Study regarding elaboration of a theoretical model of the grinding process and its influence on the environment

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Abstract: The analysis of a cutting process through grinding and its influence on the environment involves the creation of a physical model, an idealized approximation of reality, which reflects the development of principle and implies the admission of simplifying hypotheses. Based on the logic model, it is necessary to be able to study the phenomena that accompany the technological process under investigation, starting from the beginning of its design phase. In order to be able to analyze more precisely the abrasion process, was considered the sequence in which an abrasive granule is in full action of detaching a chip.

The paper proposes the mathematical modeling of the roughness of the surfaces processed by grinding based on the spherical model, which is a complex problem when it is desired to determine the influence of the technological parameters on the quality of the processed parts and on the environment.

Key-Words: - abrasion process, mathematical modeling, surface roughness, environment.

1 Introduction

The paper proposes the mathematical modeling of the roughness of the surfaces processed by grinding based on the spherical model, which is a complex problem when it is desired to determine the influence of the technological parameters on the quality of the machined parts and on the environment.

The analysis of a cutting process involves the creation of a physical model, an idealized approximation of reality, which reflects the development of principle and implies the admission of simplifying hypotheses. Based on the logic model, we must be able to study the phenomena that accompany the technological process under investigation, from its design phase. In order to be able to analyze more precisely the abrasion process, the sequence in which an abrasive granule is in full action of detaching a chip was considered.

Grinding fluid should be considered a defining element of the technological system aimed at the processing by cutting with abrasive tool. If in the 80s of the last century the specialists neglected the importance of the grinding fluid in the manufacturing system today the role of the cutting liquid is reconsidered. Almost all of the consulted materials - articles from specialized journals, reports and scientific communications, university treatises, theses and online information - have surprised the fact that there is a consensus of the specialists regarding the essential role that the cutting fluid plays in the
interaction with the other factors that influence the performance of the technological processing system.

2 A theoretical model of the abrasive grain in grinding process in the presence of the cutting liquid

The analysis of a cutting process involves the creation of a physical model, an idealized approximation of reality, which reflects the development of principle and implies the admission of simplifying hypotheses. Based on the logic model, we must be able to study the phenomena that accompany the technological process under investigation, from its design phase.

This physical model must meet the essential conditions of any theoretical model:
- To be simple;
- To represent as accurately as possible the essence of the process being analyzed.

In order to be able to analyze more precisely the abrasion process, the sequence in which an abrasive grain is in full action of removing a chip was considered.

It has been considered that the processed surface is the result of the simultaneous action of the geometrical factors characteristic of the abrasion process and of the plastic deformations that accompany it.

The grain edge is usually rounded; it reaches after some wear at radii of the order of $\rho = 20-30\mu\text{m}$ and the grain begins to move on the cutting surface with radial pressure and gradually increasing frictional force, producing a cold hardening of the surface layer (Fig.1a). Then, as the cutting layer expands, the grain penetrates into the material, initially producing only a scratch with a slight discharge of material in front and side (Fig.1b) and only a little later it removes the chip (Fig.1c). The phenomena of crushing and scratching in the first phases are more intense as the smaller the chip thickness and the larger is the radius of rounding $\rho$ (the ratio $a/\rho$ is smaller).

From a microgeometric point of view, roughness is formed as a result of copying the traces left by the abrasive grains. Due to the fact that an abrasive grain raises the micro-chips with high specific forces, intense deformations occur which deform the microrelief obtained as a result of the action of the geometrical parameters that characterize the abrasive granules.

The smoothness of the surface processed by abrasion is determined by the density of the scratches on the surface unit and the shape of an abrasive grain that proceeds the abrasion has a little influence.

It is obvious that improving the smoothness of the surface by grinding is achieved by increasing the
number of passes, decreasing the feedrate and
reducing the abrasive tool size.

