\sum_{j=1}^{n} b_j(s)\int_{-T_j}^{0} K_j(\theta)x(\theta+s)\,\Delta\theta \\
&+\sum_{j=1}^{n} c_j(s)x^{\Delta}(s-\sigma_j(s))\Big]\,\Delta s \\
&+\sum_{k:t_k\in[t,t+\omega)_{\mathbb{T}}} G(t,t_k)e_r(\sigma(t_k),t_k)I_k(x(t_k)) \\
\le\;& BR^2\int_t^{t+\omega}\Big[a(s)+\sum_{j=1}^{n}a_j(s)+\sum_{j=1}^{n}b_j(s) \\
&+\sum_{j=1}^{n}c_j(s)\Big]\,\Delta s+BM|x|_0 := H
\end{aligned}
$$

and

$$
\begin{aligned}
|(\Phi x)^{\Delta}(t)| =\;& \Big|r(t)(\Phi x)(t)-x(t)\Big[a(t)x(t) \\
&+\sum_{j=1}^{n} a_j(t)x(t-\tau_j(t)) \\
&+\sum_{j=1}^{n} b_j(t)\int_{-T_j}^{0} K_j(\theta)x(\theta+t)\Delta\theta \\
&+\sum_{j=1}^{n} c_j(t)x^{\Delta}(t-\sigma_j(t))\Big]\Big| \\
\le\;& r^M H+R^2\sum_{j=1}^{n}(a^M+a_j^M+b_j^M+c_j^M).
\end{aligned}
$$

Applying the Arzela-Ascoli Theorem, we know that $\Phi(S)$ is precompact in C_ω^0. Then, there is a finite family of subsets $\{S_{ijk}\}$ of S_{ij} such that $S_{ij}=\bigcup_k S_{ijk}$ and

$$|\Phi x-\Phi y|_0\le\varepsilon,\forall x,y\in S_{ijk}. \tag{15}$$

From (8), (10), (13)-(15) and (H_1), for any $x,y\in S_{ijk}$, we obtain

$$
\begin{aligned}
&|(\Phi x)^{\Delta}-(\Phi y)^{\Delta}|_0 \\
=\;& \max_{t\in[0,\omega]_{\mathbb{T}}}\Big\{\Big|r(t)(\Phi x)(t)-r(t)(\Phi y)(t) \\
&-x(t)\Big[a(t)x(t)+\sum_{j=1}^{n}a_j(t)x(t-\tau_j(t)) \\
&+\sum_{j=1}^{n} b_j(t)\int_{-T_j}^{0} K_j(\theta)x(\theta+t)\Delta\theta \\
&+\sum_{j=1}^{n} c_j(t)x^{\Delta}(t-\sigma_j(t))\Big] \\
&+y(t)\Big[a(t)y(t)+\sum_{j=1}^{n}a_j(t)y(t-\tau_j(t)) \\
&+\sum_{j=1}^{n} b_j(t)\int_{-T_j}^{0} K_j(\theta)y(\theta+t)\Delta\theta \\
&+\sum_{j=1}^{n} c_j(t)y^{\Delta}(t-\sigma_j(t))\Big]\Big|\Big\} \\
\le\;& \max_{t\in[0,\omega]_{\mathbb{T}}}\{|r(t)[(\Phi x)(t)-(\Phi y)(t)]|\} \\
&+\max_{t\in[0,\omega]_{\mathbb{T}}}\Big\{\Big|x(t)\Big[a(t)x(t) \\
&+\sum_{j=1}^{n}a_j(t)x(t-\tau_j(t)) \\
&+\sum_{j=1}^{n} b_j(t)\int_{-T_j}^{0} K_j(\theta)x(\theta+t)\Delta\theta \\
&+\sum_{j=1}^{n} c_j(t)x^{\Delta}(t-\sigma_j(t))\Big] \\
&-y(t)\Big[a(t)y(t)+\sum_{j=1}^{n}a_j(t)y(t-\tau_j(t)) \\
&+\sum_{j=1}^{n} b_j(t)\int_{-T_j}^{0} K_j(\theta)y(\theta+t)\Delta\theta \\
&+\sum_{j=1}^{n} c_j(t)y^{\Delta}(t-\sigma_j(t))\Big]\Big|\Big\} \\
\le\;& r^M|(\Phi x)-(\Phi y)|_0 \\
&+\max_{t\in[0,\omega]_{\mathbb{T}}}\Big\{\Big|x(t)\Big[\Big(a(t)x(t) \\
&+\sum_{j=1}^{n}a_j(t)x(t-\tau_j(t)) \\
&+\sum_{j=1}^{n} b_j(t)\int_{-T_j}^{0} K_j(\theta)x(\theta+t)\Delta\theta \\
&+\sum_{j=1}^{n} c_j(t)x^{\Delta}(t-\sigma_j(t))\Big) \\
&-\Big(a(t)y(t)+\sum_{j=1}^{n}a_j(t)y(t-\tau_j(t)) \\
&+\sum_{j=1}^{n} b_j(t)\int_{-T_j}^{0} K_j(\theta)y(\theta+t)\Delta\theta \\
&+\sum_{j=1}^{n} c_j(t)y^{\Delta}(t-\sigma_j(t))\Big)\Big]\Big|\Big\} \\
&+\max_{t\in[0,\omega]_{\mathbb{T}}}\Big\{\Big|y(t)\Big[a(t)y(t)
\end{aligned}
$$

