Approximation of Minimum Initial Capital of the Discrete Time Surplus Process using Separated Claim Technique for Motor Insurance

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Abstract: - In this paper, we compute what minimum initial capital an insurance company has to hold to ensure that the ruin probability (insolvency probability) is not greater than the given quantity of the discrete time surplus process using separating claim technique. 365 claims of motor insurance are separated into standard and large claims. The criteria of separation is at the 80th percentile; if claim is less than or equal the 80th percentile, it is called standard claim and claim is called large claim if it is greater than the 80th percentile. We also perform some simulations to estimate the ruin probability as well as calculate the minimum initial capital using regression analysis.

Key-Words: - ruin probability, discrete time surplus process, minimum initial capital.

1 Introduction

The basic knowledge of a surplus process of non-life insurance can be defined as

Surplus = Initial capital + Income – Outflow.

For the classical surplus process, we consider claim Y_n occurred and the time T_n such that $0 \le T_1 \le T_2 \le \dots$. The discrete time surplus process is defined by

$$U_n = u + cT_n - \sum_{k=1}^{n} Y_k, \quad U_0 = u, \; n \in \mathbb{N} \qquad (1)$$

where $u \ge 0$ is initial capital, $c > 0$ is premium rate.

Nyrhinen (2001), Cai and Dickson (2004), and Gao, et al. studied the discrete time surplus process in cooperating a constant interest rate r in order to obtain the upper bound of the ruin probabilities by constructing the corresponding Lundberg's bounds for the ruin probabilities. The surplus process is of the form

$$U_n = U_{n-1}(1+r) - Y_n, \quad U_0 = u, \; n \in \mathbb{N} \qquad (2)$$

where r is a constant interest rate.

Chan and Zhang (2006) studied the discrete time surplus process (1) under the assumption that $T_n = n, n \in \mathbb{N}$ and a set of claims $\{Y_n, n \in \mathbb{N}\}$ is independent and identically distributed (i.i.d.). They proposed the recursive and explicit formulas of ruin probability with only exponential and geometric claim. Sattayatham et al. (2013) generalized the model of Chan and Zhang that $\{Y_n, n \in \mathbb{N}\}$ is i.i.d. but not necessary a set of exponential or geometric claims. Since the formula of ruin probability was difficult to find explicitly, they proposed the ruin probability in the recursive form and applied Newton-Raphson method to compute the minimum initial capital.

We observe that model of Chan and Zhang did not include the interest rate r in the model and according to the model (2) of Nyrhinen, Cai and Dickson that they were not able to express the ruin probabilities explicitly. In this paper, we want to include interest rate r in our model and then find the ruin probabilities by simulations. Moreover, we observe that, in general, initial capitals and ruin probabilities have nonlinear relationship between each other. In this paper, we investigate a linear relationship between the two variables by separate the claims into two categories; standard claims and large claims. The criteria used to separate the claims is at 80th percentile; if claim is less than or equal 80th percentile, we call standard claims, V_n and we call large claims, W_n, if it is greater than 80th percentile. The model is constructed under the condition that the standard claim and large claim do not occur at the same time. We assume that an insurance company is allowed to invest in a risk-free asset with a constant interest rate r. Let $\{T_n^L, n \in N\}$ be an arrival time process of large claims. Let $Z_n^L = T_n^L - T_{n-1}^L,\ n \in N,$ the inter arrival time process $\{Z_n^L, n \in N\}$ of the arrival time process $\{T_n^L, n \in N\}$ is assumed to be i.i.d such that $Z_1^L \sim \text{Poisson}(\lambda^L)$. Let premium rate

$$c = (1+\theta)\left(\frac{EW_1}{EZ_1^L} + EV_1\right),$$

where θ is a safety loading parameter. The discrete time surplus process in this paper is given by

