

Efficiency and Stochastic Dominance in the European Equity Mutual Fund Market

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Abstract: - In order to analyze and assess the behavior of the European equity mutual funds market, we present a method for evaluating and selecting them, using two different approaches, which vary from the conventional measures of performance: Data Envelopment Analysis (DEA) and stochastic dominance.

The analysis suggests a strong relation between the results that we obtain with each method. Likewise, we demonstrate that both indicators are highly correlated with the expected returns and that they have a high explanatory power. Additionally, we have included alphas as a right-hand variable and we confirm that they have a strong relation with DEA and stochastic dominance.

Key-Words: - Equity mutual funds; DEA; efficiency; stochastic dominance; variable returns to scale.

1. Introduction

The performance of mutual funds and its determining factors are key elements in the decision-making process. The aim of this paper is studying the behavior of European equity funds and their differences by categories, analyzing their efficiency and detecting the mutual funds that have persistently obtained better results compared to their competitors. Therefore, the aim of this paper is to show and to evaluate a model for measuring the performance of European equity mutual funds, by comparing and ranking the funds in sample according to two methodologies: Data Envelopment Analysis (hereinafter DEA), and Stochastic Dominance analysis (hereinafter SD).

Additionally, the power of alpha, in order to measure the performance of the funds will also be studied. The improvement of the current methodologies used to rank investments, allows identifying the best funds and strategies, but also displays alternative risk measures in order to control the risk profile of funds. Nevertheless, the implications of this research are not only practical but are also related to general equilibrium, since the

ability of some managers to persistently get higher returns than market, would hardly fit with market efficiency theory.

2. Theoretical Framework

2.1 DEA models

DEA models allow measuring the efficiency of a set of decision making units (hereinafter DMUs), generating an efficient frontier without explicitly specifying any production function, that is, without defining any function that relates inputs and outputs. DEA models use the volume of inputs consumed and outputs generated by a set of DMU in order to estimate the best choices, comparing each DMU with all the linear combinations of the units in the sample.

One of the first papers that use DEA to study the performance of mutual funds is [10], followed by [9], [2], [1] and [8]. [9] study 135 equity funds and they show that DEA, as a multifactorial tool, is more powerful than classic performance indexes, which

usually consider only one or two factors. [4] study 614 hedge funds and compare the performance of several strategies, concluding that DEA is a suitable tool with non-Gaussian returns.

To measure the efficiency of European equity funds it is necessary to specify the sort of return to scale assumed. Variable returns to scale (VRS) models are more flexible, as [3] and [5] state. In this paper we use VRS, since we assume that DMUs – mutual funds – generally do not operate on an optimal scale. Likewise, we use input orientation in order to study whether DMUs can reduce their inputs maintaining, at the same time, the outputs at the current level. In this framework, maximum efficiency can be estimated through linear programming. Thus, using the fractional formulation of VRS:

$$Max_{(u_r, v_i)} h_0 = \frac{\sum_{r=1}^s u_r y_{r0} + u_0}{\sum_{i=1}^m v_i x_{i0}} \quad (1)$$

Subject to:

$$\frac{\sum_{r=1}^s u_r y_{rj} + u_0}{\sum_{i=1}^m v_i x_{ij}} \leq 1, j = 1 \dots n \quad (2)$$

$$u_r, v_i \geq 0 \quad \forall r, i$$

where:

y_{r0} : Amount of output (1, 2... r) produced by the DMU.

u_r : Weights, equivalent to the prices of the output ($y_{10}, y_{20} \dots y_{r0}$).

x_{i0} : Amount of input (1, 2... i) consumed by the DMU.

v_i : Weights ($v_1, v_2 \dots v_i$) generated by the model, equivalent to the prices of each input.

Thus, for each DMU, the program will try to find the set of prices that maximize the value of the DMU output, with respect to the cost of the inputs, resulting in the efficiency ratio of each DMU.

Starting from the weights (u_r, v_i) of each DMU, the constraints ensure that the result of expression [1] is not greater than 1 for every DMUs. Likewise, a DMU is efficient when h_0 is 1 and the rest of the units do not have a higher result. So DMUs with values for h_0 between 0 and 1 are inefficient.

Since we assume an input orientation, the numerator of expression [1] is constant and therefore:

$$\text{Min } \theta \quad (3)$$

Subject to:

$$x_i \theta - \chi \lambda \geq 0 \quad (4)$$

$$Y \lambda - y_r \geq 0 \quad (5)$$

$$\lambda \geq 0 \quad (6)$$

where θ is the distance in input units to the data envelope, χ is the matrix of inputs of order $m \times n$, Y is the matrix of outputs of order $s \times n$, λ is the vector of

weights of order n , and x_i and y_r are the vectors of inputs and outputs, respectively.

2.2 SD models

SD models allow choosing between different random variables, so that one variable dominates another if its probability distribution is less or equal in the range where those variables are defined. SD is not restricted to a specific risk profile, but it can be applied to any sort of Von Neumann-Morgenstern utility functions, that is, increasing and concave. SD does not require any assumption about the distribution of returns, but it works as long as a cumulative probability distribution can be defined. This point is crucial since it makes SD consistent with DEA, hence justifying the present analysis.

The requirements of SD about the utility functions of the investors depend on the order of dominance. We use second-order SD (SD2) which implies non-satiety and risk aversion.