The correctness of the theoretical model presented
above is based on the following observations:

- All cutting edges of the abrasive grains have a
certain radius $\rho$ whose value is increased during the
use of the cutting tool;
- The formation of the chip begins only after the
abrasive granule has penetrated to a certain depth in
the part material, depth depending on the size of the
radius $\rho$, the geometrical configuration of the
trajectory described by the grain and the elasto-
plastic properties of the processed material;
- At shallow depths of penetration, only elastic and
plastic deformations occur, as well as strong heat
releases due to friction;
- The thickness of the layer of effectively removed
material, $t_A$, is smaller than the theoretical size $a$ of,
according to the relation (1).

$$t_A = a - t_e - t_p, \quad (1)$$

where $t_A$ represents the thickness of the effectively
removed material layer, $t_e$ - the thickness of the
elastic deformed material layer, $t_p$ - the thickness of
the plastic deformed material layer, $a$ - the theoretical
size cutting depth.

- The separation point (Fig.2), i.e. the point at which
the part material breaks and begins to expel it, is
higher than the tip of the cutting edge.

This model of a single abrasive grain also has certain
limits that can be synthesized as follows:

- The grain edges are randomly oriented and are
located at different depths relative to a reference
surface;
- The geometry of the edges changes over time;
- Under the action of the normal forces that appear
between the workpiece and the abrasive body, the
grains at the periphery will give up elasticity and
”sink” in the abrasive body, resulting in the increase
of the number of grains (edges) that are in the
working area;
- Not all the edges that are in the working area cut;
there are negative clearance angles, which do not
allow the cutting;
- There was a reduction to 1 of the cutting edges;
- When assimilating the cutting edges with a
continuous surface the tip of the grain assimilated
with a sphere is not in fact a continuous surface;
- The tip of the grain assimilated with a sphere
allows to obtain a permanent orthogonal orientation
on the piece, which does not correspond to reality.
On a simple analysis it can be seen that this model of the "single grain" which cut during grinding process neglects the active presence in the process of the cutting liquid. All explanations refer to the dry rubbing, the dry shearing and the dry removing between the abrasive grain and the workpiece material respectively the chip removed from it.

The comparative observations of the grinding and superfinishing operations (both essentially abrasion processes) reveal to us a different "behavior" of the elements that act in the process, some of the differences arising from the interactions that the cutting liquid introduces as an active element of the machine technological system. tool-piece-cutting tool.
As a result, the "single grain" model in the case of grinding should be improved as follows:

1. Both the cutting area comprising the chip with the separation point, the deformation zone and the area that was processed are "submerged" in a continuous medium of cutting fluid, specific to the grinding.

2. If the cutting liquid is the emulsion, a dispersed system consisting of water and oil in a concentration of 3-5%, the drops of dispersed oil have the size of 0.1-1 μm can be assimilated with some balls.

3. A first role in the process is the oil droplets with surface-active properties that adhere to the solid metal and abrasive surfaces forming an adsorption layer with an active role in balancing the forces that are developed in the cut. By means of this liquid layer, the dry friction between the grains and the part is replaced with the wet friction, which means the significant reduction of the friction coefficients, the size of the frictional forces and implicitly the developed heat. The layer formed by the surface agent (oil) has elastic properties, so that when the abrasive granule is pressed into the work material, the cutting liquid acts as an elastic cushion, which opposes compression resistance, decreasing the penetration depth of the abrasive grain in the workpiece; the surface thus obtained will have a lower roughness.

4. The oil particles are deposited to the same extent on the surface of the chip removed from the part, take the chip into the mass of the cutting liquid; thus preventing the entry of the chip into the pores of the abrasive body, a situation found in the case of the
absence of the cutting fluid. The rapid removal of the chips from the processing area will be made by water, liquid present in the emulsion and which has a higher flow rate than the oil.

5. Grinding is a process that occurs with the release of a large amount of heat and therefore a liquid with maximum cooling capacity – water - is used as a cutting medium.

6. It should be noted that reducing friction by the presence of oil between the contact surfaces does not cancel the frictional forces.