$$U_n = \begin{cases} U_{n-1}(1+r_0) + c - V_n,\ n \neq T_k^L, \\ \quad \text{for all } k = 1,2,3,\cdots, \\ U_{n-1}(1+r_0) + c - W_n,\ n = T_k^L, \\ \quad \text{for some } k = 1,2,3,\cdots, \end{cases} \qquad (3)$$
$$U_0 = u,\ n \in \mathbb{N},$$

where $\{V_n, n \in \mathbb{N}\}$ and $\{W_n, n \in \mathbb{N}\}$ are assumed to be i.i.d., r_0 is a daily interest rate which is given by $r_0 = (1+r)^{1/365} - 1$ and r is compound interest rate with $r = 2\%$ per annum.

2 Methodology

2.1 Data

We use 365 motor insurance claims from some insurance company in Thailand (see Figure 1). According to the data, we then find the 80th percentile, $P_{80} = 143618$, such that the set of standard claims has 292 claims. The set of large claims has 73 claims. The plots of standard and large claims are shown in Figure 2 and Figure 3.

2.2 Parameter Estimation

By the chi-squared goodness of fit test, we obtain that standard claims accept Weibull with parameters $\alpha = 1.6215, \beta = 64262$ whereas large claims accept Pareto with parameters $\alpha = 1.8312, \beta = 143780$ at 95% confidence. We assume $Z_1^L \sim \text{Poisson}(\lambda^L)$ and computed $\lambda^L = 5$, i.e., the expected value of large claim happens once per 5 days.

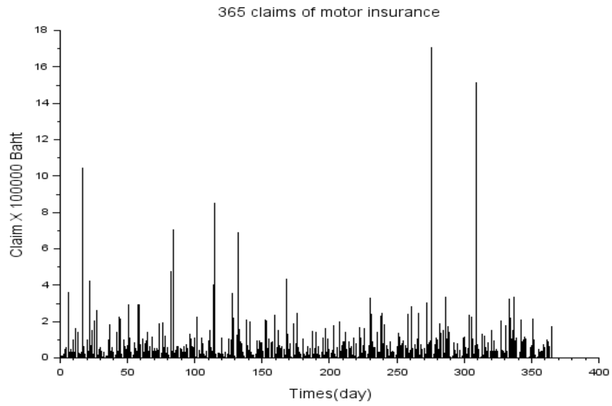

Fig. 1. The 365 Claims of motor insurance

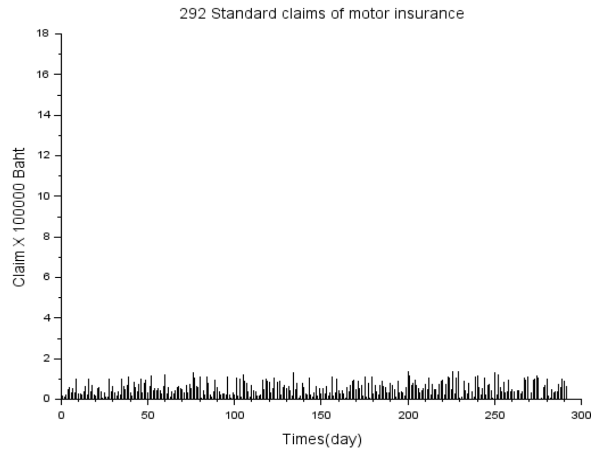

Fig. 2. The 292 Standard claims of motor insurance

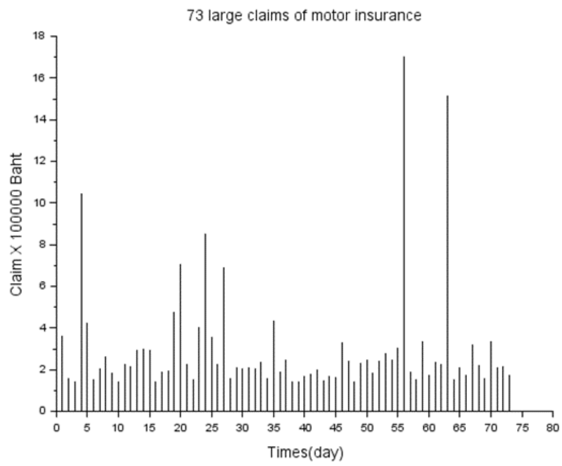

Fig. 3. The 73 claims of motor insurance

2.3 Simulation Results

In this part, we perform simulations using equation in (3) to obtain $\Phi_n(u)$, where

$$\Phi_n(u) = P\big(\{\, U_i < 0 \text{ for some } i \in \{1,2,\ldots,n\} | U_0 = u\big).$$

