Effect of varying Number of Spacing between Antenna Elements, Snapshots, SNR on AP ML, AP-SSF and ESPRIT Algorithm

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Abstract: - The direction of arrival estimation is the main key problem in array signal processing. In this paper, the alternating projection maximum Likelihood (AP-ML), Alternating projection sub space framework (AP-SSF) and ESPRIT algorithm are studied. The simulation is performed in MATLAB for single and multiple sources. The effect of the varying number of spacing between antenna elements, number of snapshots and SNR are studied. The performance comparison shows that ESPRIT algorithm performs better as compared to the AP-ML and AP-SSF.

Key-Words: - AP-ML, AP-SSF, Direction of Arrival, ESPRIT, Snapshots, SNR.

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1 Introduction

The main aim of the array signal processing is to process the incoming signal from the different directions and increase the signal strength by removing the noise and interference signal by collecting the desired signal parameter information. The ML methods have elite yet computationally costly. The subspace techniques are performed well and having computationally effective variations. The old style strategies are basic yet offer humble or lackluster showing and furthermore require an enormous number of calculations. In this the three algorithms are studied and the performance comparison of the three is studied in details.

2. Alternating Projection Maximum Likelihood

The AP-ML is a iterative based techniques for reducing the maximization problem from a multidimensional to single dimensional problem. This may be achieved by maximization according to a single parameter keeping other parameters fixed. The maximum likelihood provides an optimum solution at an array of sensors for the direction finding of multiple signals. The computational complexity is high in maximum likelihood [2,4].

By assuming an array of sensors (p) having different locations and q be the uncorrelated narrowband signals that are impinging on the array from the directions $Q_1, Q_2, Q_3 ....Q_q$. the received signal at the antenna array is given by:

$$x(t) = \sum_{m=1}^{q} a(Q_m) S_m(t) + n(t) \qquad (1)$$

Where, $x(t)$ is a $p \times 1$ received signal vector, $n(t)$ is a $p \times 1$ white noise vector with zero mean and covariance matrix of $\sigma^2 I$. σ^2 and I be the unknown noise variance and identity matrix. $S_m(t)$ be the signal emitted by the m^{th} source and received at a reference point. $a(\theta_m)$ be the $p \times 1$ steering vector corresponding to the direction θ_m. The ML estimate is to solves the 1-D problem. The estimated angle is given by the following equation:

$$\theta_i^{\hat{k}+1} = \max_{\theta_i} \frac{a(\theta_i)^H_{A(\theta_i)k} Ra(\theta_i)_{A\theta_i^k}}{a(\theta_i)^H_{A\theta_i^k} a(\theta_i)_{A\theta_i^k}} \qquad (2)$$

where, $a(\theta_i)_{A(\theta_i^k)}$ is a $p \times 1$ vector.

Given by $a(\theta_i)_{A(\theta_i^k)} \equiv (I - PA_{\theta_i^k})a(\theta_i)$ (3)

Where, $PA_{\theta_i^k}$ is a projection operator on to the subspace spanned by the columns of $A(\hat{\theta}_i^{k})$ and H denotes the Hermitian Matrix.

3. Alternating Projection SSF (Subspace Fitting Framework)

This alternating projection is used to estimate the different parameters in subspace fitting framework and gives the relationship between different algorithms. This type of framework is generally used to designed numerical algorithms and used in obtaining new methods. The extension of the ESPRIT algorithm is designed from the SSF method. The subspace fitting approach was firstly presented by [2] and it was formalized by [3] which is written as,

$$\left\lfloor \hat{A}, \hat{T} \right\rfloor = \arg\min_{A,T} \left\| M - A(\eta)T \right\|_F^2 \quad (4)$$

In this equation, M is a $m \times q$ matrix that may be obtained from the given data. The $m \times p$ matrix A is calculated from η and T is the $p \times q$ matrix. The parameter estimation vector η is the argument of $\hat{A}$. The matrix M and dimension of it can be chosen in the different ways to give different estimation. The fitting problem is separated with the help of A and T [Capon]. Now putting $\hat{A} = A * M$ in to the above equation, we may get the new equation,

$$\hat{A} = \arg\max_{A} T_r \left\{ P_A M M^* \right\} \quad (5)$$

Where, $P_A = A(A * A)^{-1} A *$ is the projection matrix which is used to projects on the column space of A. Different algorithms and methods are explore like Deterministic Maximum Likelihood (DML), Beamforming, MUSIC, Multi dimensional MUSIC(MD-MUSIC), ESPRIT are proposed based on subspace fitting framework.

Alternating projection is used to transform the multivariate non-linear maximization problem in to a sequence of 1-d maximization. [6] introduced a new algorithm by maximizing the likelihood function given below:

$$L = -KdLog\sigma^2 - \frac{1}{\sigma^2} \sum_{l=0}^{k-1} \left| x_i(l) - A(\theta)S_i \right|^2 \quad (6)$$

This method is known as alternating projection method. The flow chart and basic step follows in this method are given below. In this, firstly one dimensional projection is find out that is further used to maximize L.

4. ESPRIT (Estimation of Signal Parameters Via Rotational Invariance Techniques)

[Roy and Kailath] [1] proposed a new algorithm named ESPRIT for DOA estimation. Array doublets are formed by N/2 pairs which further form a displacement vector. The starting two elements of the doublet are separated and grouped to make two N/2 sub arrays. The vectors x and y are the data vectors corresponding to each of the sub arrays. The output of the sub arrays z and y can be expressed as:

$$x_k[n] = \sum_{i=0}^{r-1} s_i[n] a_k(\theta_i) + v_k^{(x)}[n], \quad (7)$$

$$y_k[n] = \sum_{i=1}^{r-1} s_i[n] e^{j2\pi\delta \sin\theta_k} a_k(\theta_i) + v_k^{(y)}[n], \quad (8)$$

Where similar notation has been used and δ is displacement magnitude in wavelengths. The estimated angle by ESPRIT algorithm relative to the displacement vector. The sub arrays, x and y, output is given in matrix form is:

$$x_n = As_n + v_n^x \quad (9)$$

$$y_n = A\phi s_n + v_n^y \quad (10)$$

The matrix ϕ is a diagonal $r \times r$, matrix having diagonal elements are $\{\exp(j2\pi\delta \sin\theta_0), \exp(j2\pi\delta \sin\theta_1), ..., \exp(j2\pi\delta \sin\theta_{r-1})\}$. The phase delay may be represented by the complex exponentials between the r signals and doublet pair. The data vectors may be concatenated from sub arrays to make a single 2N-2 data vector, like,

$$z_n = \left\lfloor \begin{matrix} x_n \\ y_n \end{matrix} \right\rfloor = A_b S_n \quad (11)$$

$$A_b = \begin{bmatrix} A \\ A\phi \end{bmatrix}, V_n = \begin{bmatrix} v_n^{(x)} \\ v_n^{(y)} \end{bmatrix} \tag{12}$$

The columns of *A*A_b occupy the signal subspace of the new array. Let V_s be the column matrix depending upon the signal subspace as Z_n, A_b and V_s are related with $r \times r$ transformation T is written as:

$$V_s = A_b T, \tag{13}$$

and can portioned as follows:

$$V_s = \begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} AT \\ A\phi T \end{bmatrix} \tag{14}$$

From this step, the range of E_x, E_y and A will be equal as E_x, E_y have the same range, the rank r matrix E_{xy} as follows:

$$E_{xy} = [E_x \quad E_y] \tag{15}$$

To find $r \times 2r$ rank r matrix having null space of E_{xy} to form matrix F, and is written as:

$$[E_x \quad E_y]F = E_x F_x + E_y F_y = ATF_x + A\phi TF_y \tag{16}$$

Assume ψ is:

$$\psi = -F_x [F_y]^{-1} \tag{17}$$

Reshuffling the above equations gives:

$$E_x \psi = E_y \tag{18}$$

Now by substituting we get the results:

$$AT\psi = A\phi T \Rightarrow AT\psi T^{-1} = A\phi \Rightarrow T\psi T^{-1} = \phi \tag{19}$$

The given equations means that the Eigen values of ψ is same as diagonal elements of ϕ. Once the Eigen values, λ, of ϕ have been calculated, the angle of arrival is calculated as:

$$\lambda_k = e^{j2\pi\delta \sin\theta_k} \tag{20}$$

$$\theta_k = \arcsin\left(\frac{\arg(\lambda_k)}{2\pi\delta}\right) \tag{21}$$

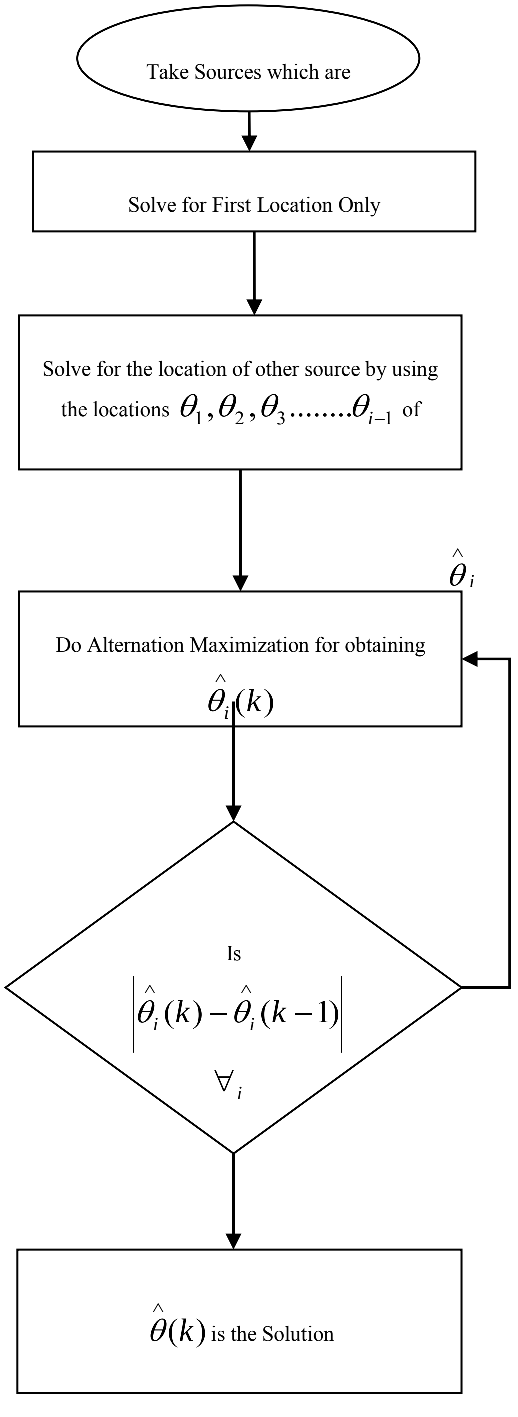

Figure 1: Flow chart of AP-SSF algorithm

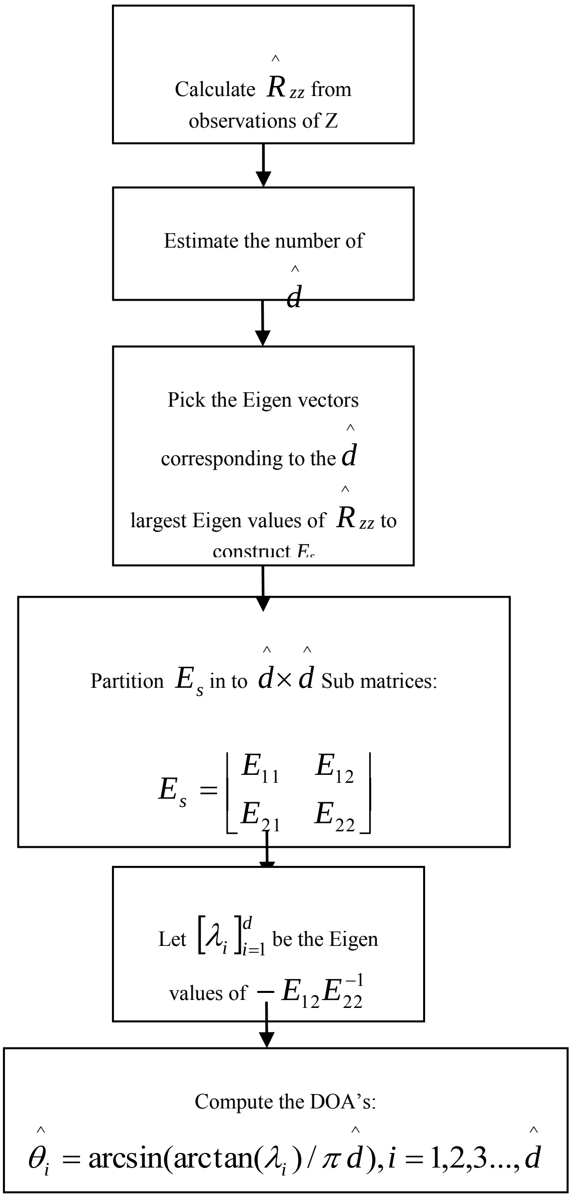

Figure 2: ESPRIT Algorithm Flow Chart

If A be the full rank matrix, then the Eigen values of the matrix ψ are the diagonal elements of ϕ and the Eigen vectors of ψ are the columns of the T. in practical, the signal subspace is not known exactly, the only estimate is from sample covariance matrix R_{xx} or from a sub space tracking algorithm. There for, $E_x\psi = E_y$, will not be exactly satisfied and we will have to resort to a least square solution to computes ψ. The least square process assumes that the columns in E_x are known exactly whereas the data in E_y is noisy. If the assumptions is made that E_x and E_y are equally noisy, the total least square criteria is used to solve, which gives better results.

The above algorithms are studied in detailed and the comparison is given here for one, two, three, four and five signals.

Table 1: Estimated Signal Directions at receiving antenna array when number of elements = 10, number of samples = 100 and SNR = 20db

Algorithm	**Signal Directions** $(40°)$
Alternating Projection ML	40.0120
Alternating Projection SSF	16.0040
ESPRIT	40.0177

The above table gives the signal estimation for single source that is coming from the direction 40° and the three algorithms the signal directions. The signal estimated from the AP-SSF gives wrong signal estimation while as comparison to the AL-ML, the ESPRIT algorithm gives better accuracy.

Table 2: Estimated Signal Directions at receiving antenna array when number of elements = 10, number of samples = 100 and SNR = 20db

Algorithm	**Signal Directions** $(-10°)$	**Signal Directions** $(-20°)$
Alternating Projection ML	-9.8600	-19.9680
Alternating Projection SSF	-9.9480	-19.912
ESPRIT	-10.0312	-19.9641

The above table gives the description of the estimated signal when two signals from the directions -10° and -20° are coming at the receiver side and the number of antenna elements are take as 10, number of samples are 100 and SNR is 20dB.

Table 3: Estimated Signal Directions at receiving antenna array when number of elements=10, number of samples=100 and SNR=20db

Algorithm	**Signal Directions** (-10°)	**Signal Directions** (-20°)	**Signal Directions** (-30°)
Alternating Projection ML	-3.3820	-26.0600	-30.0680
Alternating Projection SSF	-7.0800	-21.4840	-30.4840
ESPRIT	-10.0677	-20.4922	-30.1516

The above table gives the description of the estimated signal when three signals from the directions -10°, -20° and -30° are coming at the receiver side and the number of antenna elements are take as 10, number of samples are 100 and SNR is 20dB.

Table 4: Estimated Signal Directions at receiving antenna array when number of elements = 10, number of samples = 100 and SNR = 20db

Algorithm	**Signal Directions** (40°)	**Signal Directions** (50°)	**Signal Directions** (70°)
Alternating Projection ML	40.0120	50.0080	70.0040
Alternating Projection SSF	16.0040	66.0040	70.0040
ESPRIT	40.0177	50.0052	69.9985

The above table gives the Discription of the estimated signal when three signals from the directions 40°, 50° and 70° are coming at the receiver side and the number of antenna elements are take as 10, number of samples are 100 and SNR is 20dB.

Table 5: Estimated Signal Directions at receiving antenna array when number of elements = 10, number of samples = 64 and SNR = 20db

Algorithm	**Signal Directions** (60°)	**Signal Directions** (80°)	**Signal Directions** (90°)	**Signal Directions** (100°)
Alternating Projection ML	54.0120	75.9960	92.9920	99.9920
Alternating Projection SSF	60.0040	76.0000	87.0000	100.0080
ESPRIT	60.0131	80.0166	89.9850	100.0023

In the above table, the simulations are performed for the four signals when the number of array elements at the receiving antenna is considered 10, the samples taken at the receiving antenna are 64 and the signal to noise ratio at is 20dB.

Table 6: Estimated Signal Directions at receiving antenna array when number of elements=10, number of samples=100 and SNR=20db

Algorithm	**Signal Directions** (60°)	**Signal Directions** (80°)	**Signal Directions** (90°)	**Signal Directions** (100°)
Alternating Projection ML	54.0040	77.9800	94.0200	99.9920
Alternating Projection SSF	60.0080	77.9920	89.0080	99.9880
ESPRIT	60.0067	79.9891	90.0006	99.9832

In the above table, the simulations are performed for the four signals when the number of array elements at the receiving antenna is considered 10, the samples taken at the receiving antenna are 100 and the signal to noise ratio at is 20dB. When we increase the number of snapshots, the accuracy of the algorithms is increased and the incoming signal direction is measured more accurate.

Table 7: Estimated Signal Directions at receiving antenna array when number of elements=10, number of samples=100 and SNR=20db

Algorithm	**Signal Directions** (40°)	**Signal Directions** (50°)	**Signal Directions** (70°)	**Signal Directions** (90°)	**Signal Directions** (120°)
Alternating Projection ML	36.0240	42.0120	77.3600	104.9920	120.2400
Alternating Projection SSF	39.9920	47.9960	64.0200	85.9880	119.9960
ESPRIT	40.0224	49.9976	70.0088	89.9977	120.0021

In the above table, the simulations are performed for the five signals when the number of array elements at the receiving antenna is considered 10, the samples taken at the receiving antenna are 100 and the signal to noise ratio at is 20dB.

Table 8: Estimated Signal Directions at receiving antenna array when number of elements=100, number of samples=100 and SNR=20db

Algorithm	**Signal Directions** (60°)	**Signal Directions** (80°)	**Signal Directions** (90°)	**Signal Directions** (100°)
Alternating Projection ML	54.0000	80.0000	100.0000	100.0160
Alternating Projection SSF	54.0000	54.0000	64.0000	100.0000
ESPRIT	60.0009	80.0007	89.9998	99.9988

In the above table, the simulations are performed for the four signals when the number of array elements at the receiving antenna is considered 100, the samples taken at the receiving antenna are 100 and the signal to noise ratio at is 20dB.

Table 9: Estimated Signal Directions at receiving antenna array when number of elements=5, number of samples=100 and SNR=20db

Algorithm	**Signal Directions** (60°)	**Signal Directions** (80°)	**Signal Directions** (90°)	**Signal Directions** (100°)
Alternating Projection ML	23.9640	33.9960	95.5400	99.9640
Alternating Projection SSF	60.0320	73.9520	90.6720	99.8840
ESPRIT	60.0550	79.7319	90.0373	100.0341

In the above table, the simulations are performed for the four signals when the number of array elements at the receiving antenna is considered 5, the samples taken at the receiving antenna are 100 and the signal to noise ratio at is 20dB.

5. Effect of increasing the number of elements, number of samples and SNR at antenna array

We studied the detailed comparison of the three algorithms when we increase the number of antenna elements at the receiver array, number of samples taken at the receiver side for signal estimation and by increasing the signal to noise ratio at the receiver side. The signals taken are two here form 45° and 60° directions respectively. The estimation through ESPRIT algorithm gives good results in all conditions when we increase the antenna elements, spacing between the antenna elements and SNR level.

Table 10: Estimated Signal Directions at receiving antenna array when numbers of elements are increasing keeping other parameters fixed.

Algorithm	**Number of Elements=5** **Number of Samples =100** **SNR=20 dB**		**Number of Elements=10** **Number of Samples =100** **SNR=20 dB**		**Number of Elements=20** **Number of Samples =100** **SNR=20 dB**		**Number of Elements=100** **Number of Samples =100** **SNR=20 dB**	
	Signal Directions (45°)	**Signal Directions (60°)**	**Signal Directions (45°)**	**Signal Directions (60°)**	**Signal Directions (45°)**	**Signal Directions (60°)**	**Signal Directions (45°)**	**Signal Directions (60°)**
Alternating Projection ML	45.0600	60.0360	45.0120	59.9960	45.0000	60.0000	45.0000	60.0000
Alternating Projection SSF	45.0520	60.0320	45.0120	59.9960	45.0000	60.0000	45.0000	60.0000
ESPRIT	45.0626	60.0099	45.0177	59.9968	45.0000	60.0000	45.0000	60.0000

In the above table, the simulations are performed for the two signals incoming from the direction 45° and 60° when the number of array elements at the receiving antenna is increasing from 5, 10, 20, 100, the samples taken at the receiving antenna are 100 and the signal to noise ratio at is 20dB. From the above table it is clear that when we increase the number of samples the accuracy of the estimation is increased and the ESPRIT algorithm gives better results.

Table 11: Estimated Signal Directions at receiving antenna array when number of samples is increasing keeping other parameters fixed.

Algorithm	**Number of Elements=10** **Number of Samples =1** **SNR=20 dB**		**Number of Elements=10** **Number of Samples =10** **SNR=20 dB**		**Number of Elements=10** **Number of Samples =20** **SNR=20 dB**		**Number of Elements=10** **Number of Samples =100** **SNR=20 dB**	
	Signal Directions (45°)	**Signal Directions (60°)**	**Signal Directions (45°)**	**Signal Directions (60°)**	**Signal Directions (45°)**	**Signal Directions (60°)**	**Signal Directions (45°)**	**Signal Directions (60°)**
Alternating Projection ML	45.0480	59.9240	44.9840	60.0160	45.0000	60.0080	45.0120	59.9960
Alternating Projection SS	46.2280	65.1680	44.9760	60.0160	45.0000	60.0120	45.0120	59.9960

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ESPRIT	49.3730	99.2465	44.9512	60.0240	44.9957	60.0201	45.0000	60.0000

In the above table, the simulations are performed for the two signals incoming from the direction 45° and 60° when the number of samples taken at the receiver side is increasing from 1, 10, 20, 100, the number of antenna elements are fixed as 10 and the signal to noise ratio at is 20dB. From the above table it is clear that when we increase the number of antenna elements the accuracy of the estimation is increased.

Table 12: Estimated Signal Directions at receiving antenna array when SNR are increasing keeping other parameters fixed.

Algorithm	**Number of Elements =10** **Number of Samples= 100** **SNR=1dB**		**Number of Elements =10** **Number of Samples= 100** **SNR=10dB**		**Number of Elements =10** **Number of Samples= 100** **SNR=20dB**		**Number of Elements =10** **Number of Samples= 100** **SNR=100 dB**	
	Signal Directions (45°)	**Signal Directions (60°)**	**Signal Directions (45°)**	**Signal Directions (60°)**	**Signal Directions (45°)**	**Signal Directions (60°)**	**Signal Directions (45°)**	**Signal Directions (60°)**
Alternating Projection ML	45.1720	59.9760	45.0440	59.9920	45.0120	59.9960	45.0000	60.0000
Alternating Projection SSF	45.1680	59.9720	45.0480	59.9840	45.0120	59.9960	45.0000	60.0000
ESPRIT	45.1246	59.9748	45.0526	59.9940	45.0000	60.0000	45.0000	60.0000

In the above table, the simulations are performed for the two signals incoming from the direction 45° and 60° when the SNR at the receiver side is increasing from 1, 10, 20, 100, the number of antenna elements are fixed as 10 and the samples taken are 100. When the SNR level is increased the estimation accuracy is decreased.

Conclusion: The simulation is performed in the MATLAB environment and the results obtained at different conditions. The performance comparison shows that ESPRIT algorithm performs better as compared to the AP-ML and AP-SSF.

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