Formally, we can define the cumulative distributions F and G of two risky assets, where x is and uncertain return and U is the utility function. The investors are not satisfied and F dominates G in SD1 if:

$$F(x) \leq G(x) \text{ for every } x \quad (7)$$

Adding risk aversion, F dominates G in SD2 if:

$$\int_{-\infty}^x [G(x) - F(x)] dx \geq 0 \text{ for every } x \quad (8)$$

Finally, adding decreasing ARA, F dominates G in SD3 if:

$$\int_{-\infty}^x \int_{-\infty}^v [G(x) - F(x)] dv dx \geq 0 \text{ for every } x \quad (9)$$

The research on SD in the framework of the performance of mutual funds starts in the seventies. [7] show that efficient portfolios generated according to mean-variance space are reasonably similar to those derived from concave utility functions. They also suggest SD in order to reduce the number of alternatives in a first data compilation. [11] generate a set of efficient portfolios and run SD1, SD2, and SD3 tests. They analyze 140 stocks for the period 1960-1963, concluding that there are no significant differences between mean-variance approach and SD2.

[6] show that SD is a powerful tool in order to model investment preferences and to detect opportunities in the market of mutual funds. Using an iterative process, the authors reduce a large set of mutual funds in the period 1985-2000, to a subset of

non-dominated funds. They suggest SD to detect funds capable to beat market indexes.

3. Efficiency and SD of european equity funds

3.1 Data

Data used in this paper, are provided by Morningstar, with 2,101 European equity funds having a monthly frequency. In order to simplify the sample, we have considered only the funds with observations along the entire period considered, 2000-2016. Additionally, we have used average volume to choose only the largest funds. Finally, we have used the weight of each investment strategy in order to consider them proportionally in the sample.

The final sample includes 50 funds. We have divided the interval into three sub-periods to verify the results under different market cycles: (i) the whole sample, 2000-2016, (ii) bear market period, 2007-2009, and (iii) positive mean returns period, 2007-2012. Table 1 and Figures 1-6 show the summary of statistics and the distribution of the sample for each period.

The Table 1 shows that period 2007-2009 is characterized by: (i) a negative return due the fall of the markets in that period, and (ii) a high volatility, a slightly negative asymmetry and a positive kurtosis. In the period 2007-2012, the market raises, improving returns and volatility, although the negative asymmetry and kurtosis persist. Finally, in the whole period 2000-2016 the returns are higher and the risk is lower. The asymmetry is similar to other period but the kurtosis is greater, which suggests distributions with sharper peaks.

Table 1: Summary statistics

Period	Mean	Standard deviation	Median	Skewness	Kurtosis	Jarque-Bera	Min	Max	Sharpe
2000-2016	0,342	4,980	0,863	-0,415	2,023	40,250	35,080	-24,777	-0,553
2007-2012	0,012	5,560	0,571	-0,479	1,448	9,044	27,558	-23,296	-0,545
2007-2009	-0,469	6,383	0,269	-0,389	1,163	2,934	27,558	-23,296	-0,670

Fig. 1: Histogram 2000-2016

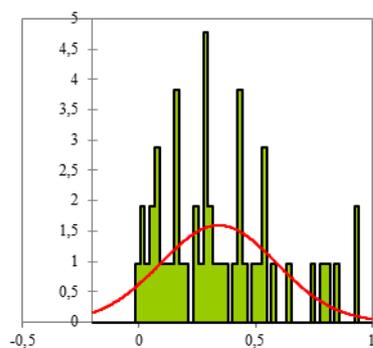


Fig. 2: Q-Q plot 2000-2016

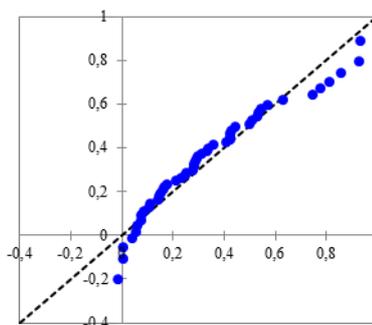


Fig. 3: Histogram 2007-2012

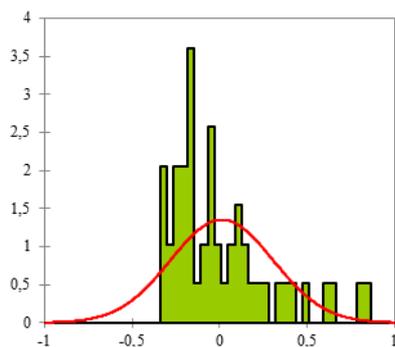


Fig. 4: Q-Q plot 2007-2012

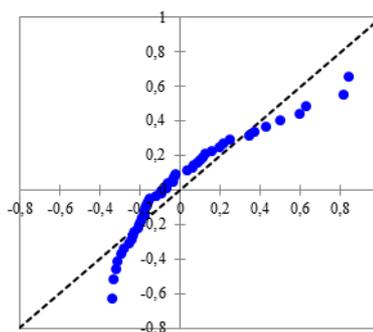


Fig. 5: Histogram 2007-2009

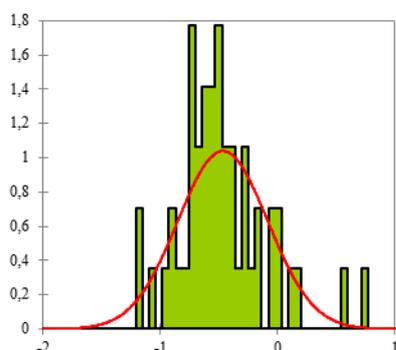
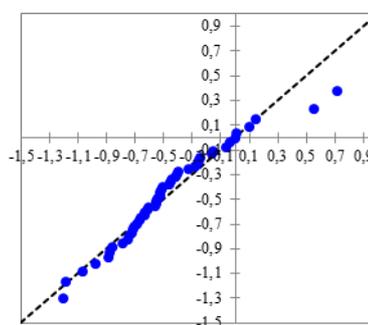