The flowchart showing simulation process of ruin probabilities is shown in the Figure 4. The initial capital is set as 0, 20,000, 40,000, …, 1,000,000 Baht for 10,000 simulations.

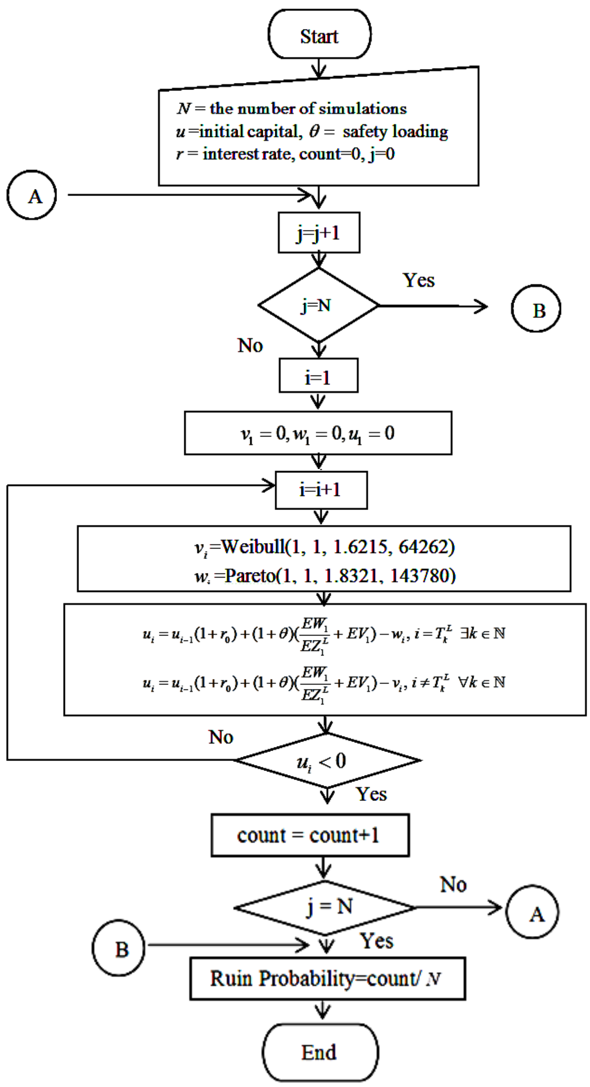

Fig. 4. Flowchart of the simulation process of the ruin probabilities.

3 Results

The simulation results of the discrete time surplus process U_n are shown in Figure 5 that shows relationship between the ruin probability and initial capital. The top curve (blue color) is plotted based on parameter $\theta =$ 0.1, the second curve (green color) is for the case where $\theta = 0.2$, and so on, the bottom curve (red color) is plotted with $\theta = 1.0$.

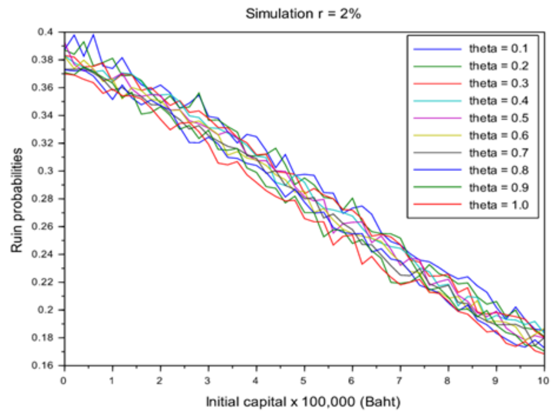

Fig. 5. Simulation results

3.1 Regression analysis and minimum initial capital

From Figure 5, we can consider the relationship between ruin probability $\Phi(u,365) := y$ and initial capital u as a linear function,

$$y = mu + b.$$

Now we know initial capital u and the ruin probability y. By applying the least squares linear regression method, we obtain the approximated m and b as the following:

$$m = \frac{n\sum_{i=1}^{n} u_i y_i - \left(\sum_{i=1}^{n} u_i\right)\left(\sum_{i=1}^{n} y_i\right)}{n\sum_{i=1}^{n} u_i^2 - \left(\sum_{i=1}^{n} u_i\right)^2}$$

$$b = \frac{\left(\sum_{i=1}^{n} u_i^2\right)\left(\sum_{i=1}^{n} y_i\right) - \left(\sum_{i=1}^{n} u_i\right)\left(\sum_{i=1}^{n} u_i y_i\right)}{n\sum_{i=1}^{n} u_i^2 - \left(\sum_{i=1}^{n} u_i\right)^2}$$

where u_i is initial capital and y_i is the ruin probability at initial capital u_i. The results is shown in Figure 6.

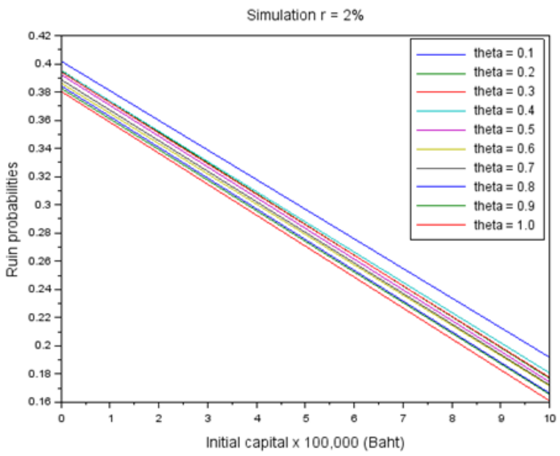

Fig. 6. The relationship between ruin probability and initial capital

Finally, if the ruin probability cannot exceed α (given quantity), i.e.,

$$\Phi(u,365) \le \alpha,$$

or

$$mu + b \le \alpha.$$

Now m is the negative number, thus

$$u \ge \frac{\alpha - b}{m}.$$

Therefore, the required minimum initial capital (MIC) is

$$\text{MIC} = \frac{\alpha - b}{m}.$$

Minimum initial capital (MIC) as shown in Table 1 in the case where we set the interest rate $r = 2\%$, $\alpha = 0.01$ and premium rate $c = (1+\theta)(\frac{EW_1}{EZ_1^L} + EV_1)$ where

$$\frac{EW_1}{EZ_1^L} + EV_1 = 120861.39.$$

Table 1 MIC with $r_0 = 2\%,\ \alpha = 0.01$

Safety loading θ	MIC (Baht)
0.1	1865766
0.2	1796507
0.3	1773452
0.4	1764155
0.5	1753505
0.6	1751932
0.7	1749046
0.8	1717627
0.9	1716763
1.0	1687530

4 Conclusion

From Table 1 if an insurance company carries the ruin probability less than 0.01, the company has to hold the minimum initial capital 1,865,766 Baht under safety loading 0.1. If an insurance company carries the ruin probability is less than 0.01, the company has to hold the minimum initial capital 1,687,530 Baht under safety loading 1. We can see that the safety loading is increasing while the minimum initial capital is decreasing. Moreover, we can see from Figure 6 and Table 1 that, by separating the claims into 2 parts, the range of MIC is not that large, in other words MIC has small decreasing with θ being increasing. This is good for an insurance company in terms of financial management.

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