$$
\begin{aligned}
&+\sum_{j=1}^{n} a_j(t)y(t-\tau_j(t))\\
&+\sum_{j=1}^{n} b_j(t)\int_{-T_j}^{0} K_j(\theta)y(\theta+t)\Delta\theta\\
&+\sum_{j=1}^{n} c_j(t)y^{\Delta}(t-\sigma_j(t))\Big][x(t)-y(t)]\Big|\Big\}\\
\leq\ & r^M\varepsilon + R\max_{t\in[0,\omega]_{\mathbb{T}}}\Big\{a(t)|x(t)-y(t)|\\
&+\sum_{j=1}^{n} a_j(t)|x(t-\tau_j(t))-y(t-\tau_j(t))|\\
&+\sum_{j=1}^{n} b_j(t)\int_{-T_j}^{0} K_j(\theta)|x(\theta+t)\\
&-y(\theta+t)|\Delta\theta\\
&+\sum_{j=1}^{n} c_j(t)|x^{\Delta}(t-\sigma_j(t))-y^{\Delta}(t-\sigma_j(t))|\Big\}\\
&+\varepsilon\max_{t\in[0,\omega]_{\mathbb{T}}}\Big\{a(t)y(t)+\sum_{j=1}^{n} a_j(t)y(t-\tau_j(t))\\
&+\sum_{j=1}^{n} b_j(t)\int_{-T_j}^{0} K_j(\theta)y(\theta+t)\Delta\theta\\
&+\sum_{j=1}^{n} c_j(t)|y^{\Delta}(t-\sigma_j(t))|\Big\}\\
\leq\ & r^M\varepsilon + R\varepsilon\big(a^M+\sum_{j=1}^{n} a_j^M+\sum_{j=1}^{n} b_j^M\big)\\
&+R(\eta+\varepsilon)\big(\sum_{j=1}^{n} c_j^M\big)\\
&+R\varepsilon\big(a^M+\sum_{j=1}^{n} a_j^M+\sum_{j=1}^{n} b_j^M+\sum_{j=1}^{n} c_j^M\big)\\
=\ & \big(R\eta\sum_{j=1}^{n} c_j^M\big)+\hat{H}\varepsilon, \qquad (16)
\end{aligned}
$$

where $\widehat{H}=r^M+2R[a^M+\sum\limits_{j=1}^{n} a_j^M+\sum\limits_{j=1}^{n} b_j^M+\sum\limits_{j=1}^{n} c_j^M]$.