3 Theoretical modeling of the roughness of the grinded surfaces

For the mathematical modeling of the roughness obtained from the processing by grinding, we consider the granular model with spherical tip (Fig.4). Generally unanimously accepted and used mainly when approaching theoretical researches in the processing with abrasive tools (grinding, superfinishing, honing, polishing, magnetoabrasive processing).

The abrasive grain, with the cone angle $\beta$ is considered to have the free height $h_g$, the embedding width $b$, the radius at the peak $r$ (obtained as a result of wear in the machining process). Let $L_g$ be the distance between the grains. Angle $\gamma$ and $\alpha$ represent the relief and rake angles $\gamma$ of the abrasive grain.

Fig.4 Spherical model of the grain for abrasive machining
According to the spherical model (Fig.4), by geometrical calculations, considering the radius at the tip of the grain \( r = 0 \), the following relation was reached:

\[
b = 4 \cdot s_g \cdot \tan \frac{\beta}{2} \cdot \sqrt{\frac{t}{D}},
\]

(1)

where \( s_g \) is the feed of the abrasive grain, \( t \) - the cutting depth, \( D \) - the diameter of the abrasive wheel.

Considering that the parameter \( b \) represents with good approximation the angular pitch of the edges of the abrasive grains and assimilating the processing of grinding with the one by cylindrical milling, we propose an empirical relation of calculation of the surface roughness in the form below:

\[
h = \frac{s_g^2 \cdot \delta}{4 \cdot D \cdot v_s},
\]

(2)

Where \( h \) is the height of the microirregularities, \( \delta \) - the angular pitch of the cutting edges [rad], \( s_g \) - workpiece feed [mm / s] - the feedrate, \( v_s \) - the angular velocity of the grinding wheel [m/min].

Based on the notations presented in Fig.3, considering that \( b \) represents the angular pitch, the following relation of calculation of the height of the microneregularities at grinding results:

\[
h = \frac{t \cdot s_g \cdot \tan \frac{\beta}{2}}{v_s \cdot D \cdot \sqrt{D}}
\]

(3)

If the following grinding dependencies are considered [2]:

\[
s_g = \frac{s_l}{N_g},
\]

(4)

where \( s_l \) is the longitudinal feed, \( N_g \) - the number of grains that cut during one rotation of grinding wheel, given by the relation (5):

\[
N_g = \frac{\pi \cdot D}{L_g} \cdot k_1,
\]

(5)

where \( k_1 \) there is a coefficient that takes into account the fact that only a certain number of grains effectively cut.

From the combination of relations (3) - (5), the following relation (6) results:

\[
h = \frac{t \cdot s_l \cdot L_g \cdot \tan \frac{\beta}{2}}{\pi \cdot k_1 \cdot D^2 \cdot \sqrt{D} \cdot v_s}
\]

(6)

The sliding condition of the grain on the processed surface is of the form [5]:

\[
\alpha \leq \arctg \mu,
\]

(7)

where \( \alpha \) is the relief angle of the abrasive grain, \( \mu \) - the friction angle on the relief face.

Based on Fig.4 it can be written:

\[
\gamma + \frac{\beta}{2} + 2 \cdot \alpha = \pi
\]

(8)

After a series of trigonometric calculations it follows that:

\[
tg \frac{\beta}{2} = ctg(\gamma + 2 \cdot \alpha)
\]

(9)

\[
\mu = tg \alpha
\]

(10)

Taking into account the condition (10), relation (6) becomes:
The distance between abrasive grains $L_g$ depends on the size of the abrasive grains of which the abrasive body is made and on its structure index.

For the evaluation of the theoretical distance between the abrasive grains the following are admitted:

1. The abrasive grains are of the same size, corresponding to granulation characteristic according to DIN ISO 6344;

2. The abrasive grains have a regular spherical shape (Model 1, Fig.5a) or a cubic shape (Model 2, Fig.5b);

3. The percentage of the total volume of the abrasive body occupied by the abrasive grains is given by the characteristic structure of the abrasive body.

