# Transportation Subsidy Models Involving Operator and Manufacturer: A Case Study in Indonesia

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*Abstract:* - In order to reduce a high amount of traffic congestion in a city, a government adopted fiscal subsidy to encourage the use of public transportation, especially buses. This paper deals with two government's subsidy models: a subsidy for purchasing buses from the manufacturers and a subsidy for reducing ticket price for passengers. From both subsidy models, we determine the maximum profit of the operator and manufacturer using non-cooperative solution game theory. By Analyzing both models and making numerical examples using data from Indonesia public transport, it is expected that the influence of subsidy to the profit of the operator and manufacturers can be revealed. The result indicates that reducing ticket price will give higher profit both to the operator and manufacturers.

*Key-Words:* - Subsidy model, transportation operator, non-cooperative solution

## 1 Introduction

The number of vehicles that rapidly grow every year is one of the reasons why traffic congestion happens. According to the Indonesia's Central Bureau of Statistics (BPS), in 2016 there are 124,348,224 vehicles in Indonesia. In Bandung, there are about 1,25 million vehicles that are used every day. With only 1,236.48 km length of the road, we cannot avoid traffic congestion. A traffic congestion produces unwanted situations in many big cities, e.g. it has contributed to air pollution and hampered economic activity. Furthermore, it can cause health problems, such as respiratory diseases. According to Indonesia's transportation ministry report, vehicles contribute to 70% pollutant in Jakarta. With 9.9 million vehicles in 2009, the pollutant treat is getting worst.

In order to reduce traffic congestion or traffic jam in big cities in Indonesia, the government offers the subsidy policy for transportation sector. In 2016, Indonesia's transportations ministry provided the subsidy to the public transport operator for supplying buses in 11 big cities. With this subsidy, the government expects that people choose to use bus rather than they own vehicle for their dailly transportation mode.

The study of subsidy model has been done by many authors in many different areas/sectors (see [11] and the references therein for details). In [12], the authors discussed a government subsidy applied to green products in which a retailer takes a product from manufacturer and sells to customer. Recently, the authors in [3,11] developed mathematical models to study the government subsidy in public transport sector. Unlike the authors in [11] and the references therein, the authors in [3] analysed two different kinds of subsidy: (i) the subsidy in purchasing bus from an appointed public transport manufacturer, and (ii) the subsidy for reimbursing reduced ticket price for passengers. They showed that a cooperative solution gives a higher profit for both the public transport operator and the manufacturer. They use the two-parameter Weibull failure intensity function in their analysis.

The present paper deals with government subsidy model for one of the government transportation corporations in Indonesia. There are two subsidy models that will be analyzed, that is the subsidy for purchasing bus and for reducing ticket price. The goal of the paper is to answer the research question on which subsidy model is better in terms of maximum profit to the transportation operator. We will use the game theoretical approach to develop the model. In this game we choose Damri, a government transportation corporations, as a public transportation operator acts as a leader and issues a policy to initially maximize profit. The manufacturer, as a follower, maximizes its profit based on operator's policy. We consider the buses' first failure data which are obtained from one major city where Damri operates. Different to our previous work, here we fit the data to three-parameter Weibull intensity function and estimate the parameters with the MLE method. There are some known methods to analyze data of reliable systems [8]. We consider non-cooperative solution in order to maximize profit.

The work in [2,4, 9] give good reviews on how the game theoretical approach is applied to obtain solution for general equipment models. The authors in [2] explained non-cooperative and cooperative solution in order to maximize profit on a lease contract problem. The authors in [4,9] explained the game theory approach in the presence of a maintenance service agent. This paper is organized as follows. Section 2 gives model formulation and section 3 gives the solution and analyzes both models. Section 4 gives result in processing first failure data and section 5 gives numerical examples to elaborate the models in more details. Finally, conclusion is presented in section 6.

# 2 Model Formulation

In this section we formulate the problem buy introducing several notations and definitions that will be used in the subsequent sections. The concepts such as operator's revenue and expense, preventive and corrective maintenance, and operator's and manufacturer's profit functions are defined.

