Robust finite-time boundedness of discrete-time neural networks with time-varying delays

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Abstract: This paper is concerned with the problem of robust finite-time boundedness for the discrete-time neural networks with time-varying delays. By constructing an appropriate Lyapunov-Krasovskii functional, we proposed the sufficient conditions which ensure the robust finite-time boundedness of the discrete-time neural networks with time-varying delay in terms of linear matrix inequalities. Then the sufficient conditions of robust finite-time stability for the discrete-time neural networks with time-varying delays are given. Finally, a numerical example is present to illustrate the efficiency of proposed methods.

Key-Words: - Finite-time boundedness; Lyapunov-Krasovskii functional; discrete-time systems; neural networks; time-varying delays

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1 Introduction

In recent years, neural networks have been widely used in associative memory, pattern recognition, model identification, signal processing, static image processing, optimization control problems and other aspects [1-4]. Many applications of neural networks rely on dynamic behavior. Therefore the stability of neural networks has incurred extensive attention from scholars. In [5], the stability criteria for neural networks were given. Assimakis et al. [6] investigated the robust exponential stability for uncertain recurrent neural networks.

With the rapid development of technology, computer technology is introduced into the field of engineering research. In fact, when the computer processes the input and output control signals, the resulting signals are discrete-time. Therefore, discrete-time systems have attracted extensive attention of researchers [7-10]. On the other hand, in the field of practical application, the time-delay phenomenon commonly exists in the neural network system, which will not only reduce the transmission speed of the network but also lead to the instability or vibration of the network. Therefore, it is of great theoretical and practical significance to study the stability and control performance of neural network systems with delay [9-12]. In [9], Yu et al. studied the exponential stability for discrete-time recurrent neural networks with time-varying delay. In [11], Liu et al. considered the mean square exponential stability for discrete-time stochastic fuzzy neural network.

In many real systems, we are sometimes interested in the performance of the system over a finite time interval. For automobile suspension control system, the performance of short time interval is more popular. Compared with the Lyapunov asymptotic stability, the research of finite-time stability considers the behavior of the dynamic system within a finite time interval. The finite time stability theory can be applied to the design of wheeled robots and the attitude tracking and attitude cooperative control technology of spacecraft. In [13], Dorato proposed the definition of short-time stability. In recent years, finite time stability and stabilization have attracted much attention [14-19]. In [14], the finite-time stability analysis of neutral-type neural networks with random time varying delays was given. In [16], via a new argument Lyapunov-Krasovskii functional, the finite-time stability of neural networks with time-varying delays was studied. In [19], Ren et al. considered the finite-time stabilization for uncertain positive Markovian jumping neural networks. In [20], Amato et al. extended the concept of finite time stability introducing the definition of finite-time boundedness for the state of a system. In [21], by using reciprocally convex approach, Tuan investigated the finite-time boundedness for discrete-time delay neural networks. In [22], the criterion of finite-time boundedness for the nonlinear switched neutral system was presented.

To the best of our knowledge, the problem of finite time boundedness of discrete time neural networks is seldom studied. In this paper, we consider the robust finite-time boundedness problem for a class of uncertain discrete-time neural networks. We

3. Main Results

This section provides some criteria for the finite-time boundedness of system (1).

Theorem 1. Suppose that Assumption 1 holds. Given positive constants $\chi, N, c_1 < c_2, \sigma > 1,$ a symmetric positive-definite matrix R, system (1) is robustly finite-time bounded with respect to (c_1, c_2, χ, R, N), if there exist symmetric positive-definite matrices P, Q, T_1, T_2, diagonal matrices $U_1 > 0, U_2 > 0$, and positive scalars μ_1, μ_2, λ_i $(i = 1, 2, \cdots, 5)$, such that the following LMIs hold:

$$\Lambda = \begin{bmatrix} \Lambda_{11} & \Lambda_{12} \\ * & \Lambda_{22} \end{bmatrix} < 0, \tag{10}$$

$$\lambda_1 R < P < \lambda_2 R, Q < \lambda_3 R, T_1 < \lambda_4 R, T_2 < \lambda_5 R, \tag{11}$$

$$\begin{bmatrix} \mu_2 \sigma^{N-1} \chi - c_2 \lambda_1 & \upsilon_2 \lambda_2 & \upsilon_3 \lambda_3 & \upsilon_4 \lambda_4 & \upsilon_5 \lambda_5 \\ * & -\upsilon_2 \lambda_2 & 0 & 0 & 0 \\ * & * & -\upsilon_3 \lambda_3 & 0 & 0 \\ * & * & * & -\upsilon_4 \lambda_4 & 0 \\ * & * & * & * & -\upsilon_5 \lambda_5 \end{bmatrix} < 0, \tag{12}$$

where

$$\Lambda_{11} = \begin{bmatrix} \Theta_1 & 0 & 0 & 0 & \Phi_2 U_1 \\ * & -\sigma^{d_1} Q - \Psi_1 U_2 & 0 & 0 & 0 \\ * & * & -\sigma^{d_1} T_2 & 0 & 0 \\ * & * & * & -\sigma^{d_2} T_1 & 0 \\ * & * & * & * & -U_1 \end{bmatrix},$$

$$\Theta_1 = -\sigma P + T_1 + T_2 + (d_2 - d_1 + 1)Q - \Phi_1 U_1,$$

$$\Lambda_{12} = \begin{bmatrix} 0 & 0 & -A^T P & -A^T PD & -\mu_1 N_1^T \\ \Psi_2 U_2 & 0 & A_d{}^T P & A_d{}^T PD & \mu_1 N_2^T \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & G^T P & G^T PD & \mu_1 N_3^T \end{bmatrix},$$

$$\Lambda_{22} = \begin{bmatrix} -U_2 & 0 & H^T P & H^T PD & \mu_1 N_4^T \\ * & -\mu_2 I & C^T P & C^T PD & \mu_1 N_5^T \\ * & * & -P & 0 & 0 \\ * & * & * & D^T PD - \mu_1 I & 0 \\ * & * & * & * & -\mu_1 I \end{bmatrix},$$

$$\upsilon_2 = c_1 \sigma^N, \upsilon_4 = c_1 \sigma^N d_2 \sigma^{d_2 - 1}, \ \upsilon_5 = c_1 \sigma^N d_1 \sigma^{d_1 - 1},$$

$$\upsilon_3 = c_1 \sigma^N \left[d_2 \sigma^{d_2 - 1} + \sigma^{d_2 - 2} \frac{d_2^2 - d_1^2 - d_1 - d_2}{2} \right].$$