Fig. 6: Q-Q plot 2007-2009



3.2 DEA results

Table 2 shows inputs and outputs initially chosen, considering the measures most extensively used for studying the performance of mutual funds. We have used principal component analysis to choose only the most useful factors in order to explain the maximum amount of variance with the least loss of information. Table 3 shows the results for the period 2000-2016. Regarding the inputs, LPM_2 is the factor with greatest impact on component 1 with a load of 0.9835, while kurtosis is the most important factor for component 2, with a load of 0.9001. Components 1 and 2 together, represent 81.61% of the variance of the inputs. On the outputs side, HPM_3 (with a load of 0.8163 in component 1) and asymmetry (with a load of 0.9225 in component 2) have the greatest impact on the principal component analysis. Components 1 and 2 explain 81.35% of the total variance of the outputs. Therefore, we use these variables as inputs and outputs, respectively, for the three periods.

It is important to note that in some cases there is a problem of negative observations. However, this

drawback is solved by the property of the invariance, which means, adding a constant to the corresponding variable, turning its value from negative to positive without changing the efficient frontier.

We have sorted the sample by strategies and we have determined which funds are efficient. Table 4 shows that in the period 2000-2016 most of the funds are inefficient (72%), with only a 28% of efficient funds. In this period it is especially remarkable the category "Europe ex-UK Large-Cap Equity". Likewise, in the period 2007-2012 efficient funds weigh 22% of the sample (11 funds), with the categories "Europe ex-UK Large-Cap Equity" and "Europe Large-Cap Blend Equity" being the most efficient. Finally, in the period 2007-2009 there are 10 efficient funds, which represent 20% of the sample. "Europe Large-Cap Blend Equity" is the most efficient category in this period. In summary, "Europe ex-UK Large-Cap Equity" and "Europe Large-Cap Blend Equity" are the categories that most often show as efficient in the sample.

Table 2: Inputs and outputs

Possible inputs	Possible outputs
Lower Partial Moments 1 (LPM ₁)	High Partial Moments 1 (HPM ₁)
Lower Partial Moments 2 (LPM ₂)	High Partial Moments 2 (HPM ₂)
Lower Partial Moments 3 (LPM ₃)	High Partial Moments 3 (HPM ₃)
Standard deviation (SD)	Annualized average (AA)
Kurtosis (K)	Maximum return (RMax)
Value at Risk (VaR)	Minimum return (RMax)
Conditional Value at Risk (CVaR)	Skewness (A)
Modified Value at Risk (MVaR)	
Maximum Drawdown (MaxD)	
Average Drawdown (MD)	
Standard deviation of Drawdown (SD D)	

Table 3: Principal component analysis of inputs and outputs, 2000-2016

Input	Component 1	Component 2	Output	Component 1	Component 2
LPM ₁	0,9819	-	HPM ₁	0,8131	0,5468
LPM ₂	0,9835	-	HPM ₂	0,8157	0,5395
LPM ₃	0,9828	-	HPM ₃	0,8163	0,5370
Standard deviation	0,8982	-	Annualized average	-	0,6366
Kurtosis	-	0,9001	Maximum return	-	0,7877
VaR	0,9084	-	Minimum Return	-0,8486	-
CVaR	0,9538	-	Skewness	-	0,9225
MVaR	-	-0,8833			
Maximum Drawdown	0,5904	-			
Average Drawdown	0,8408	-			
Standard deviation of Drawdown	0,7427	-			
Explained variance (%)	65,57%	81,61%	Explained variance (%)	65,90%	81,35%

Table 4: Number of efficient funds by category

Morningstar classification	2000-2016				2007-2012				2007-2009			
	Efficient	Non Efficient										
Europe Equity Income	0	0%	1	2%	0	0%	1	2%	0	0%	1	2%
Europe ex-UK Large-Cap Equity	4	8%	1	2%	4	8%	1	2%	3	6%	2	4%
Europe ex-UK Small/Mid-Cap Equity	1	2%	0	0%	0	0%	1	2%	0	0%	1	2%
Europe Flex-Cap Equity	1	2%	2	4%	1	2%	2	4%	2	4%	1	2%
Europe Large-Cap Blend Equity	3	6%	14	28%	4	8%	13	26%	4	8%	13	26%
Europe Large-Cap Growth Equity	0	0%	2	4%	0	0%	2	4%	0	0%	2	4%
Europe Large-Cap Value Equity	0	0%	3	6%	1	2%	2	4%	1	2%	2	4%
Europe Mid-Cap Equity	1	2%	1	2%	0	0%	2	4%	0	0%	2	4%
Europe Small-Cap Equity	1	2%	1	2%	0	0%	2	4%	0	0%	2	4%
Eurozone Flex-Cap Equity	0	0%	1	2%	0	0%	1	2%	0	0%	1	2%
Eurozone Large-Cap Equity	2	4%	10	20%	1	2%	11	22%	0	0%	12	24%
Eurozone Mid-Cap Equity	1	2%	0	0%	0	0%	1	2%	0	0%	1	2%
Total European equity funds	14	28%	36	72%	11	22%	39	78%	10	20%	40	80%

DEA also allows determining which funds are benchmarks for inefficient funds, so that it is possible to calculate the number of times that an efficient fund has been benchmarked for inefficient funds. This provides useful information, since the higher the

number of times a fund is benchmarked for others, the higher the optimality of that investment. Likewise, this point allows using the benchmark as a way to improve the performance. For the periods 2000-2016, 2007-2012 and 2007-2009, the

categories that constitute benchmarks are "Europe ex-UK Large-Cap Equity", "Europe ex-UK Large-Cap Equity", and "Europe Large-Cap Blend Equity", respectively.