## 2.1 Notations

The following notations will be used in model formulation

$q$: Number of Passengers per year
$n$: Number of Buses
$K$: Bus Operating Time
$\gamma_i$: Demand's function parameters
$N$: Number of PMs
$\tau$: Time between PM
$\delta$: Degree of Repair
$\lambda_0(t)$: Failure Intensity before PM
$\lambda(t)$: Failure Intensity after PM
$p$: Passenger's ticket price
$C_m$: Total Cost of CM
$C_f$: Cost for Every CM Action
$C_p$: Total Cost of PM
$a,b$: Cost Component for Every PM
$C_r$: Bus Production Cost
$u$: Subsidy Amount
$w$: Bus Price
$\Psi_{d_1}$: Operator's Profit for First Model
$\Psi_{d_2}$: Operator's Profit for Second Model
$\Psi_{m_1}$: Manufacturer's Profit for First Model
$\Psi_{m_2}$: Manufacturer's Profit for First Model.

## 2.2 Operator's revenues

In *ceteris paribus* condition the law of demand states that if the product price increases, demand will decrease, and if the product price decreases then demand will increase. In this case, if the number of passengers per year is $q$ and ticket price is $p$, we have a linear relation demand function

$$q(p)=\gamma_0+\gamma_1 p \text{ with } \gamma_1<0 \text{ and } \gamma_0>0 . \tag{1}$$

The number of buses per year $n$ can be obtained by dividing $q$ with bus capacity $m$ so that $n=\frac{q(p)}{m}$ ; $n,m>0$. Operator's revenue per year, $R_d$, is obtained from the total passengers multiplied by ticket price

$$R_d(p,K)=q(p).p . \tag{2}$$

In this paper, we will use two government subsidy models. First, subsidy for purchasing bus from manufacturer and the second one is subsidy for reducing ticket price. To make it simple, we use index 1 for first model and index 2 for the second model function. For the first model, subsidy does not influence demand function [12] so $q(p_1)=\gamma_0+\gamma_1 p_1$. For the second model, subsidy amount $u$ influences demand function so

$q(p_2,u)=\gamma_0+\gamma_1 p_2+\gamma_2 u$ where $u,\gamma_2>0$. Thus, based on equation (2) we have two revenue functions $R_d$

$$R_d=\begin{cases}(\gamma_0+\gamma_1 p_1)\,p_1 & \text{; for first model}\\ (\gamma_0+\gamma_1 p_2+\gamma_2 u)\,p_2 & \text{; for second model}\end{cases} \tag{3}$$

and two number of bus functions

$$n=\begin{cases}\dfrac{(\gamma_0+\gamma_1 p_1)}{m}\\ \dfrac{(\gamma_0+\gamma_1 p_2+\gamma_2 u)}{m}\end{cases} \tag{4}$$

### 2.3 Operator's expenses

The main operator's expenses are for maintaining and purchasing bus. We use two kinds of maintenance, preventive maintenance (PM) and corrective maintenance (CM). Maintenance is needed to reduce failure intensity. We define these two different maintenance types as the following.

#### 2.3.1 Preventive maintenance

Let the operator does $N$ times PM in $K$-year period. Then, the interval time between two PM $\tau$ is formulated by $\tau=\dfrac{K}{N+1}$ year. The authors in [1] show that there are two models relate the PM with the failure intensity: Kijima-Type I model and Kijima-Type II model [6,7]. The Kijima-Type I model assumes that the *n*th repair can only remove the damage incurred between the (*n*-1)th and the *n*th failures; therefore it partially reduces the additional age of the system. Meanwhile, the Kijima-Type II model assumes that the *n*th repair will remove the cumulative damage from both current and previous failures. The *n*th repair modifies the virtual age that has been accumulated till to the repair time. Here we assume that due to some reasons, the PM in our case can only remove the damage incurred between the (*n*-1)th and the *n*th failures, and hence we use the Kijima-Type I model in the subsequent discussion.