4. An elementary volume of abrasive body consisting of an ordered network is considered: 8 abrasive granules (spherical or cubic) disposed in the 8 corners of a cube.

Let $L_g$ = the distance between the centers of two neighboring abrasive grains.

The two models show:

**Model 1**: abrasive grains are spherical in shape.

The volume of the abrasive body is calculated by the formula (12):

$$V_{Ca} = L^3 \quad (12)$$
The volume of abrasive grain (diameter $D_g$) contained in the elemental volume $V_{CA}$ is calculated by the formula (13):

$$V_{GA} = 8 \cdot \frac{1}{8} \cdot \frac{4 \cdot \pi}{3} \cdot D_g^3$$  \hspace{1cm} (13)

The ratio between the volume of the abrasive grains and the volume of the abrasive body represents the structure index of the respective abrasive body.

Let $i$ be the part of the volume of the abrasive body which is made of abrasive grains.

$$i = \frac{V_{GA}}{V_{CA}}$$  \hspace{1cm} (14)

Substituting in the expression (14) the relations (12) and (13) the following formula of the distance between the abrasive grains is obtained (15):

$$L = \sqrt[3]{\frac{\pi}{6 \cdot i}} \cdot D_g$$  \hspace{1cm} (15)

**Model 2**: the abrasive grains are cubic in shape.

The volume of the abrasive body is calculated by the formula (16):

$$V_{CA} = L^3$$  \hspace{1cm} (16)

The volume of abrasive granules (side "l") contained in the elemental volume $V_{CA}$ is calculated by the formula (17):

$$V_{GA} = 8 \cdot \frac{1}{8} \cdot l^3 = l^3$$  \hspace{1cm} (17)

It is considered that the side of the cube representing the theoretical grain is equal to the side of the screen through which the grains were selected when forming abrasive bodies (18).

$$l = D_g$$  \hspace{1cm} (18)

The ratio between the volume of the abrasive grains and the volume of the abrasive body represents the index structure of the abrasive body.

Let $i$ be the part of the volume of the abrasive body which is made of abrasive grains (19).

$$i = \frac{V_{GA}}{V_{CA}}$$  \hspace{1cm} (19)

Substituting in the expression (19) the relations (16) and (17) the following formula of the distance between the abrasive grains is obtained (20):

$$L = \sqrt[3]{\frac{l}{i}} \cdot l$$  \hspace{1cm} (20)

Substituting (18) into (20) we obtained:

$$L = \sqrt[3]{\frac{l}{i}} \cdot D_g$$  \hspace{1cm} (21)

Substituting in the relation (11) the expression of the theoretical distance between the abrasive grains, we obtain the relations (22) and (23) for the model of the cubic and spherical shape (22):
If the influence of the tip radius of the abrasive grain on the angular pitch \( b \) is also taken into account, it results on the basis of Fig.4 that:

\[
h = \frac{t \cdot s_i \cdot \sqrt[3]{\frac{\pi}{6 \cdot i} \cdot D_s}}{\pi \cdot k_1 \cdot D^2 \cdot \sqrt{D \cdot v_s}} \cdot \frac{2 \cdot \mu \cdot \left(1 - \mu^2\right) \cdot \text{ctg} \gamma - 1}{1 - \mu^2 + 2 \cdot \mu \cdot \text{ctg} \gamma}
\]

(22)

\[
h = \frac{t \cdot s_i \cdot \sqrt[3]{\frac{1}{i} \cdot D_s}}{\pi \cdot k_1 \cdot D^3 \cdot \sqrt{D \cdot v_s}} \cdot \frac{2 \cdot \mu \cdot \left(1 - \mu^2\right) \cdot \text{ctg} \gamma - 1}{1 - \mu^2 + 2 \cdot \mu \cdot \text{ctg} \gamma}
\]

(23)