Tables 5-7 detail several statistics for each category, in order to clarify the differences between efficient and inefficient funds. Table 5 (period 2000-2016) shows 14 efficient funds, out of a total of 50 funds. Regarding the categories, 8 out of 12 include efficient funds. In general, efficient funds have better results according to their mean returns, standard deviations and Sharpe ratios. Furthermore, all inefficient funds have negative skewness and higher kurtosis. In summary, efficient funds use to have higher returns, lower risk, lower kurtosis and higher Sharpe ratios.

Table 6 (period 2007-2012) shows 11 efficient funds and only 5 out of 12 categories have efficient funds. The returns in this period are higher for the efficient funds, with a slightly higher risk, but actually very similar, independently of the efficiency level. Inefficient funds have a generally negative skewness. Kurtosis is higher in the non efficient funds, and although Sharpe ratio is negative in some cases, it is also higher in the case of efficient funds. Therefore, in this period, efficient funds overcome inefficient ones in terms of risk and return, and have lower negative skewness.

Finally, Table 7 shows the period 2007-2009 results, being the time interval with the lowest number of efficient funds, only 10 funds in 4 categories. Returns are frequently negative but higher in the case of efficient funds. In any case, the risk is greater and skewness is close to zero or positive in these cases, while it is always negative for inefficient funds. Non efficient funds have a higher kurtosis. Likewise, Sharpe ratio is higher in the efficient funds, despite it is negative in most of the cases. In summary, efficient funds are more profitable, have higher levels of risk and higher Sharpe ratios.

In order to check the quality of management, we have used Tracking Error to sort funds according to their active/passive management profile in the three periods. This allows us concluding that higher returns commonly correspond to higher risks with respect to the benchmark. Although we do not find funds with pure active management, as the sample goes through the crisis period there is a trend to active management. The only strategy that clearly provides non-directional results is "Europe Large-Cap Blend Equity".

Tables 5-7 allow concluding that, as time interval goes through the crisis period: (i) the efficiency level falls, both in number of funds and in number of categories; (ii) the returns are lower and even negative depending on the period; (iii) the risk increases; (iv) efficient funds still have a higher Sharpe ratio than inefficient funds; and (v) funds tend to active management in order to reduce directional movements. Additionally, non-efficient funds always show negative skewness while efficient funds do not have a clear pattern; mainly because of extreme events. Usually returns are leptokurtic, with values closer to the mean.

Figure 7 shows risk and returns for the three periods of the mutual funds in the sample, with efficient funds highlighted with a higher size. In general, efficient funds in the period 2007-2009 are riskier, while in the period 2007-2012 funds are more concentrated. The results of sample 2000-2016 are more homogeneous and have positive returns in most of the cases.

According to these results we can conclude that: (i) the period 2007-2009 it is characterized by low returns and a high risk; (ii) in the bearish period the skewness and kurtosis happen to be positive and the Sharpe ratios worsen, although the results are better for efficient funds; and (iii) DEA model is consistent with the fact that the most efficient funds use to have better results.

3.3 SD Results

In addition to DEA model, we have studied the performance of the mutual funds in our sample through SD analysis. For this purpose, we have sorted the sample into dominant and dominated funds using SD2. Taking these results we have calculated an SD index (SDI) in order to get values comparable with those of DEA. The index allows establishing a ranking of the mutual funds according to their SD and it follows:

$$SDI_i = \frac{D_{A,i} - D_{B,i}}{N} \quad (10)$$

where:

SDI_i : SD index value of fund i .

$D_{A,i}$: Number of funds dominated by the fund i .

$D_{B,i}$: Number of funds that dominate fund i .

N : Number of funds.

Table 5: Statistics by category (period 2000-2016)