According to Kijima-Type 1 model, PM turns age of bus $t$ into virtual age $v(t)<t$. Assumed that every PM has the same degree of repair $0\le\delta\le 1$ where $\delta=1$ means minimal repair and $\delta=0$ means perfect repair [2]. As can be seen in Fig. 1, PM reduces failure intensity function to become $\lambda(t)<\lambda_0(t)$ and normal age by $(1-\delta)i\tau$. As a result, the bus virtual age for $i\tau\le t<(i+1)\tau$ is $v(t)=t-(1-\delta)i\tau=i\delta\tau+t-i\tau$. So, the failure intensity function of bus will become $\lambda(t)=\lambda_0(v(t))$. If every cost PM is $C_p^*=a+(1-\delta)\tau b$ then the total cost for $N$ times PM is

$$C_p=Na+N(1-\delta)\tau b=Na+\frac{NbK(1-\delta)}{(N+1)}. \tag{5}$$

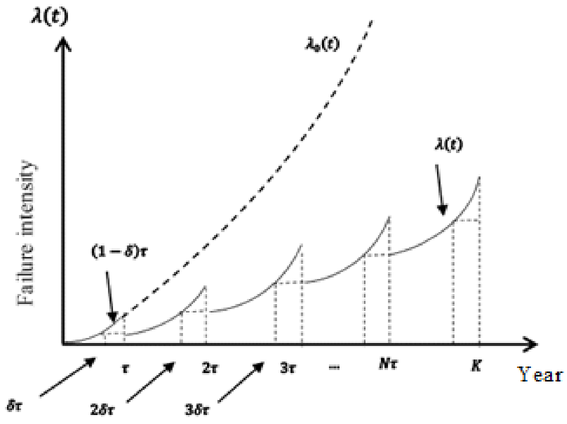



Fig 1. Failure intensity curve after Preventive Maintenance (PM). At $i\tau$, $i=1,2,3,...,N$ the operator does a PM action.

#### 2.3.2 Corrective maintenance

While operating, buses may experience failure at a random time. When failures occur, buses need to be repaired. In this paper, every failure is assumed minimally to be repaired so that the failure intensity is just the same as that just before the failure. Without any PM, failure is Non-Homogenous Poisson Process (NHPP) with failure intensity $\lambda_0(t)$ [5]. After PM, failure process in interval $[i\tau,(i+1)\tau)$ for $i=0,1,2,...,N$ is still NHPP with intensity function $\lambda(t)=\lambda_0(v(t))$. A useful result from NHPP theory is that the expected number of failures to have occurred by a given time is equal to the cumulative intensity function [10]. Thus, the expected total number of failures is $\sum_{i=0}^{N}\int_{i\tau}^{(1+i)\tau}\lambda_0(v(t))dt$. If the cost for every CM is $C_f$ then following [2] the total cost is

$$C_m=C_f.\sum_{i=0}^{N}\int_{i\tau}^{(1+i)\tau}\lambda_0(v(t))dt. \tag{6}$$

### 2.3.3 Bus prices

Another expense for operator is for purchasing bus. If bus price is $w$, then for first subsidy model the operator must pay $w-u$. For the second model, the operator must pay $w$ for purchasing bus from manufacturer.

## 2.4 Operator's profit function

Profit function is the difference between revenue (3) and expense for PM (5), CM (6), and purchasing bus. Operator's profit function for the first subsidy model is given by

$$\Psi_{d_1} = R_{d_1} - \left(w - u + C_p + C_m\right) n_1 \tag{7}$$

and for the second model is

$$\Psi_{d_2} = R_{d_2} - \left(w + C_p + C_m\right) n_2 . \tag{8}$$

## 2.5 Manufacturer's profit function

If the production cost for every bus is $C_r$ then manufacture's profit function for the first model is given by

$$\Psi_{m_1} = \left(w - C_r\right) n_1 , \tag{9}$$

while for second model is given by

$$\Psi_{m_2} = \left(w - C_r\right) n_2 . \tag{10}$$

# 3 Non-cooperative Solution

In the non-cooperative solution, an operator will act as a leader and issue profit policy. Manufacturer will act as a follower and make profit policy based on operator's policy. In the first subsidy model, we determine ticket price $p_1$ that maximize profit function (7) by differentiating $\frac{\partial \Psi_{d_1}}{\partial p_1} = 0$ and $\frac{\partial^2 \Psi_{d_1}}{\partial p_1^2} < 0$ yielding in

$$p_1 = \frac{1}{2m}\left( w - u + C_p + C_m - \frac{\gamma_0 m}{\gamma_1} \right). \tag{11}$$