From (15) and (16) we have

$$|\Phi x-\Phi y|_1\leq \big(R\sum_{j=1}^{n} c_j^M\big)\eta+\hat{H}\varepsilon, \forall x,y\in S_{ijk}.$$

As ε is arbitrary small, it follows that

$$\alpha_{C_\omega^1}(\Phi(S))\leq \big(R\sum_{j=1}^{n} c_j^M\big)\alpha_{C_\omega^1}(S).$$

Therefore, Φ is strict-set-contractive. This completes the proof. □

3 Main Result

Our main result of this paper is as follows:

Theorem 9. *Assume that $(H_1)-(H_2)$, (H_4) hold.*

(i) *If $r^M\leq 1$, then system (2) has at least one positive ω-periodic solutions.*

(ii) *If (H_3) holds and $r^M>1$, then system (2) has at least one positive ω-periodic solutions.*

Proof: We only need to prove (i), since the proof of (ii) is similar. Let $R=\frac{1}{A\Theta\Gamma}$ and $0<r<\frac{\Theta-BM}{B\Pi}$. Then we have $0<r<R$. From Lemma 7 and Lemma 8, we know that Φ is strict-set-contractive on $K_{r,R}$. In view of (10), we see that if there exists $x^*\in K$ such that $\Phi x^*=x^*$, then x^* is one positive ω-periodic solution of system (2). Now, we shall prove that condition (ii) of Lemma 6 hold.

First, we prove that $\Phi x\not\geq x$, $\forall x\in K$, $|x|_1=r$. Otherwise, there exists $x\in K$, $|x|_1=r$ such that $\Phi x\geq x$. So $|x|>0$ and $\Phi x-x\in K$, which implies that

$$(\Phi x)(t)-x(t)\geq \Theta|\Phi x-x|_1\geq 0, \forall t\in[0,\omega]_{\mathbb{T}}. \qquad (17)$$

Moreover, for $t\in[0,\omega]_{\mathbb{T}}$, by (H_4), we have

$$
\begin{aligned}
&(\Phi x)(t)\\
=\ & \int_t^{t+\omega} G(t,s)x(s)\Big[a(s)x(s)\\
&+\sum_{j=1}^{n} a_j(s)x(s-\tau_j(s))\\
&+\sum_{j=1}^{n} b_j(s)\int_{-T_j}^{0} K_j(\theta)x(\theta+s)\,\Delta\theta\\
&+\sum_{j=1}^{n} c_j(s)x^{\Delta}(s-\sigma_j(s))\Big]\,\Delta s\\
&+\sum_{k:t_k\in[t,t+\omega)_{\mathbb{T}}} G(t,t_k)e_r(\sigma(t_k),t_k)I_k(x(t_k))\\
\leq\ & Br|x|_0\int_0^{\omega}\Big[a(s)+\sum_{j=1}^{n} a_j(s)\\
&+\sum_{j=1}^{n} b_j(s)+\sum_{j=1}^{n} c_j(s)\Big]\,\Delta s+BM|x|_0\\
=\ & B(r\Pi+M)|x|_0\\
<\ & \Theta|x|_0. \qquad (18)
\end{aligned}
$$

In view of (17) and (18), we have

$$|x|_0 \leq |\Phi x| < \Theta |x|_0 < |x|_0,$$

which is a contradiction.

Finally, we prove that $\Phi x \nleq x$, $\forall x \in K$, $|x|_1 = R$ also holds. For this case, we only need to prove that

$$\Phi x \not< x,\ x \in K,\ |x|_1 = R.$$

Suppose, for the sake of contradiction, that there exists $x \in K$ and $|x|_1 = R$ such that $\Phi x < x$, thus $x - \Phi x \in K \setminus \{0\}$. Furthermore, for any $t \in [0, \omega]_{\mathbb{T}}$, we have