Proof. Choose the following Lyapunov-Krasovskii functional candidate

$$V(k) = V_1(k) + V_2(k) + V_3(k) + V_4(k) + V_5(k), \tag{13}$$

where

$$V_1(k) = x^T(k) P x(k),$$
$$V_2(k) = \sum_{i=k-d(k)}^{k-1} \sigma^{k-1-i} x^T(i) Q x(i),$$
$$V_3(k) = \sum_{i=k-d_2}^{k-1} \sigma^{k-1-i} x^T(i) T_1 x(i),$$
$$V_4(k) = \sum_{i=k-d_1}^{k-1} \sigma^{k-1-i} x^T(i) T_2 x(i),$$
$$V_5(k) = \sum_{j=-d_2+1}^{-d_1} \sum_{i=k+j}^{k-1} \sigma^{k-1-i} x^T(i) Q x(i).$$

Let us define the forward difference of $V(k)$ as

$$\Delta V(k) = V(k+1) - V(k).$$

We have

$$\begin{aligned} \Delta V_1(k) &= V_1(k+1) - V_1(k) \\ &= (\sigma - 1)V_1(k) + V_1(k+1) - \sigma V_1(k) \\ &= (\sigma - 1)V_1(k) + \xi^T(k) \Upsilon_1(k) P \Upsilon_1^T(k) \xi(k) \\ &\quad - \sigma x^T(k) P x(k), \end{aligned} \tag{14}$$

where

$$\Upsilon_1^T(k) = \left[-A(k) \ \ A_d(k) \ \ 0 \ \ 0 \ \ G(k) \ \ H(k) \ \ C(k) \right],$$

$$\xi(k) = \left[x^T(k) \ \ x^T(k - d(k)) \ \ x^T(k - d_1) \ \ x^T(k - d_2) \ \ g^T(x(k)) \ \ h^T(x(k - d(k))) \ \ \omega^T(k) \right]^T.$$

Denote

$$\Upsilon_1^T = \left[-A \ \ A_d \ \ 0 \ \ 0 \ \ G \ \ H \ \ C \right],$$

$$\Delta \Upsilon^T(k) = \left[-\Delta A(k) \ \ \Delta A_d(k) \ \ 0 \ \ 0 \ \ \Delta G(k) \ \ \Delta H(k) \ \ \Delta C(k) \right].$$

By Lemma 2, we have

$$\begin{aligned} \Upsilon_1(k) P \Upsilon_1{}^T(k) &= (\Upsilon_1 + \Delta \Upsilon(k)) P (\Upsilon_1 + \Delta \Upsilon(k))^T \\ &= (\Upsilon_1 + \Upsilon_2 F^T(k) D^T) P (\Upsilon_1 + \Upsilon_2 F^T(k) D^T)^T \\ &\le \Upsilon_1 P \Upsilon_1^T + \Upsilon_1 PD(\mu_1 I - D^T PD)^{-1} D^T P \Upsilon_1^T + \mu_1 \Upsilon_2 \Upsilon_2^T, \end{aligned} \tag{15}$$

where $\Upsilon_2^T = [-N_1 \ N_2 \ 0 \ 0 \ N_3 \ N_4 \ N_5]$.

According to (14) and (15), we have

$$\begin{aligned} \Delta V_1(k) &\le (\sigma - 1)V_1(k) - \sigma x^T(k) P x(k) \\ &\quad + \xi^T(k)[\Upsilon_1 P \Upsilon_1^T + \Upsilon_1 PD(\mu_1 I - D^T PD)^{-1} \\ &\quad \times D^T P \Upsilon_1^T + \mu_1 \Upsilon_2 \Upsilon_2^T] \xi(k), \end{aligned} \tag{16}$$

$$\begin{aligned}\Delta V_2(k) &= V_2(k+1)-V_2(k)\\
&=(\sigma-1)V_2(k)+\sum_{i=k+1-d(k+1)}^{k}\sigma^{k-i}x^T(i)Qx(i)\\
&\quad-\sum_{i=k-d(k)}^{k-1}\sigma^{k-i}x^T(i)Qx(i)\\
&=(\sigma-1)V_2(k)+x^T(k)Qx(k)\\
&\quad+\sum_{i=k-d_1+1}^{k-1}\sigma^{k-i}x^T(i)Qx(i)\\
&\quad+\sum_{i=k+1-d(k+1)}^{k-d_1}\sigma^{k-i}x^T(i)Qx(i)\\
&\quad-\sum_{i=k-d(k)+1}^{k-1}\sigma^{k-i}x^T(i)Qx(i)\\
&\quad-\sigma^{d(k)}x^T(k-d(k))Qx(k-d(k))\\
&\le(\sigma-1)V_2(k)+x^T(k)Qx(k)\\
&\quad+\sum_{i=k+1-d_2}^{k-d_1}\sigma^{k-i}x^T(i)Qx(i)\\
&\quad-\sigma^{d_1}x^T(k-d(k))Qx(k-d(k)),
\end{aligned}\tag{17}$$

