lie in $|z| < 1$ for every real or complex number α with $|\alpha| \leq 1$ and $R > r \geq 1$. Applying Lemma 19 to the polynomial $G(z)$ and noting that B is a linear operator, it follows that all the zeros of the polynomial

$$\begin{aligned} T(z) &= B[G(z)] = B[F(Rz)] - \alpha B[F(rz)] \\ &= B[P(Rz)] - \alpha B[P(rz)] - \delta\left(R^n - \alpha r^n\right) B[z^n] \end{aligned} \tag{42}$$

lie in $|z| < 1$ for every real or complex number δ with $|\delta| < 1$ and $R > r \geq 1$, which implies

$$|B[P(Rz)] - \alpha B[P(rz)]| \geq m\,|R^n - \alpha r^n|\,|B[z^n]|.$$

for $|z| \geq 1$. If above inequality is not true, then there is a point $z = w$ with $|w| \geq 1$ such that

$$\begin{aligned} &|\{B[P(Rz)] - \alpha B[P(rz)]\}_{z=w}| \\ &< m\,|R^n - \alpha r^n|\,|\{B[z^n]\}_{z=w}|. \end{aligned}$$

Since all the zeros of $B[z^n]$ lie in $|z| < 1$, therefore, $\{B[z^n]\}_{z=w} \neq 0$. We take

$$\delta = \frac{\{B[P(Rz)] - \alpha B[P(rz)]\}_{z=w}}{m(R^n - \alpha r^n)\,\{B[z^n]\}_{z=w}},$$

then δ is well defined real or complex number with $|\delta| < 1$ and with choice of δ, from (42) we get, $T(w) = 0$ with $|w| \geq 1$, which contradicts the fact that all the zeros of $T(z)$ lie in $|z| < 1$. Thus

$$|B[P(Rz)] - \alpha B[P(rz)]| \geq m\,|R^n - \alpha r^n|\,|B[z^n]|$$

for every real or complex number α with $|\alpha| \leq 1$, $R > r \geq 1$ and $|z| \geq 1$. This completes the proof of Theorem 14.

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