$$x(t) - (\Phi x)(t) \geq \Theta |x - \Phi x|_1 > 0. \tag{19}$$

In addition, for any $t \in [0, \omega]_{\mathbb{T}}$, we find

$$\begin{aligned}
&(\Phi x)(t)\\
= &\int_t^{t+\omega} G(t,s)x(s)\Big[a(s)x(s)\\
&+\sum_{j=1}^n a_j(s)x(s-\tau_j(s))\\
&+\sum_{j=1}^n b_j(s)\int_{-T_j}^0 K_j(\theta)x(\theta+s)\,\Delta\theta\\
&+\sum_{j=1}^n c_j(s)x^{\Delta}(s-\sigma_j(s))\Big]\,\Delta s\\
&+\sum_{k:t_k\in[t,t+\omega)_{\mathbb{T}}} G(t,t_k)e_r(\sigma(t_k),t_k)I_k(x(t_k))\\
\geq\ & A\Theta|x|_1^2\int_0^{\omega}\Big[\Theta a(s)+\sum_{j=1}^n\Theta a_j(s)\\
&+\sum_{j=1}^n\Theta b_j(s)-\sum_{j=1}^n c_j(s)\Big]\,\Delta s\\
= &A\Theta\Gamma R^2\\
= &R.
\end{aligned} \tag{20}$$

From (19) and (20), we obtain

$$|x| > |\Phi x|_0 \geq R,$$

which is a contradiction. Therefore, conditions (i) and (ii) hold. By Lemma 6, we see that Φ has at least one nonzero fixed point in K. Therefore, system (2) has at least one positive ω-periodic solutions. This completes the proof. □

Remark 10. *From the proof of our results, one can see that if all of or some of $T_j (j = 1, 2, \ldots, n)$ replaced by ∞, the conclusion of Theorem 9 remains true.*

4 An example

Consider the following dynamic system on time scale $\mathbb{T}$,

$$\begin{cases} x^{\Delta}(t) = x(t)\Big[\frac{2+\cos t}{8\pi} - (5-2\sin t)x(t)\\ \qquad -(2+\sin t)x(t-\tau(t))\\ \qquad -(1-\frac{2}{3}\sin t)\\ \qquad \times\int_{-T_1}^0 K_1(s)x(t+s)\Delta s\\ \qquad -\frac{1-\sin t}{20}x^{\Delta}(t-\sigma(t))\Big],\ t\neq t_k,\\ x(t_k^+) = x(t_k^-) + 0.01\sin(x(t_k)), k = 1, 2, \cdots, \end{cases} \tag{21}$$

where $\tau, \sigma \in C(\mathbb{T}, \mathbb{T})$ are 2π-periodic functions with respect to their first arguments, respectively. $T_1 \in (0, \infty)_{\mathbb{T}}$, $K_1 \in C([-T_1, 0]_{\mathbb{T}}, (0, \infty))$, $\int_{-T_1}^0 K_1(s)\Delta s = 1$. Obviously,

$$r(t) = \frac{2+\cos t}{8\pi},\ a(t) = 5 - 2\sin t,$$
$$a_1(t) = 2 + \sin t,\ b_1(t) = 1 - \frac{2}{3}\sin t,$$
$$c_1(t) = \frac{1-\sin t}{20}.$$

Let $\mathbb{T} = \mathbb{R}, p = 2$, by a direct calculation, we can get $\Theta = e^{-\frac{1}{2}}, \Gamma = 30.1734, \Pi = 30.6781$, and

(H_1) $\Theta a(t) + \sum_{j=1}^n \Theta a_j(t) + \sum_{j=1}^n \Theta b_j(t) - \sum_{j=1}^n c_j(t)$ $= e^{-\frac{1}{2}}(8.05 + \frac{5}{3}\sin t) - \frac{1-\sin t}{20} \geq 3.7717 > 0.$

(H_2) $(1 + r^m)A_0\Theta\Gamma = 29.3335 \geq 9.7667 = \max_{t\in[0,2\pi]_{\mathbb{T}}}\{a(t) + \sum_{j=1}^n a_{ij}(t) + \sum_{j=1}^n b_{ij}(t) + \sum_{j=1}^n c_{ij}(t)\}.$

Hence, $(H_1), (H_2)$ hold. For $|x|_0 > 0$, then there exists a sufficient large number $M > 0$ such that (H_4) holds. Moreover, $r^M = \frac{3}{8\pi} \leq 1$. According to Theorem 9, system (21) has at least one positive 2π-periodic solution.

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