$$\begin{aligned}\Delta V_3(k) &= V_3(k+1)-V_3(k)\\
&=(\sigma-1)V_3(k)+x^T(k)T_1x(k)\\
&\quad+\sum_{i=k-d_2+1}^{k-1}\sigma^{k-i}x^T(i)T_1x(i)\\
&\quad-\sum_{i=k-d_2+1}^{k-1}\sigma^{k-i}x^T(i)T_1x(i)\\
&\quad-\sigma^{d_2}x^T(k-d_2)T_1x(k-d_2)\\
&=(\sigma-1)V_3(k)+x^T(k)T_1x(k)\\
&\quad-\sigma^{d_2}x^T(k-d_2)T_1x(k-d_2),
\end{aligned}\tag{18}$$

$$\begin{aligned}\Delta V_4(k) &= V_4(k+1)-V_4(k)\\
&=(\sigma-1)V_4(k)+V_4(k+1)-\sigma V_4(k)\\
&=(\sigma-1)V_4(k)+\sum_{i=k-d_1+1}^{k}\sigma^{k-i}x^T(i)T_2x(i)\\
&\quad-\sum_{i=k-d_1}^{k-1}\sigma^{k-i}x^T(i)T_2x(i)\\
&=(\sigma-1)V_4(k)+x^T(k)T_2x(k)\\
&\quad-\sigma^{d_1}x^T(k-d_1)T_2x(k-d_1),
\end{aligned}\tag{19}$$

$$\begin{aligned}\Delta V_5(k) &= V_5(k+1)-V_5(k)\\
&=(\sigma-1)V_5(k)\\
&\quad+\sum_{j=-d_2+1}^{-d_1}\sum_{i=k+j+1}^{k}\sigma^{k-i}x^T(i)Qx(i)\\
&\quad-\sum_{j=-d_2+1}^{-d_1}\sum_{i=k+j}^{k-1}\sigma^{k-i}x^T(i)Qx(i)\\
&=(\sigma-1)V_5(k)+\sum_{j=-d_2+1}^{-d_1}\Big[x^T(k)Qx(k)\\
&\quad+\sum_{i=k+j+1}^{k-1}\sigma^{k-i}x^T(i)Qx(i)\\
&\quad-\sum_{i=k+j+1}^{k-1}\sigma^{k-i}x^T(i)Qx(i)\\
&\quad-\sigma^{-j}x^T(k+j)Qx(k+j)\Big]\\
&=(\sigma-1)V_5(k)+(d_2-d_1)x^T(k)Qx(k)\\
&\quad-\sum_{i=k-d_2+1}^{k-d_1}\sigma^{k-i}x^T(i)Qx(i).
\end{aligned}\tag{20}$$

From Assumption 1, for any $i=1,2,\cdots,n$, we have

$$(g_i(x_i(k))-\alpha_i^+x_i(k))(g_i(x_i(k))-\alpha_i^-x_i(k))\le 0,\tag{21}$$

which is equivalent to

$$\begin{bmatrix}x(k)\\ g(x(k)\end{bmatrix}^T\begin{bmatrix}\alpha_i^+\alpha_i^-e_ie_i^T & -\dfrac{\alpha_i^++\alpha_i^-}{2}e_ie_i^T\\ -\dfrac{\alpha_i^++\alpha_i^-}{2}e_ie_i^T & e_ie_i^T\end{bmatrix}\begin{bmatrix}x(k)\\ g(x(k)\end{bmatrix}\le 0,\tag{22}$$

where e_i denotes the units column vector having element 1 on its i th row and zeros elsewhere. Let $U_1>0, U_2>0$, be any $n\times n$ diagonal matrices. Then we have

$$\begin{bmatrix}x(k)\\ g(x(k)\end{bmatrix}^T\begin{bmatrix}\Phi_1U_1 & -\Phi_2U_1\\ -\Phi_2U_1 & U_1\end{bmatrix}\begin{bmatrix}x(k)\\ g(x(k)\end{bmatrix}\le 0,\tag{23}$$

$$\begin{bmatrix}x(k-d(k))\\ h(x(k-d(k))\end{bmatrix}^T\begin{bmatrix}\Psi_1U_2 & -\Psi_2U_2\\ -\Psi_2U_2 & U_2\end{bmatrix}\begin{bmatrix}x(k-d(k))\\ h(x(k-d(k))\end{bmatrix}\le 0,\tag{24}$$

where

$$U_1 = diag\{\theta_{11}, \theta_{12}, \cdots, \theta_{1n}\},$$
$$\Phi_1 = diag\{\alpha_1^- \alpha_1^+, \alpha_2^- \alpha_2^+, \cdots, \alpha_n^- \alpha_n^+\},$$
$$\Phi_2 = diag\{\frac{\alpha_1^- + \alpha_1^+}{2}, \frac{\alpha_2^- + \alpha_2^+}{2}, \cdots, \frac{\alpha_n^- + \alpha_n^+}{2}\},$$
$$U_2 = diag\{\theta_{21}, \theta_{22}, \cdots, \theta_{2n}\},$$
$$\Psi_1 = diag\{\beta_1^- \beta_1^+, \beta_2^- \beta_2^+, \cdots, \beta_n^- \beta_n^+\},$$
$$\Psi_2 = diag\{\frac{\beta_1^- + \beta_1^+}{2}, \frac{\beta_2^- + \beta_2^+}{2}, \cdots, \frac{\beta_n^- + \beta_n^+}{2}\}.$$

Combine (16)-(20), (23) and (24), we get

$$\begin{aligned}
\Delta V(k) \le\ & (\sigma - 1)V(k) - \sigma x^T(k)Px(k) + \xi^T(k)[\Upsilon_1 P \Upsilon_1^T \\
& + \Upsilon_1 PD(\mu_1 I - D^T PD)^{-1} D^T P \Upsilon_1^T + \mu_1 \Upsilon_2 \Upsilon_2^T] \\
& \times \xi(k) + x^T(k)T_1 x(k) + x^T(k) T_2 x(k) \\
& + (d_2 - d_1 + 1)x^T(k)Qx(k) - \sigma^{d_1} x^T(k - d(k)) \\
& \times Qx(k - d(k)) - \sigma^{d_2} x^T(k - d_2) T_1 x(k - d_2) \\
& - \sigma^{d_1} x^T(k - d_1) T_2 x(k - d_1) - x^T(k)\Phi_1 U_1 x(k) \\
& + 2x^T(k)\Phi_2 U_1 g(x(k)) - g^T(x(k)) U_1 g(x(k)) \\
& - x^T(k - d(k))\Psi_1 U_2 x(k - d(k)) \\
& + 2x^T(k - d(k))\Psi_2 U_2 g(x(k - d(k))) \\
& - g^T(x(k - d(k)))U_2 g(x(k - d(k))) \\
& - \mu_2 \omega^T(k)\omega(k) + \mu_2 \omega^T(k)\omega(k) \\
=\ & (\sigma - 1)V(k) + \xi^T(k)[\Xi + \tilde{\Upsilon}_1 P \Upsilon_1^T + \tilde{\Upsilon}_1 PD \\
& (\mu_1 I - D^T PD)^{-1} D^T P \tilde{\Upsilon}_1^T + \mu_1 \tilde{\Upsilon}_2 \tilde{\Upsilon}_2^T]\xi(k) \\
& + \mu_2 \omega^T(k)\omega(k),
\end{aligned} \tag{25}$$

where

$$\Xi = \begin{bmatrix} \Xi_{11} & 0 & 0 & 0 & \Phi_2 U_1 & 0 & 0 \\ * & \Xi_{22} & 0 & 0 & 0 & \Psi_2 U_2 & 0 \\ * & * & \Xi_{33} & 0 & 0 & 0 & 0 \\ * & * & * & \Xi_{44} & 0 & 0 & 0 \\ * & * & * & * & -U_1 & 0 & 0 \\ * & * & * & * & * & -U_2 & 0 \\ * & * & * & * & * & * & \Xi_{77} \end{bmatrix},$$

$$\Xi_{11} = -\sigma P + T_1 + T_2 + (d_2 - d_1 + 1)Q - \Phi_1 U_1,$$
$$\Xi_{22} = -\sigma^{d_1} Q - \Psi_1 U_2,\ \Xi_{33} = -\sigma^{d_1} T_2,$$
$$\Xi_{44} == -\sigma^{d_2} T_1,\ \Xi_{77} = -\mu_2 I.$$

From Lemma 1, we deduce that the inequality $\Xi + \Upsilon_1 P \Upsilon_1^T + \Upsilon_1 PD(\mu_1 I - D^T PD)^{-1} D^T P \Upsilon_1^T + \mu_1 \Upsilon_2 \Upsilon_2^T < 0$ is satisfied if (10) is feasible.

Hence, we get

$$\Delta V(k) \le (\sigma - 1)V(k) + \mu_2 \omega^T(k)\omega(k). \tag{26}$$

From (26), it follows that

$$V(k+1) - \sigma V(k) \le \mu_2 \omega^T(k)\omega(k). \tag{27}$$

By iteration, and take (2) into account, inequality (27) implies

$$\begin{aligned}
V(k) &\le \sigma^k V(0) + \mu_2 \sum_{i=0}^{k-1} \sigma^{k-i-1} \omega^T(i)\omega(i) \\
&\le \sigma^N V(0) + \mu_2 \sigma^{N-1} \sum_{i=0}^{k-1} \omega^T(i)\omega(i) \\
&\le \sigma^N V(0) + \mu_2 \sigma^{N-1} \chi, \qquad k \le N.
\end{aligned} \tag{28}$$

Denote

$$\tilde{P} = R^{-\frac{1}{2}} P R^{-\frac{1}{2}}, \quad \tilde{Q} = R^{-\frac{1}{2}} Q R^{-\frac{1}{2}}, \quad \tilde{T}_1 = R^{-\frac{1}{2}} T_1 R^{-\frac{1}{2}},$$
$$\tilde{T}_2 = R^{-\frac{1}{2}} T_2 R^{-\frac{1}{2}}.$$

Using condition (12), we can get

$$\begin{aligned}
V(0) =\ & x^T(0)Px(0) + \sum_{i=-d(0)}^{-1} \sigma^{-1-i} x^T(i)Qx(i) \\
& + \sum_{i=-d_2}^{-1} \sigma^{-1-i} x^T(i) T_1 x(i) + \sum_{i=-d_1}^{-1} \sigma^{-1-i} x^T(i) T_2 x(i) \\
& + \sum_{j=-d_2+1}^{-d_1} \sum_{i=j}^{-1} \sigma^{-1-i} x^T(i)Qx(i) \\
\le\ & \lambda_{\max}(\tilde{P}) x^T(0)Rx(0) \\
& + \sigma^{d_2-1} \lambda_{\max}(\tilde{Q}) \sum_{i=-d(0)}^{-1} x^T(i)Rx(i) \\
& + \sigma^{d_2-1} \lambda_{\max}(\tilde{T}_1) \sum_{i=-d_2}^{-1} x^T(i)Rx(i) \\
& + \sigma^{d_1-1} \lambda_{\max}(\tilde{T}_2) \sum_{i=-d_1}^{-1} x^T(i)Rx(i) \\
& + \sigma^{d_2-2} \lambda_{\max}(\tilde{Q}) \sum_{j=-d_2+1}^{-d_1} \sum_{i=j}^{-1} x^T(i)Rx(i) \\
\le\ & \Big[\lambda_2 + (d_2 \sigma^{d_2-1} + \sigma^{d_2-2} \frac{d_2^2 - d_1^2 + d_1 - d_2}{2}) \\
& \times \lambda_3 + d_2 \sigma^{d_2-1} \lambda_4 + d_1 \sigma^{d_1-1} \lambda_5 \Big] c_1.
\end{aligned} \tag{29}$$

From (28) and (29), we obtain

$$V(k) \le \sigma^N \delta + \mu_2 \sigma^{N-1} \chi, \tag{30}$$

where

$$\delta = \big[\lambda_2 + (d_2 \sigma^{d_2-1} + \sigma^{d_2-2} \frac{d_2^2 - d_1^2 + d_1 - d_2}{2}) \lambda_3 + d_2 \sigma^{d_2-1} \lambda_4 + d_1 \sigma^{d_1-1} \lambda_5 \big] c_1.$$

According to (13), we get

$$V(k) \ge x^T(k)Px(k) \ge \lambda_1 x^T(k)Rx(k). \tag{31}$$

Note that (12) is equivalent to

$$\mu_2\sigma^{N-1}\chi - c_2\lambda_1 + c_1\sigma^N\lambda_2 + c_1\sigma^N\Big[d_2\sigma^{d_2-1} + \sigma^{d_2-2} \times \frac{d_2^2 - d_1^2 + d_1 - d_2}{2}\Big]\lambda_3 + c_1\sigma^N d_2\sigma^{d_2-1}\lambda_4 + c_1\rho^N d_1\rho^{d_1-1}\lambda_5 < 0,$$

that is

$$\mu_2\sigma^{N-1}\chi - c_2\lambda_1 + \sigma^N\delta < 0. \tag{32}$$

Consequently, it can be obtained from (30) - (32)

$$x^T(k)Rx(k) \le \frac{1}{\lambda_1}\Big[\sigma^N\delta + \mu_2\sigma^{N-1}\varepsilon^2\Big] < \frac{1}{\lambda_1}\lambda_1 c_2 = c_2. \tag{33}$$

which indicates that system (1) is robustly finite-time bounded with respect to (c_1, c_2, χ, R, N). This completes the proof.

Consider the following discrete-time neural networks

$$\begin{aligned} x(k+1) &= -(A+\Delta A(k))x(k) + (C+\Delta C(k))\omega(k) \\ &\quad + (A_d + \Delta A_d(k))x(k-d) \\ &\quad + (G+\Delta G(k))g(x(k)) \\ &\quad + (H+\Delta H(k))h(x(k-d)), \end{aligned} \tag{34}$$

$$x(\theta) = \phi(\theta),\quad \theta\in\Gamma = \{-d, -d+1, \cdots, 0\},$$

where d is a positive integer.

The following corollary can be obtained.

Corollary 1. Suppose that Assumption 1 holds. Given positive constants $\chi, N, c_1 < c_2, \sigma > 1,$ a symmetric positive-definite matrix $R,$ system (34) is finite-time bounded with respect to (c_1, c_2, χ, R, N), if there exist symmetric positive definite matrices $P, Q,$ diagonal matrices $U_1 > 0, U_2 > 0,$ positive scalars $\mu_1, \mu_2, \lambda_1, \lambda_2, \lambda_3,$ such that the following LMIs hold:

$$\tilde{\Lambda} = \begin{bmatrix} \tilde{\Lambda}_{11} & \tilde{\Lambda}_{12} \\ * & \tilde{\Lambda}_{22} \end{bmatrix} < 0, \tag{35}$$

$$\lambda_1 R < P < \lambda_2 R, Q < \lambda_3 R, \tag{36}$$

$$\begin{bmatrix} \mu_2\sigma^{N-1}\chi - c_2\lambda_1 & c_1\sigma^N\lambda_2 & c_1\sigma^N d\sigma^{d-1}\lambda_3 \\ * & -c_1\sigma^N\lambda_2 & 0 \\ * & * & -c_1\sigma^N d\sigma^{d-1}\lambda_3 \end{bmatrix} < 0, \tag{37}$$

where

$$\tilde{\Lambda}_{11} = \begin{bmatrix} \tilde{\Theta}_1 & 0 & \Phi_2 U_1 & 0 \\ * & -\sigma^d Q - \Psi_1 U_2 & 0 & \Psi_2 U_2 \\ * & * & -U_1 & 0 \\ * & * & * & -U_2 \end{bmatrix},$$

$$\tilde{\Theta}_1 = -\sigma P + Q - \Phi_1 U_1,$$

$$\tilde{\Lambda}_{12} = \begin{bmatrix} 0 & -A^T P & -A^T PD & -\mu_1 N_1^T \\ 0 & A_d^T P & A_d^T PD & \mu_1 N_2^T \\ 0 & G^T P & G^T PD & \mu_1 N_3^T \\ 0 & H^T P & H^T PD & \mu_1 N_4^T \end{bmatrix},$$

$$\tilde{\Lambda}_{22} = \begin{bmatrix} -\mu_2 I & C^T P & C^T PD & \mu_1 N_5^T \\ * & -P & 0 & 0 \\ * & * & D^T PD - \mu_1 I & 0 \\ * & * & * & -\mu_1 I \end{bmatrix}.$$

Proof. Choose the following Lyapunov-Krasovskii functional candidate

$$V(k) = x^T(k)Px(k) + \sum_{i=k-d}^{k-1}\sigma^{k-1-i}x^T(i)Qx(i).$$

We have

$$\begin{aligned} \Delta V(k) &= (\sigma-1)V(k) + V(k+1) - \sigma V(k) \\ &= (\sigma-1)V(k) + \tilde{\xi}^T(k)\Upsilon_1(k)P\Upsilon_1^T(k)\tilde{\xi}(k) \\ &\quad + x^T(k)Qx(k) - \sigma^d x^T(k-d)Qx(k-d) \\ &\quad - \sigma x^T(k)Px(k) \\ &\le (\sigma-1)V(k) + \tilde{\xi}^T(k)[\Upsilon_1 P\Upsilon_1^T \\ &\quad + \Upsilon_1 PD(\mu_1 I - D^T PD)^{-1}D^T P\Upsilon_1^T \\ &\quad + \mu_1\Upsilon_2\Upsilon_2^T]\tilde{\xi}(k) + x^T(k)Qx(k) \\ &\quad - \sigma^d x^T(k)Qx(k) - \sigma x^T(k)Px(k), \end{aligned} \tag{38}$$

where

$$\tilde{\Upsilon}_1^T = \big[-A\ \ A_d\ \ G\ \ H\ \ C\big],$$

$$\tilde{\Upsilon}_2^T = [-N_1\ N_2\ N_3\ N_4\ N_5],$$

$$\tilde{\xi}(k) = \Big[x^T(k)\ \ x^T(k-d)\ \ g^T(x(k))\ \ h^T(x(k-d(k)))\ \ \omega^T(k)\Big]^T.$$

Combine (38), (23) and (24), we get

$$\begin{aligned} \Delta V(k) &\le (\sigma-1)V(k) + \tilde{\xi}^T(k)[\Upsilon_1 P\Upsilon_1^T \\ &\quad + \Upsilon_1 PD(\mu_1 I - D^T PD)^{-1}D^T P\Upsilon_1^T \\ &\quad + \mu_1\Upsilon_2\Upsilon_2^T]\tilde{\xi}(k) + x^T(k)Qx(k) \\ &\quad - \sigma^d x^T(k)Qx(k) - \sigma x^T(k)Px(k) \\ &\quad - x^T(k)\Phi_1 U_1 x(k) + 2x^T(k)\Phi_2 U_1 g(x(k)) \\ &\quad - g^T(x(k))U_1 g(x(k)) - x^T(k-d(k))\Psi_1 \\ &\quad \times U_2 x(k-d(k)) + 2x^T(k-d(k))\Psi_2 U_2 \\ &\quad \times g(x(k-d(k))) - g^T(x(k-d(k)))U_2 \\ &\quad \times g(x(k-d(k))) - \mu_2\omega^T(k)\omega(k) \\ &\quad + \mu_2\omega^T(k)\omega(k) \end{aligned}$$

$$= (\sigma-1)V(k) + \tilde{\xi}^T(k)[\tilde{\Xi} + \tilde{\Upsilon}_1 P \tilde{\Upsilon}_1^T + \tilde{\Upsilon}_1 PD (\mu_1 I - D^T PD)^{-1} D^T P \tilde{\Upsilon}_1^T + \mu_1 \tilde{\Upsilon}_2 \tilde{\Upsilon}_2^T]\tilde{\xi}(k) + \mu_2 \omega^T(k)\omega(k), \tag{39}$$

where

$$\tilde{\Xi} = \begin{bmatrix} \tilde{\Theta}_1 & 0 & \Phi_2 U_1 & 0 & 0 \\ * & -\sigma^d Q - \Psi_1 U_2 & 0 & \Psi_2 U_2 & 0 \\ * & * & -U_1 & 0 & 0 \\ * & * & * & -U_2 & 0 \\ * & * & * & * & -\mu_2 I \end{bmatrix},$$

$\tilde{\Theta}_1 = -\sigma P + Q - \Phi_1 U_1.$

From Lemma 1, we get that

$$\tilde{\Xi} + \tilde{\Upsilon}_1 P \tilde{\Upsilon}_1^T + \tilde{\Upsilon}_1 PD(\mu_1 I - D^T PD)^{-1} D^T P \tilde{\Upsilon}_1^T + \mu_1 \tilde{\Upsilon}_2 \tilde{\Upsilon}_2^T < 0$$

is satisfied if (35) is feasible.

Hence, we get

$$\Delta V(k) \le (\sigma-1)V(k) + \mu_2 \omega^T(k)\omega(k). \tag{40}$$

According to the proof of the Theorem 1, we have

$$\begin{aligned} V(k) &\le \sigma^k V(0) + \mu_2 \sum_{i=0}^{k-1} \sigma^{k-i-1} \omega^T(i)\omega(i) \\ &\le \sigma^N V(0) + \mu_2 \sigma^{N-1} \sum_{i=0}^{k-1} \omega^T(i)\omega(i) \\ &\le \sigma^N V(0) + \mu_2 \sigma^{N-1} \chi, \qquad k \le N. \end{aligned} \tag{41}$$

Using condition (36), we can get

$$\begin{aligned} V(0) &= x^T(0)Px(0) + \sum_{i=-d}^{-1} \sigma^{-1-i} x^T(i)Qx(i) \\ &\le \lambda_{\max}(\tilde{P}) x^T(0)Rx(0) \\ &\quad + \sigma^{d-1} \lambda_{\max}(\tilde{Q}) \sum_{i=-d}^{-1} x^T(i)Rx(i) \\ &\le (\lambda_2 + d\sigma^{d-1}\lambda_2)c_1. \end{aligned} \tag{42}$$

So we have

$$V(k) \le \sigma^N(\lambda_2 + d\sigma^{d-1})c_1 + \mu_2 \sigma^{N-1}\chi. \tag{43}$$

From (35). it follows that

$$V(k) \ge x^T(k)Px(k) \ge \lambda_1 x^T(k)Rx(k). \tag{44}$$

From (37), we have

$$\mu_2 \sigma^{N-1}\chi + c_1\sigma^N(\lambda_2 + d\sigma^{d-1}\lambda_3) < c_2\lambda_1. \tag{45}$$

It can be obtained from (43) - (45)

$$x^T(k)Rx(k) \le \frac{1}{\lambda_1} V(k) < \frac{1}{\lambda_1}\lambda_1 c_2 = c_2, \tag{46}$$

which implies that the system (34) is robustly finite-time bounded with respect to (c_1, c_2, χ, R, N). This completes the proof.

When $\Delta A(k) = 0,\ \Delta A_d(k) = 0, \Delta G(k) = 0, \Delta H(k) = 0, \Delta C(k) = 0,$ the system (1) reduced to the following neural networks

$$\begin{aligned} x(k+1) &= -Ax(k) + A_d x(k-d(k)) + C\omega(k) \\ &\quad + Gg(x(k)) + Hh(x(k-d(k))), \\ x(\theta) &= \phi(\theta), \quad \theta \in \Gamma = \{-d_2, -d_2+1, \cdots, 0\}. \end{aligned} \tag{47}$$

According to the similar ideas in the proof of Theorem 1, we can obtain the following corollary.

Corollary 2. Suppose that Assumption 1 holds. Given positive constants $\chi, N, c_1 < c_2, \sigma > 1,$ a symmetric positive-definite matrix R, system (47) is finite-time bounded with respect to (c_1, c_2, χ, R, N), if there exist symmetric positive definite matrices P, Q, T_1, T_2, diagonal matrices $U_1 > 0, U_2 > 0$, positive scalars $\mu, \lambda_i\ (i = 1, 2, \cdots, 5)$, such that the following LMIs hold:

$$\bar{\Lambda} = \begin{bmatrix} \bar{\Lambda}_{11} & \bar{\Lambda}_{12} \\ * & \bar{\Lambda}_{22} \end{bmatrix} < 0, \tag{48}$$

$$\lambda_1 R < P < \lambda_2 R, Q < \lambda_3 R, T_1 < \lambda_4 R, T_2 < \lambda_5 R, \tag{49}$$

$$\begin{bmatrix} \mu\sigma^{N-1}\chi - c_2\lambda_1 & \upsilon_2\lambda_2 & \upsilon_3\lambda_3 & \upsilon_4\lambda_4 & \upsilon_5\lambda_5 \\ * & -\upsilon_2\lambda_2 & 0 & 0 & 0 \\ * & * & -\upsilon_3\lambda_3 & 0 & 0 \\ * & * & * & -\upsilon_4\lambda_4 & 0 \\ * & * & * & * & -\upsilon_5\lambda_5 \end{bmatrix} < 0, \tag{50}$$

where

$$\bar{\Lambda}_{11} = \begin{bmatrix} \Theta_1 & 0 & 0 & 0 \\ * & -\sigma^{d_1} Q - \Psi_1 U_2 & 0 & 0 \\ * & * & -\sigma^{d_1} T_2 & 0 \\ * & * & * & -\sigma^{d_2} T_1 \end{bmatrix},$$

$\Theta_1 = -\sigma P + T_1 + T_2 + (d_2 - d_1 + 1)Q - \Phi_1 U_1,$

$$\bar{\Lambda}_{12} = \begin{bmatrix} \Phi_2 U_1 & 0 & 0 & -A^T P \\ 0 & \Psi_2 U_2 & 0 & A_d{}^T P \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\bar{\Lambda}_{22} = \begin{bmatrix} -U_1 & 0 & 0 & G^T P \\ * & -U_2 & 0 & H^T P \\ * & * & -\mu I & C^T P \\ * & * & * & -P \end{bmatrix},$$

$\upsilon_2 = c_1\sigma^N,\ \upsilon_4 = c_1\sigma^N d_2\sigma^{d_2-1},\ \upsilon_5 = c_1\rho^N d_1\sigma^{d_1-1},$

$$\upsilon_3 = c_1\sigma^N \left[d_2\sigma^{d_2-1} + \sigma^{d_2-2} \frac{d_2^2 - d_1^2 + d_1 - d_2}{2} \right].$$

When $\omega(k)=0$, the system (1) reduced to the following discrete-time neural networks

$$\begin{aligned} x(k+1) &= -(A+\Delta A(k)x(k)+(G+\Delta G(k))g(x(k)) \\ &\quad +(A_d+\Delta A_d(k))x(k-d(k)) \\ &\quad +(H+\Delta H(k))h(x(k-d(k))), \\ x(\theta) &= \phi(\theta), \quad \theta\in\Gamma=\{-d_2,-d_2+1,\cdots,0\}. \end{aligned} \tag{51}$$

From Remark 2, we know that when $\omega(k)=0$, the definition of finite-time boundedness can become finite-time stability. According to the similar ideas in the proof of Theorem 1, we can get the following corollary.

Corollary 3. Suppose that Assumption 1 holds. Given positive constants $N,\sigma>1,c_1<c_2$, a symmetric positive-definite matrix R, system (51) is finite-time stable with respect to (c_1,c_2,R,N) , if there exist symmetric positive-definite matrices P,Q,T_1,T_2, diagonal matrices $U_1>0,U_2>0$, positive scalars μ,λ_i $(i=1,2,\cdots,5)$, such that the following LMIs hold:

$$\hat{\Lambda}=\begin{bmatrix}\Lambda_{11} & \hat{\Lambda}_{12}\\ * & \hat{\Lambda}_{22}\end{bmatrix}<0, \tag{52}$$

$$\lambda_1 R<P<\lambda_2 R, Q<\lambda_3 R, T_1<\lambda_4 R, T_2<\lambda_5 R, \tag{53}$$

$$\begin{bmatrix} -c_2\lambda_1 & \upsilon_2\lambda_2 & \upsilon_3\lambda_3 & \upsilon_4\lambda_4 & \upsilon_5\lambda_5 \\ * & -\upsilon_2\lambda_2 & 0 & 0 & 0 \\ * & * & -\upsilon_3\lambda_3 & 0 & 0 \\ * & * & * & -\upsilon_4\lambda_4 & 0 \\ * & * & * & * & -\upsilon_5\lambda_5 \end{bmatrix}<0, \tag{54}$$

where

$$\Lambda_{11}=\begin{bmatrix} \Theta_1 & 0 & 0 & 0 & \Phi_2 U_1 \\ * & \Pi & 0 & 0 & 0 \\ * & * & -\sigma^{d_1}T_2 & 0 & 0 \\ * & * & * & -\sigma^{d_2}T_1 & 0 \\ * & * & * & * & -U_1 \end{bmatrix},$$

$$\Theta_1=-\sigma P+T_1+T_2+(d_2-d_1+1)Q-\Phi_1 U_1,$$

$$\Pi=-\sigma^{d_1}Q-\Psi_1 U_2,$$

$$\hat{\Lambda}_{12}=\begin{bmatrix} 0 & -A^T P & -A^T PD & -\mu N_1^T \\ \Psi_2 U_2 & A_d{}^T P & A_d{}^T PD & \mu N_2^T \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & G^T P & G^T PD & \mu N_3^T \end{bmatrix},$$

$$\hat{\Lambda}_{22}=\begin{bmatrix} -U_2 & H^T P & H^T PD & \mu N_4^T \\ * & -P & 0 & 0 \\ * & * & D^T PD-\mu I & 0 \\ * & * & * & -\mu I \end{bmatrix},$$

$$\upsilon_2=c_1\sigma^N, \upsilon_4=c_1\sigma^N d_2\sigma^{d_2-1},\ \upsilon_5=c_1\sigma^N d_1\sigma^{d_1-1},$$

$$\upsilon_3=c_1\sigma^N\left[d_2\sigma^{d_2-1}+\sigma^{d_2-2}\frac{d_2^2-d_1^2+d_1-d_2}{2}\right].$$

4. Numerical example

In this section, we present one example to demonstrate the effectiveness of our results.

Example 1. Consider the system (1) with the following parameters

$$A=\begin{bmatrix}0.4 & 0\\ 0 & 0.5\end{bmatrix},\quad A_d=\begin{bmatrix}-0.12 & 0.1\\ 0.15 & 0.1\end{bmatrix},$$

$$C=\begin{bmatrix}0.2 & 0.1\\ 0.2 & 0.25\end{bmatrix},\quad G=\begin{bmatrix}0.01 & 0.01\\ 0.01 & 0.01\end{bmatrix},$$

$$H=\begin{bmatrix}0.01 & 0.01\\ 0.01 & 0.01\end{bmatrix},\quad R=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},$$

$$F(k)=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},\quad D=\begin{bmatrix}0.01 & 0\\ 0 & 0.01\end{bmatrix},$$

$$N_1=\begin{bmatrix}0.1 & 0\\ 0 & 0.3\end{bmatrix},\quad N_2=\begin{bmatrix}0.1 & 0.4\\ 0.2 & 0.1\end{bmatrix},$$

$$N_3=\begin{bmatrix}0.1 & 0.3\\ 0.3 & 0.2\end{bmatrix}, N_4=\begin{bmatrix}0.2 & 0.3\\ 0.2 & 0.3\end{bmatrix},$$

$$N_5=\begin{bmatrix}0.1 & 0.1\\ 0.1 & 0.1\end{bmatrix},\quad \omega(k)=\begin{pmatrix}e^{-0.1k}\sin(k)\\ e^{-0.1k}\cos(k)\end{pmatrix},$$

$$g(x(k))=\begin{pmatrix}\tanh(0.08x_1(k))\\ \tanh(0.06x_2(k))\end{pmatrix},$$

$$h(x(k-d(k)))=\begin{pmatrix}\tanh(0.08x_1(k-d(k)))\\ \tanh(0.08x_2(k-d(k)))\end{pmatrix},$$

$$d(k)=2+\sin(\frac{k\pi}{2}),\quad \chi=1,$$

$$\sigma=1.001, c_1=1, c_2=7, N=10.$$

By using Matlab LMI control Toolbox to solve LMIs (10)-(12), we have

$$P=\begin{bmatrix}0.0884 & 0.0013\\ 0.0013 & 0.0703\end{bmatrix},\quad Q=\begin{bmatrix}0.0098 & 0.0004\\ 0.0004 & 0.0066\end{bmatrix},$$

$$T_1=\begin{bmatrix}0.0027 & -0.0008\\ -0.0008 & 0.0008\end{bmatrix}, U_1=\begin{bmatrix}6.2862 & 0\\ 0 & 6.2862\end{bmatrix},$$

$$T_2 = \begin{bmatrix} 0.0031 & -0.0009 \\ -0.0009 & 0.0009 \end{bmatrix}, U_2 = \begin{bmatrix} 0.1043 & 0 \\ 0 & 0.1043 \end{bmatrix},$$

$\lambda_1 = 0.0687, \lambda_2 = 0.1039, \lambda_3 = 0.0114, \lambda_4 = 0.0086,$

$\lambda_5 = 0.0159, \mu_1 = 2.5568 \times 10^{-5}, \mu_2 = 0.1936.$

According to Theorem 1, the system (1) is robustly finite-time bounded with respect to $(1, 7, 1, I, 10)$. The state trajectory of system (1) is shown in Fig.1

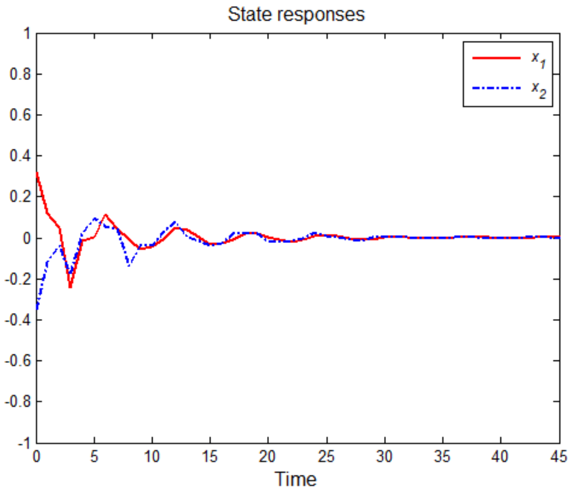

Fig 1. The state trajectory of system (1)

5 Conclusion

The paper has investigated the robust finite-time boundedness for discrete-time neural networks with time-varying delays. Through constructing a Lyapunov-Krasovskii functional, based on the linear matrix inequality technique, robust finite-time boundedness criteria for the discrete-time neural networks with time-varying delays have been established. Furthermore, robust finite-time stability criterion for the discrete-time neural networks with time-varying delays has been given. At the end of the article, an example has been given to verify the validity of the stability criterion. The finite time H_∞ control for the discrete-time stochastic neural networks with mixed time delays is a very meaningful topic that deserves further exploration.

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