If the influence of the tip radius of the abrasive grain on the angular pitch \( b \) is also taken into account, it results on the basis of Fig.4 that:

\[
b = 2 \cdot h_{g} \cdot \text{tg} \frac{\beta}{2} + \frac{r \cdot \left(1 - \sin \frac{\beta}{2}\right)}{\cos \frac{\beta}{2}}
\]

(24)

The relationship (2) finally becomes:

\[
h = \frac{s_i}{2 \cdot D \cdot v_s} \left[ h_{g} \cdot \text{tg} \frac{\beta}{2} + \frac{r \cdot \left(1 - \sin \frac{\beta}{2}\right)}{\cos \frac{\beta}{2}} \right]
\]

(25)

The value of the optimum friction coefficient, which ensures the minimum height of the microneregularities, is obtained by canceling the partial derivative of the relations (24) respectively (25) with respect to \( \mu \):

\[
\frac{\partial h}{\partial \mu} = 0 ,
\]

(26)

which leads to a series of calculations to solve the following equation:

\[
\mu^4 \cdot \text{ctg} \gamma - \mu^3 \cdot (\text{ctg} \gamma - \text{ctg}^2 \gamma) - \mu^2 \cdot (6 \cdot \text{ctg}^2 \gamma + 3 \cdot \text{ctg} \gamma - 2)
\]

\[
- 3 \cdot \mu \cdot \text{ctg} \gamma + \text{ctg} \gamma = 0
\]

(27)

If we consider the very small values, of the order of the hundreds of \( \mu \), the terms of the order 3 and 4 of the equation can be neglected and the equation becomes:

\[
(6 \cdot \text{ctg}^2 \gamma + 3 \cdot \text{ctg} \gamma - 2) \cdot \mu^2 + 3 \cdot \mu \cdot \text{ctg} \gamma - \text{ctg} \gamma = 0
\]

(28)

The convenient solution of the equation is:

\[
\mu_{\text{optim}} = -3 \cdot \text{ctg} \gamma + \sqrt{24 \cdot \text{ctg}^3 \gamma + 9 \cdot \text{ctg}^2 \gamma + 12 \cdot \text{ctg} \gamma - 8}
\]

(29)

4 Conclusions

1. Although there is a unanimity of the opinion of the specialists regarding the important role that the cutting fluid plays in the grinding manufacturing process, the old theoretical model of the grinding does not take it into consideration.

2 Starting from the composition of the grinding fluid (homogeneous mixture of water-soluble chemical fluids, water-soluble oils, synthetic oils, and petroleum-based oils) and from the surface active properties of the oil, it was considered that the oil droplets will form an adsorption layer on the surface of the intervening solid bodies (abrasive grain, workpiece, and chip removed), assimilable with a layer of small balls that prevents the direct contact
between the grains and the workpiece. Due to the bonding of the oil particles with the removed chip, it is "pulled" into the liquid and removed with it from the cutting area to avoid clogging of the abrasive body. For this thing the cutting fluid must have very good flow properties.

3. The role of cutting fluid the grinding process is to rapidly evacuate the heat from the cutting area (fluid with high cooling capacity), to quickly remove the formed chip (fluid with polar constituents) and to form a separation layer between the grain and workpiece, reducing the friction and the amount of heat produced.

4. The mathematical modeling of the surface roughness machined grinding on the basis of the spherical model is a rather complex problem when we want to understand the action of the parameters with major influence on the quality of the grinding surfaces. These parameters are: technological parameters, constructive parameters and the cutting fluid.

5. The influence of the cutting fluid on the surfaces quality through the friction coefficient \( \mu \) is of parabolic type, which requires the determination of the optimum value that makes the height of the geometric microasperities smaller possible.

6. Have been established the theoretical relationships of the distance between the abrasive grains according to the index structure of the abrasive body and the dimensions of the abrasive grains, assuming that all the abrasive grains have the same size and are located at the same distance. Two models were considered: in one the shape of abrasive granules were considered as spheres and in another cubes.

References:


