

Measuring Durability for Insurance Company Based on Probability of Ruin

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Abstract: In this paper, we present the process of the measuring durability of insurance company, in which, this study focus on the discrete-time under the limited time the company must reserve sufficient initial capital to ensure that probability of ruin does not exceed the given quantity of risk. Therefore the illustration of the minimum initial capital under the specified period for the claim size process to the exponential distribution and the weibull distribution has explained.

Key-Words: Surplus process, Measuring Durability, Minimum Initial Capital, Probability of Ruin.

1 Introduction

According to the economic and world situation, business companies have been fluctuating. Consequently, business companies will face loss at a high risk if they do not pay attention on it. Currently, businesses are now considering insurance a help which tends to be a very popular investment for business company owners. On the other hand, in the insurance business, the initial capital, claim severities and premium rate controlled the fluctuation of surplus process:

$$\text{Surplus} = \text{Initial surplus} + \text{Inflow} - \text{Outflow}.$$

In this paper, we denote the probability space (Ω, F, P) . Let U_n be the surplus process for the claim happen at time $n \in \mathbb{N}$. Consequently, the surplus process at time n as the following expression,

$$U_n = u + c_0 n - \sum_{k=1}^n Y_k \quad ; \quad U_0 = u, \quad (1)$$

where $u \geq 0$ the initial capital, $c_0 \geq 0$ is the premium rate for one time unit and the claim size process $\{Y_n; n \in \mathbb{N}\}$ is the set of independent and identically distributed (i.i.d) random variables. The probability of ruin of one unit time is defined by

$$\Phi_n(u) = P(U_k < 0 \text{ for some } k = 1, 2, 3, \dots | U_0 = u) \quad (2)$$

Thus, the probability of survival for the time $n \in \mathbb{N}$ is defined by

$$\varphi_n(u) = P(U_k \geq 0 \text{ for all } n = 1, 2, 3, \dots | U_0 = u) \quad (3)$$

Many researchers are interested to study the surplus process in model (1). The general approach for studying the probability of ruin in the discrete-time surplus process is called *Gerber-Shiu discounted penalty function*; For instances, Pavlova and Willmot [7], Li [3,4] and Dickson [1]. They studied the probability of ruin as a function of the initial capital $u \geq 0$. Moreover, Chan and Zhang [5] considered in a case of discrete time surplus process obtaining the closed form for the probability of ruin with exponential claim distribution. Sattayatham et al.[6] introduced the closed form of the probability of ruin, and studied the minimum initial capital problem controlled the probability of ruin so that it was not greater than a given quantity of risk. Klongdee et al. [8] studied the model (1) for the minimum initial capital under α -regulation of the discrete-time risk process in the case of motor insurance which separated two claim severities and calculated the regression analysis. In this paper, we focus on the discrete-time under the limited time of the company which must reserve sufficient initial capital to ensure that the probability of ruin does not exceed the given quantity of risk by considering the relationship of the ruin probability, safety loading and initial capital.

2 Materials and Methods

In this section, the premium rate c_0 of the surplus process (1) can calculate by the expected value principle as the following expression,

$$c_0 = (1 + \theta)E[Y],$$

where $\theta > 0$ is the safety loadings of the insurer. The survival duration D can be calculated by the surplus process for the claim happen at time n as the following expression,

$$D = \min\{n|U_n \geq 0 \text{ and } U_{n+1} < 0\},$$

for $n \in \mathbb{N}$, and set $P(D = n)$ is the probability of survival duration at the time n . Next, we investigate the probability of the survival duration greater than the specified period L at the time n , denoted by $P(D > L)$. After that, we study the discrete-time surplus process under the regulation that the insurance company has to reserve sufficient initial capital of L to ensure that the probability of ruin does not exceed the quantity α . Consequently, the minimum initial capital corresponding to $(u, c_0, L, \alpha, \{Y_n; n \in \mathbb{N}\})$ and can be calculated as the following expression,

$$\text{MID}_\alpha(u, c_0, L) = \min\{u|P(D > L) \geq \alpha\}. \quad (4)$$

In this paper, we consider the appropriated and edited the data of the claim severities of motor insurance in Thailand provided by an insurance company. We assume that the claim size has exponential distribution and weibull distribution. The claim sizes data consist of 103 days as shown in Fig.1.

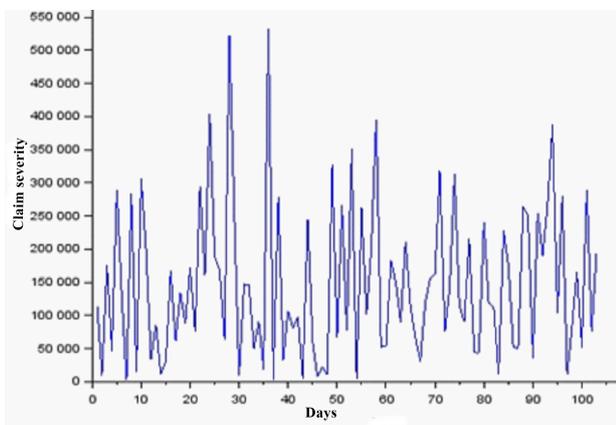


Figure 1: The claim sizes data consist of 103 days.

Next, we introduce the probability density function and the cumulative distribution function of these distributions which are described as shown below

The exponential distribution

A random variable Y is said to have exponential distribution has a positive scale parameter λ , denoted by $Y \sim \text{Exp}(\lambda)$, if its probability density function is

$$f_X(x; \lambda) = \frac{1}{\lambda}e^{-\frac{x}{\lambda}}, \quad x \geq 0.$$

The cumulative distribution function is of the form

$$F_X(x; \lambda) = 1 - e^{-\frac{x}{\lambda}}, \quad x \geq 0.$$

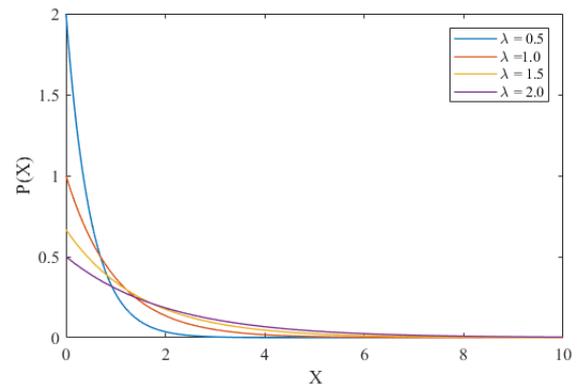


Figure 2: The probability density function of the exponential distribution.

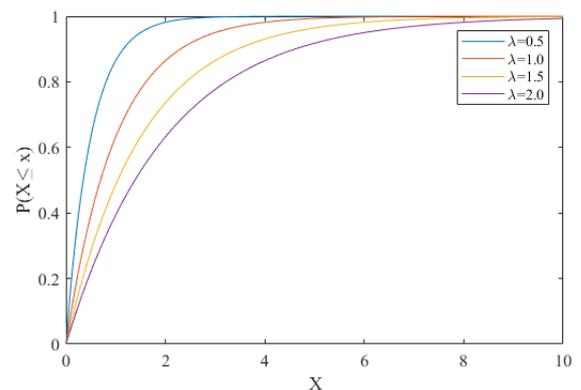


Figure 3: The cumulative distribution function of the exponential distribution.

The weibull distribution

A random variable Y is said to have weibull distribution has a positive real scale parameter λ and a positive real shape parameter α , if its probability density function is

$$f_X(x; \alpha; \lambda) = \frac{\alpha x^{\alpha-1}}{\lambda^\alpha}e^{-\left(\frac{x}{\lambda}\right)^\alpha}, \quad x \geq 0.$$

The cumulative distribution function is of the form

$$F_X(x; \alpha; \lambda) = 1 - e^{-\left(\frac{x}{\lambda}\right)^\alpha}, \quad x \geq 0.$$

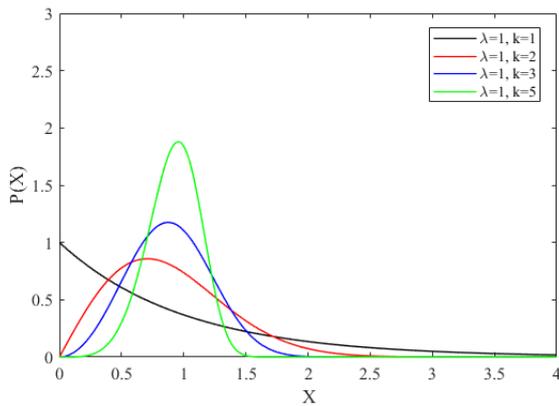


Figure 4: The probability density function of the exponential distribution.

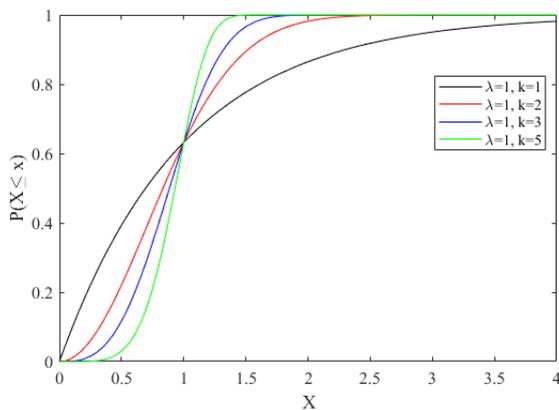


Figure 5: The cumulative distribution function of the exponential distribution.

Maximum Likelihood Estimator

Let X be a population random variable with a parameter θ . Assuming X is continuous, the density function as $f_X(x; \theta)$ in order to emphasize the dependency on θ as well as X . The likelihood function is given by

$$L(x; \theta) = L(\theta; x_1, x_2, \dots, x_n) = \prod_{i=1}^n f_X(x; \theta). \quad (5)$$

Taking logarithm in (5), $\ln L(x; \theta)$, is called log-likelihood function that solved by taking derivative of $\ln L(x; \theta)$ with respect to the parameter θ , and setting it equal to zero in order to solve for θ . The value of θ that obtained from maximizing the function $\ln L(x; \theta)$ is exactly that the same value of θ , is called maximum likelihood estimator (MLE).

Next, we approximate the minimum initial capital under the specified period L based on the probability of ruin as mentioned in (4). Firstly, we shall approximate its parameter by using the maximum likelihood

estimation (MLE). We obtain an appropriate value of the parameter of the dataset as

Table 1: Parameters of distributions of experiment

Distributions	Parameters	
	Scale	Shape
Exponential	6.805×10^5	-
Weibull	1.5642×10^5	1.0084

For the second experiment, we perform the simulation for computing the minimum initial capital under the specified period L as mentioned in (4). The simulation method can proceed as the following flowchart in Fig. 6-7

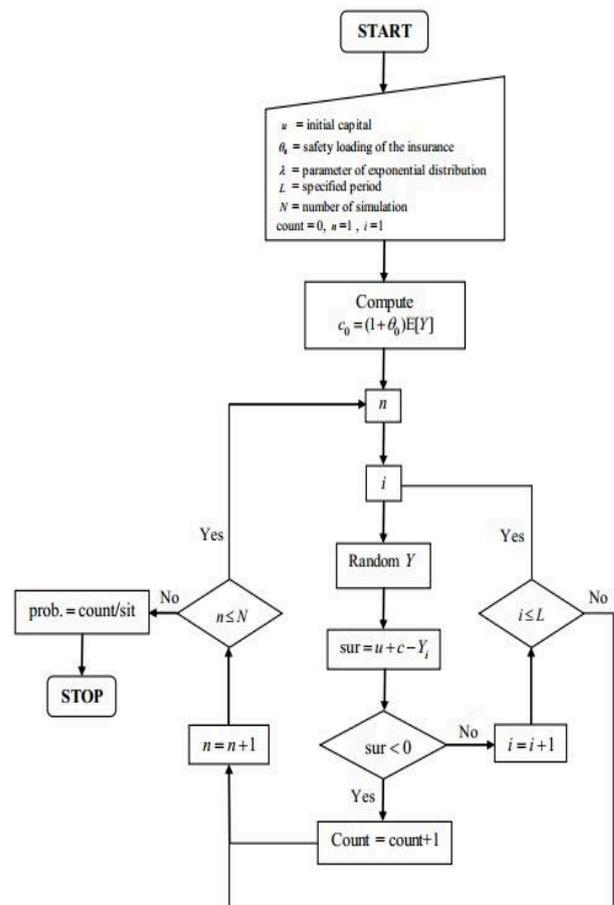


Figure 6: The flowchart of computing the ruin probability of $MID_\alpha(u, c_0, L)$

The process in Figure 6 is a calculate the probability of the survival duration greater than the specified period L at the time n .

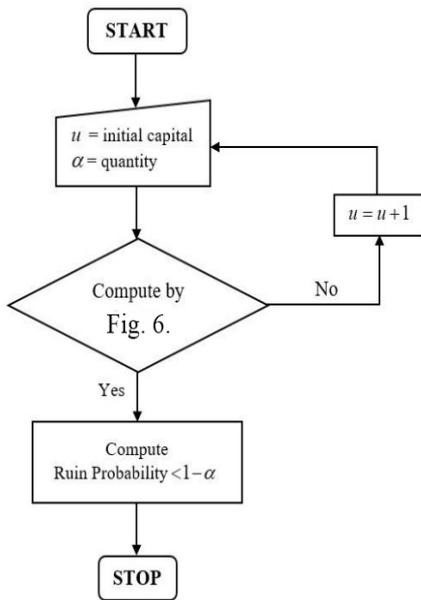


Figure 7: The flowchart of computing the ruin probability of $MID_\alpha(u, c_0, L)$ (cont.)

The process in Fig. 7 is a calculate the probability of ruin from Fig. 6 does not exceed the quantity α .

3 Simulation Results

We recall the discrete-time surplus process U_n at time n as the following expression

$$U_n = u + c_0n - \sum_{k=1}^n Y_k \quad ; \quad U_0 = u,$$

where $u \geq 0$ is initial surplus, c_0 is the premium rate and $\{Y_n; n \in \mathbb{N}\}$ is the independent and identically distribution claim size random variables. Thus, the recursive formula of the probability of ruin of the discrete times surplus process is the form

$$\Phi_n(u) = \sum_{k=1}^n \frac{(\alpha(u + kc_0))^{k-1} e^{-\alpha(u+kc_0)}}{(k-1)!} \cdot \frac{u + c_0}{u + nc_0} \tag{6}$$

for all $n = 1, 2, 3, \dots[1]$.

We consider the finite ruin probability of the surplus process $\{U_n; n \in \mathbb{N}\}$ which is driven by i.i.d. claim size process $\{Y_n; n \in \mathbb{N}\}$ and the premium rate $c_0 > 0$. Let $u \geq 0$ be an initial capital. For each $n \in \mathbb{N}$, we let

$$D = \min\{n|U_n \geq 0 \text{ and } U_{n+1} < 0\}, \tag{7}$$

denote the survival duration and $P(D)$ is the probability of survival duration. Then, we have

Theorem Let $n \in \mathbb{N}$, $c_0 > 0$ and $u \geq 0$ be given. If $\{Y_n; n \in \mathbb{N}\}$ are i.i.d. claim sizes process,

$$P(D) = \varphi_n(u) - \varphi_{n+1}(u).$$

Proof We set $S_n = \{\omega : U_n(\omega) \geq 0\}$ and $S_{n+1} = \{\omega : U_{n+1}(\omega) < 0\}$. Consider,

$$\begin{aligned} P\left(\bigcap_{k=1}^n S_k\right) &= P\left(\bigcap_{k=1}^n S_k \cap \left\{\bigcap_{k=1}^{n+1} S_k \cup R_{n+1}\right\}\right) \\ &= P\left(\bigcap_{k=1}^{n+1} S_k \cup \left\{\bigcap_{k=1}^n S_k \cap R_{n+1}\right\}\right) \\ &= P\left(\bigcap_{k=1}^{n+1} S_k\right) + P\left(\bigcap_{k=1}^n S_k \cap R_{n+1}\right) \end{aligned}$$

Then, we have $P(D) = \varphi_n(u) - \varphi_{n+1}(u)$.

Moreover, the case of probability of ruin at time n , we have

$$P(D) = \Phi_{n+1}(u) - \Phi_n(u).$$

Consequently, the closed form of the probability of ruin for the discrete time surplus process in the case of exponential distribution is of the form

$$P(D) = \frac{(\alpha(u + nc_0))^{n-1}}{(n-1)!e^{\alpha(u+nc_0)}} \cdot \frac{u + c_0}{u + nc_0} \tag{8}$$

for all n .

Next, we investigate the probability of the survival duration greater than the specified period L at the time n , denoted by $P(D > L)$. Next, we study the discrete-time surplus process under the regulation that the insurance company has to reserve sufficient initial capital of L to ensure that the probability of ruin does not exceed the quantity α . Consequently, the minimum initial capital corresponding to $(u, c_0, L, \alpha, \{Y_n; n \in \mathbb{N}\})$ and can be calculated as mentioned (4).

The simulation results for $MID_\alpha(u, c_0, L)$ of exponential distribution are achieved with 1,000 paths and set $\alpha = 0.1$, $\alpha = 0.05$ and $\theta = 0.2, \dots, 0.9$ as shown in the following Tables 2 and Fig.8.