	Min.	Max.	1 st Quartile	Med.	3 rd Quartile	Average	SD	Skewness	Kurtosis	VaR 95%	CVaR 95%	Sharpe
<i>Efficient Mutual Funds</i>												
Europe Equity Income	-	-	-	-	-	-	-	-	-	-	-	-
Europe ex-UK Large-Cap Equity	-15,79	19,06	-2,10	1,16	3,59	0,70	4,79	-0,32	1,48	7,45	10,79	1,13
Europe ex-UK Small/Mid-Cap Equity	-18,12	34,78	-1,68	1,64	4,47	0,93	5,69	0,49	7,01	8,12	12,08	1,42
Europe Flex-Cap Equity	-17,07	27,56	-1,70	1,41	3,74	0,93	5,25	0,01	3,72	8,17	11,83	1,55
Europe Large-Cap Blend Equity	-19,05	15,19	-2,43	0,70	3,30	0,19	4,57	-0,53	0,78	8,38	10,85	-0,20
Europe Large-Cap Growth Equity	-	-	-	-	-	-	-	-	-	-	-	-
Europe Large-Cap Value Equity	-	-	-	-	-	-	-	-	-	-	-	-
Europe Mid-Cap Equity	-19,32	12,75	-2,37	1,13	4,62	0,53	5,53	-0,69	1,00	9,83	13,11	0,60
Europe Small-Cap Equity	-18,64	14,06	-2,09	1,38	4,43	0,78	5,19	-0,74	1,26	8,40	12,19	1,22
Eurozone Flex-Cap Equity	-	-	-	-	-	-	-	-	-	-	-	-
Eurozone Large-Cap Equity	-16,44	14,14	-2,76	0,84	3,54	0,23	4,95	-0,52	0,74	8,48	11,76	-0,06
Eurozone Mid-Cap Equity	-24,78	35,08	-3,08	1,01	3,95	0,28	6,68	0,13	4,71	11,60	15,75	0,05
<i>Non-Efficient Mutual Funds</i>												
Europe Equity Income	-18,52	15,91	-2,51	0,63	3,09	0,14	5,25	-0,52	1,69	10,39	13,53	-0,25
Europe ex-UK Large-Cap Equity	-16,16	18,54	-2,50	0,93	3,32	0,21	5,00	-0,43	1,44	8,97	11,90	-0,10
Europe ex-UK Small/Mid-Cap Equity	-	-	-	-	-	-	-	-	-	-	-	-
Europe Flex-Cap Equity	-16,56	20,83	-1,83	1,06	3,19	0,64	4,45	-0,51	2,19	7,12	10,63	1,04
Europe Large-Cap Blend Equity	-21,40	19,01	-2,22	0,67	3,16	0,22	4,66	-0,50	1,34	8,26	11,26	-0,08
Europe Large-Cap Growth Equity	-16,88	16,12	-2,43	0,98	3,45	0,36	5,25	-0,50	1,29	9,63	12,80	0,25
Europe Large-Cap Value Equity	-17,05	23,40	-2,23	0,97	3,54	0,36	4,80	-0,49	1,80	8,32	11,63	0,26
Europe Mid-Cap Equity	-21,99	25,99	-2,33	1,02	3,77	0,43	5,84	-0,33	3,41	9,25	14,21	0,35
Europe Small-Cap Equity	-20,56	20,92	-2,68	1,27	4,43	0,57	5,37	-0,45	1,78	8,67	12,57	0,71
Eurozone Flex-Cap Equity	-19,91	18,37	-1,46	1,19	3,34	0,54	4,72	-0,79	3,14	7,48	12,19	0,74
Eurozone Large-Cap Equity	-23,30	22,80	-2,78	0,74	3,54	0,17	5,12	-0,48	1,20	9,01	12,50	-0,19
Eurozone Mid-Cap Equity	-	-	-	-	-	-	-	-	-	-	-	-

Table 6: Statistics by category (period 2007-2012)

	Min.	Max.	1st Quartile	Med.	3rd Quartile	Average	SD	Skewness	Kurtosis	VaR 95%	CVaR 95%	Sharpe
<i>Efficient Mutual Funds</i>												
Europe Equity Income	-	-	-	-	-	-	-	-	-	-	-	-
Europe ex-UK Large-Cap Equity	-15,79	16,99	-2,70	1,43	3,78	0,52	5,47	-0,48	0,64	9,56	11,85	0,57
Europe ex-UK Small/Mid-Cap Equity	-	-	-	-	-	-	-	-	-	-	-	-
Europe Flex-Cap Equity	-17,07	27,56	-2,57	0,79	3,18	0,43	6,02	0,61	5,60	10,25	12,94	0,35
Europe Large-Cap Blend Equity	-21,40	13,93	-2,58	0,12	3,23	-0,08	5,02	-0,45	0,94	9,62	11,58	-0,86
Europe Large-Cap Growth Equity	-	-	-	-	-	-	-	-	-	-	-	-
Europe Large-Cap Value Equity	-14,86	23,40	-3,19	0,36	3,76	0,06	6,01	0,32	2,58	11,09	12,79	-0,37
Europe Mid-Cap Equity	-	-	-	-	-	-	-	-	-	-	-	-
Europe Small-Cap Equity	-	-	-	-	-	-	-	-	-	-	-	-
Eurozone Flex-Cap Equity	-	-	-	-	-	-	-	-	-	-	-	-
Eurozone Large-Cap Equity	-13,34	13,91	-3,38	-0,49	3,68	-0,33	5,39	-0,24	0,15	10,17	11,94	-1,30
Eurozone Mid-Cap Equity	-	-	-	-	-	-	-	-	-	-	-	-
<i>Non-Efficient Mutual Funds</i>												
Europe Equity Income	-15,52	14,72	-2,12	0,25	2,43	-0,07	4,96	-0,59	1,87	10,63	12,51	-0,77
Europe ex-UK Large-Cap Equity	-16,16	18,54	-2,93	0,61	2,93	-0,03	5,62	-0,22	1,68	10,93	13,05	-0,61
Europe ex-UK Small/Mid-Cap Equity	-17,15	13,87	-1,84	1,55	3,97	0,60	5,13	-1,04	2,21	8,21	12,71	0,80
Europe Flex-Cap Equity	-16,56	20,83	-2,38	0,72	3,04	0,16	5,57	-0,35	1,60	10,48	12,91	-0,22
Europe Large-Cap Blend Equity	-19,05	18,33	-2,54	0,35	3,05	-0,10	5,30	-0,54	1,45	10,68	12,92	-0,80
Europe Large-Cap Growth Equity	-16,88	16,12	-1,08	1,35	3,46	0,73	5,20	-0,70	2,35	9,97	12,64	1,08
Europe Large-Cap Value Equity	-17,05	15,57	-2,70	0,22	3,39	-0,13	5,42	-0,62	1,12	10,37	13,24	-0,85
Europe Mid-Cap Equity	-21,99	20,24	-2,55	0,46	3,34	-0,04	6,15	-0,71	1,98	10,81	15,46	-0,55
Europe Small-Cap Equity	-20,56	15,55	-2,41	0,72	3,79	0,05	5,70	-0,85	2,06	9,39	14,28	-0,41
Eurozone Flex-Cap Equity	-19,91	18,37	-2,89	-0,05	3,52	-0,16	6,15	-0,42	1,60	11,26	14,29	-0,79
Eurozone Large-Cap Equity	-23,30	22,80	-3,50	0,43	3,70	-0,16	5,85	-0,51	0,97	11,78	13,91	-0,85
Eurozone Mid-Cap Equity	-17,60	11,29	-3,03	0,79	3,72	-0,03	5,43	-0,91	1,22	9,51	13,37	-0,62

Table 7: Statistics by category (period 2007-2009)

	Min.	Max.	1st Quartile	Med.	3rd Quartile	Average	SD	Skewness	Kurtosis	VaR 95%	CVaR 95%	Sharpe
<i>Efficient Mutual Funds</i>												
Europe Equity Income	-	-	-	-	-	0,46	6,52	-0,44	-	-	-	-
Europe ex-UK Large-Cap Equity	-15,79	16,99	-2,63	1,51	4,36	0,46	6,52	-0,44	0,50	11,79	13,39	0,26
Europe ex-UK Small/Mid-Cap Equity	-	-	-	-	-	-	-	-	-	-	-	-
Europe Flex-Cap Equity	-17,07	27,56	-3,94	-0,01	3,25	-0,20	7,38	0,55	2,64	12,09	14,21	-0,84
Europe Large-Cap Blend Equity	-13,14	12,92	-3,03	0,24	2,71	-0,45	5,24	-0,36	0,71	10,40	11,57	-1,71
Europe Large-Cap Growth Equity	-	-	-	-	-	-	-	-	-	-	-	-
Europe Large-Cap Value Equity	-14,86	23,40	-4,26	-0,12	1,90	-0,55	7,10	0,61	2,69	12,35	13,93	-1,44
Europe Mid-Cap Equity	-	-	-	-	-	-	-	-	-	-	-	-
Europe Small-Cap Equity	-	-	-	-	-	-	-	-	-	-	-	-
Eurozone Flex-Cap Equity	-	-	-	-	-	-	-	-	-	-	-	-
Eurozone Large-Cap Equity	-	-	-	-	-	-	-	-	-	-	-	-
Eurozone Mid-Cap Equity	-	-	-	-	-	-	-	-	-	-	-	-
<i>Non-Efficient Mutual Funds</i>												
Europe Equity Income	-15,52	14,72	-3,35	-0,35	2,16	-0,97	5,80	-0,30	1,39	11,89	13,73	-2,63
Europe ex-UK Large-Cap Equity	-16,16	18,54	-3,15	-0,01	2,66	-0,52	5,82	-0,44	1,16	11,53	13,25	-1,73
Europe ex-UK Small/Mid-Cap Equity	-17,15	13,87	-3,09	0,70	4,33	-0,32	6,19	-0,80	1,14	13,57	14,85	-1,22
Europe Flex-Cap Equity	0,00	14,30	-2,50	0,39	2,86	-0,79	6,15	-0,54	0,80	13,44	14,10	-2,12
Europe Large-Cap Blend Equity	-21,40	18,33	-3,04	0,09	2,96	-0,56	6,20	-0,47	1,20	12,33	14,21	-1,66
Europe Large-Cap Growth Equity	-16,88	16,12	-1,58	0,63	3,24	0,06	6,10	-0,59	1,80	12,06	14,18	-0,50
Europe Large-Cap Value Equity	-17,05	15,57	-2,56	0,20	3,06	-0,53	6,08	-0,65	1,24	12,71	14,51	-1,60
Europe Mid-Cap Equity	-21,99	20,24	-3,66	-0,82	3,64	-1,03	7,20	-0,59	1,39	15,92	18,24	-2,22
Europe Small-Cap Equity	-20,56	15,55	-2,62	0,83	3,52	-0,34	6,69	-0,89	1,73	14,85	16,62	-1,17
Eurozone Flex-Cap Equity	-19,91	18,37	-3,20	-0,70	3,36	-0,67	7,28	-0,28	1,21	12,97	16,33	-1,61
Eurozone Large-Cap Equity	-23,30	22,80	-3,30	0,18	3,45	-0,54	6,53	-0,45	0,91	12,57	14,51	-1,56
Eurozone Mid-Cap Equity	-17,60	11,29	-3,17	-0,18	3,72	-0,86	6,13	-0,91	0,97	13,52	15,58	-2,27

Fig. 7: Risk and returns of mutual funds

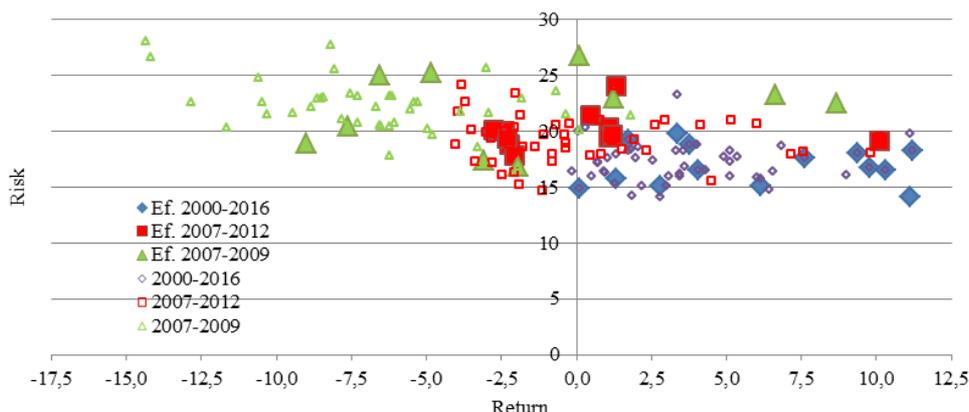


Table 8: Number of dominant and dominated funds

Morningstar category	2000-2016		2007-2012		2007-2009	
	Dominant	Dominated	Dominant	Dominated	Dominant	Dominated
Europe Equity Income	2%	26%	26%	4%	6%	0%
Europe ex-UK Large-Cap Equity	42%	8%	36%	2%	36%	8%
Europe ex-UK Small/Mid-Cap Equity	18%	12%	32%	0%	8%	4%
Europe Flex-Cap Equity	11%	19%	25%	4%	29%	1%
Europe Large-Cap Blend Equity	30%	19%	21%	13%	23%	13%
Europe Large-Cap Growth Equity	39%	4%	39%	1%	2%	40%
Europe Large-Cap Value Equity	24%	27%	16%	15%	16%	7%
Europe Mid-Cap Equity	1%	71%	5%	45%	2%	40%
Europe Small-Cap Equity	5%	19%	6%	23%	5%	22%
Eurozone Flex-Cap Equity	2%	56%	6%	66%	14%	0%
Eurozone Large-Cap Equity	16%	28%	7%	40%	7%	33%
Eurozone Mid-Cap Equity	2%	44%	8%	28%	0%	18%

The index values range between -1 (when the fund *i* is dominated by the remainder) and +1 (the fund *i* dominates the remainder). Thus, funds with a negative index have a worse position in the ranking than those with a positive index, and a result close to zero means that the fund *i* neither dominates nor is dominated. We have sorted the funds in the sample by strategy and after that we have estimated the percentage of dominant and dominated funds through SD2 analysis. Table 8 shows the results. In the period 2000-2016 “Europe ex-UK Large-Cap Equity” is the most dominant category, while “Eurozone Flex-Cap Equity” is the most dominated rank.

For the period 2007-2012 the category “Europe Large-Cap Growth Equity” is the top ranked strategy, contrasting with “Eurozone Flex-Cap Equity”, that is in the bottom of the ranking. Finally, in the period 2007-2009 the strategy “Europe ex-UK Large-Cap

Equity” is again top ranked, and “Europe Large-Cap Growth Equity” and “Europe Mid-Cap Equity” are the statistically dominated categories.

In summary, the category “Europe ex-UK Large-Cap Equity” is the strategy with the highest SDI, in concordance with DEA results. There are periods with funds that have an index value close to zero. For instance, the periods 2000-2016 and 2007-2012 have a 2% of funds that neither dominate nor are dominated, although in the period 2007-2009 there are no funds of this sort.

The results of SD are extreme in the period 2007-2009, which seems to confirm that the best funds tend to be more robust in adverse market conditions. In this framework, it is convenient to deepen the effect of risk and returns on SD index. Table 9 shows the correlation analysis between these variables and, in addition, the alpha of each fund

Table 9: Spearman's rank correlation of risk and return

		2000-2016	2007-2009	2007-2012
Return vs	Alpha	0,1792 <i>0,2124</i>	0,5571 <i>< 0,0001</i>	0,4190 <i>0,0027</i>
	DEA	0,4127 <i>0,0031</i>	0,6027 <i>< 0,0001</i>	0,5107 <i>0,0002</i>
	SDI	0,2778 <i>0,0511</i>	0,7893 <i>< 0,0001</i>	0,6777 <i>< 0,0001</i>
	Alpha	-0,8005 <i>< 0,0001</i>	-0,6646 <i>< 0,0001</i>	-0,6617 <i>< 0,0001</i>
	DEA	-0,2488 <i>0,0815</i>	-0,4104 <i>0,0033</i>	-0,2924 <i>0,0397</i>
	SDI	-0,7408 <i>< 0,0001</i>	-0,6084 <i>< 0,0001</i>	-0,6295 <i>< 0,0001</i>

The values in italics are different from 0 with a significance of 0.05.

Table 10: Spearman's rank correlation between DEA, SD and alpha

	2000-2016	2007-2009	2007-2012
SDI vs DEA	0,5346 <i>< 0,0001</i>	0,7807 <i>0,0000</i>	0,7411 <i>< 0,0001</i>
SDI vs Alfa	0,7752 <i>< 0,0001</i>	0,8003 <i>< 0,0001</i>	0,7740 <i>< 0,0001</i>
DEA vs Alfa	0,4501 <i>0,0012</i>	0,6660 <i>0,0000</i>	0,5982 <i>< 0,0001</i>

The values in italics are different from 0 with an alpha significance level of 0.05.

In the period 2007-2009 we find a positive relationship between the SD index and the returns, so that a fund with a high SDI commonly have a high return. However, this relation seems to disappear in 2000-2016 period. With respect to risk, funds with a high SDI use to have a lower risk. These results are contrary to those of DEA model, so in the next section we study the causal relationship of both methodologies on expected returns.

3.4 Relationship between DEA and SDI

Since DEA provides a score between 0 and 1 and our SDI value ranges between -1 and 1, we can easily study the relationship between the results of both models. Table 10 shows the Spearman's rank correlation between this two rankings and, additionally, with alpha parameter.

For the period 2000-2016 DEA and the SDI have a correlation of 53.5%, while for the periods 2007-2012 and 2007-2009 this correlation is 74.1% and 78.1%, respectively. The results denote a strong relationship between these methodologies, a little lower for the period 2000-2016. Anyway, in our opinion it is important to clarify the role of the managers on the value creation process. As shown in Table 10, SDI and alpha have very high correlations, above of 77% in all periods. This suggests that dominant funds use to have higher alphas and,

therefore, that the management gets a better performance for these funds. With respect to DEA, the correlations are more modest and do not reach the levels of the SDI.

Finally, we use a regression analysis in order to model the relationship between expected returns, DEA and SD, with expected returns as the left hand variable and DEA and SD as the right hand variables. The regression explicit the influence of the latter variables on the former:

$$E(R_i) = \beta_0 + \beta_1 DEA_i + \beta_2 SDI_i + \varepsilon_i \quad (11)$$

In the period 2000-2016 the model do not have multicollinearity so the right hand variables, although correlated, are not redundant with an inflation factor (VIF) of 1.532 and a tolerance of 0.65. Although there is autocorrelation –Durbin-Watson statistic has a value of 1.311– we have avoid the problem using the Cochrane-Orcutt method. Thus, the regression function results as follows:

$$E(R_i) = -1,393 + 1,848 DEA_i + 0,024 SDI_i + \varepsilon_i \quad (12)$$

The F test has a p-value of 98.58%. The p-values of the parameters of DEA and SDI variables are 98.38% and 17.57%, respectively. This means that, although expected returns depend positively and

significantly on efficiency and they depend positively on SD, this last relation is not significant.

For the period 2007-2012 there is not multicollinearity (VIF 2,725 and tolerance 0,367) and, with respect to autocorrelation, the Durbin-Watson statistic has a value of 1.457, so again variables are autocorrelated. Using the Cochrane-Orcutt method we define the following regression:

$$E(R_{it}) = -0,374 + 0,438DEA_i + 0,424SDI_i + \varepsilon_i \quad (13)$$

The F test results in a significance of 99.99%. The p-values of the parameters of DEA and SDI variables show a significance of 38.75% and 99.41%, so the expected returns depend positively on efficiency –although the parameter is not significant– and SD for the period 2007-2012.

Finally, in the period 2007-2009 there is not multicollinearity (VIF 2,577 and tolerance 0,388) and Durbin-Watson statistic is 1.909, so there is not autocorrelation. The regression function is:

$$E(R_{it}) = -0,755 + 0,323DEA_i + 0,641SDI_i + \varepsilon_i \quad (14)$$

The p-value of F test is 99.99%, while the p-values of the parameters of DEA and SDI variables are 29.40% and 99.59%, respectively. This means that, as in the previous models, expected returns depends positively but not significantly on DEA and both positively and significantly on SD, for the period 2007-2009.

These results allow concluding that: (i) in the long data series (period 2000-2016), characterized by stable and positive returns, DEA prevails over SD in explaining expected returns; (ii) analogously, in the period of crisis (2007-2009) the weighing of DEA diminishes and raise the weighing of SD; (iii) anyway, generally DEA explains a higher proportion of expected returns with respect to SD; (iv) considering the three regressions, the model corresponding to the period 2007-2012 has the higher explanatory power; (v) in summary, DEA seems to be a better tool in the long-run for performance analysis, while SD provides better results in the short-run and in periods of turbulence.

4. Conclusion

In this paper we have studied the role of efficiency, the preferences of investors in a risky environment, and performance of managers, using DEA and SD2 models. We have studied efficiency assuming variable returns to scale with an input orientation, and we have ranked the funds in our sample for three

periods: 2000-2016, 2007-2012 and 2007-2009. Additionally, we have sorted these funds according their SD using a SD index. Finally, we have estimated the alpha of each fund in order to study the quality of the management.

In general, there is a positive and significant relationship between these three indicators. We can conclude: (i) there are high and positive correlations between DEA, SD and alpha for the three periods; (ii) these indicators are positively correlated with returns and negatively correlated with risk, with a high significance level; (iii) DEA and SD are good explanatory variables of expected returns of mutual funds, with a direct relationship in both cases. Anyway, efficiency seems more effective in the long-run, while SD is more suitable in the short-run and in high volatility periods.

Finally, our sample shows a sharp linear relation between efficiency, SD and performance. This fact open some questions about related areas, as the effect of efficiency and SD on size of mutual funds in order to study possible economies of scale, or the role of DEA and SD models in asset pricing.

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