Table 2: $MID_{\alpha}(u, c_0, L)$ of exponential distribution

safety loading(θ)	$MID_{\alpha}(u, c_0, L)$	
	$\alpha = 0.1$	$\alpha = 0.05$
0.2	74,471	95,910
0.3	52,000	69,990
0.4	35,108	52,213
0.5	30,023	41,929
0.6	23,522	36,033
0.7	18,761	30,288
0.8	14,196	25,051
0.9	11,259	21,984

Table 2 shows simulation of $MID_{\alpha}(u, c_0, L)$ for exponential distribution in case of $\alpha = 0.1$ and $\alpha = 0.05$ which has the details as follows: the first column shows the safety loading θ from 0.2 to 0.9, column 2-3 shows the value of minimum initial capital under the specified period $L = 1,000$.

The illustrate of the numerical results as shown in Table 2 of column 3 can describe that under the regulation of the insurance company, $L = 1,000, \theta = 0.4$. The insurance company must reserve the minimum initial capital equals 35,108 to ensure that probability of ruin does not exceed the given quantity $\alpha = 0.1$.

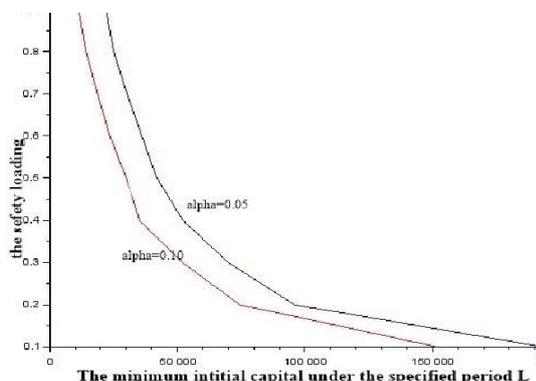


Figure 8: The simulation of $MID_{\alpha}(u, c_0, L)$ for exponential distribution.

Fig.8 shows the comparing of the experimental results between $\alpha = 0.1$ and $\alpha = 0.05$ for the safety loading and the minimum initial capital under the specified period $L = 1,000$ and we choose parameter of the exponential distribution.

The simulation results for $MID_{\alpha}(u, c_0, L)$ of weibull distribution are achieved with 1,000 paths and set $\alpha = 0.1, \alpha = 0.05$ and $\theta = 0.2, 0.25, 0.3, \dots, 0.9$ as shown in the following Tables 3 and Fig.9.

Table 3: $MID_{\alpha}(u, c_0, L)$ of weibull distribution

safety loading(θ)	$MID_{\alpha}(u, c_0, L)$	
	$\alpha = 0.1$	$\alpha = 0.05$
0.2	721,579	937,365
0.3	481,011	635,944
0.4	343,263	489,305
0.5	253,247	374,708
0.6	198,513	310,613
0.7	150,659	251,883
0.8	107,203	214,019
0.9	86,319	177,166

Table 3 shows simulation of $MID_{\alpha}(u, c_0, L)$ for weibull distribution in case of $\alpha = 0.1$ and $\alpha = 0.05$ which has the details as follows: the first column shows the safety loading θ from 0.2 to 0.9, column 2-3 shows the value of minimum initial capital under the specified period $L = 1,000$.

The illustrate of the numerical results as shown in Table 3 of column 3 can describe that under the regulation of the insurance company, $L = 1,000, \theta = 0.4$. The insurance company must reserve the minimum initial capital equals 489,305 to ensure that probability of ruin does not exceed the given quantity $\alpha = 0.05$.

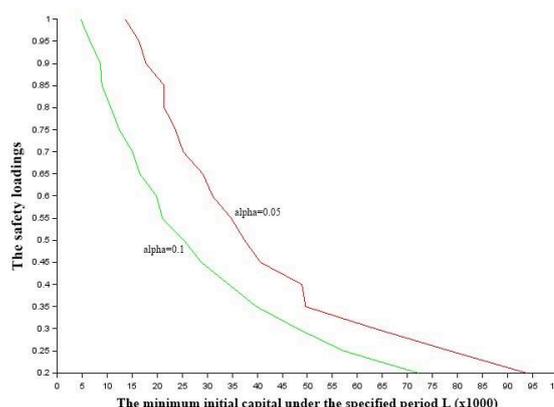


Figure 9: The simulation of $MID_{\alpha}(u, c_0, L)$ for weibull distribution.

Fig.9 shows the comparing of the experimental results between $\alpha = 0.1$ and $\alpha = 0.05$ for the safety loading and the minimum initial capital under the specified period $L = 1,000$ and we choose parameter of the weibull distribution.

4 Conclusion

In this paper, we study the measuring durability of insurance company. We computed the minimum initial capital under the specified period for exponential claim. The mentioned results can be suggest the insurance company for planning to reserve the minimum initial capital under the limited regulation to ensure that the probability of ruin does not exceed the given quantity of risk.

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