Substitute (11) into (7), we have

$$\Psi_{d_1}(\max) = -\frac{\gamma_1}{4m^2} A^2 \quad ; \; \gamma_1 < 0 , \tag{12}$$

where $A = \left( w - u + C_p + C_m + \frac{\gamma_0 m}{\gamma_1} \right)$. The number of buses in the first model becomes

$$n_1 = \frac{\left(\gamma_0 + \gamma_1 p_1\right)}{m} = \frac{\gamma_1}{2m^2} A . \tag{13}$$

Thus, manufacturer's profit will be

$$\Psi_{m_1}\left(\max\right) = \frac{\gamma_1}{2m^2}\left(w - C_r\right) A . \tag{14}$$

For the second subsidy model, subsidy is proposed to reduce passenger's ticket price. So it will influence demand function. As we see in (3) the operaor's revenue is $R_{d_2} = \left(\gamma_0 + \gamma_1 p_2 + \gamma_2 u\right) p_2$. To get the operator's maximum profit, it is determined with $p_2$ so that $\frac{\partial \Psi_{d_2}}{\partial p_2} = 0$ and $\frac{\partial^2 \Psi_{d_2}}{\partial p_2^2} < 0$, yielding in

$$p_2 = \frac{1}{2m}\left( w + C_p + C_m - \frac{\gamma_0 m}{\gamma_1} - \frac{\gamma_2 u m}{\gamma_1} \right). \tag{15}$$

This is the optimum ticket price before government subsidy. After subsidy the ticket price will be

$$\begin{aligned} p_2^* &= p_2 - \frac{u}{m} \\ &= \frac{1}{2m}\left( w + C_p + C_m - \frac{\gamma_0 m}{\gamma_1} - \frac{\gamma_2 u m}{\gamma_1} - 2u \right) \end{aligned} \tag{16}$$

Substituting (16) into (8), then the operator's maximum profit is

$$\Psi_{d_2}\left(\max\right) = -\frac{\gamma_1}{4m^2}\left(B^2 - 4u^2\right), \tag{17}$$

where

$$B = \left( w + C_p + C_m + \frac{\gamma_0 m}{\gamma_1} + \frac{\gamma_2 u m}{\gamma_1} \right). \tag{18}$$

The number of buses in the second model becomes

$$n_2 = \frac{\left(\gamma_0 + \gamma_1 p_2^* + \gamma_2 u\right)}{m} = \frac{\gamma_1}{2m^2}\left(B - 2u\right). \tag{19}$$

Thus, manufacturer's profit will be given by

$$\Psi_{m_2}(\max)=\frac{\gamma_1}{2m^2}(w-C_r)(B-2u). \tag{20}$$

Further, we obtain the following proposition regardless the amount of the subsidy.

**Proposition 1**.

The relations $n_1<n_2$ and $p_1<p_2$ always hold simultaneously, regardless the amount of the subsidy.

**Proof:**

$$\begin{aligned} n_1-n_2 &= \frac{\gamma_1}{2m^2}\left(w-u+C_p+C_m+\frac{\gamma_0 m}{\gamma_1}\right) \\ &\quad -\frac{\gamma_1}{2m^2}\left(w+C_p+C_m+\frac{\gamma_0 m}{\gamma_1}+\frac{\gamma_2 um}{\gamma_1}-2u\right) \\ &= \frac{\gamma_1}{2m^2}\left(u-\frac{\gamma_2 um}{\gamma_1}\right)<0 \quad ;\gamma_1<0,\ \gamma_2,u>0 \end{aligned}$$

$$\begin{aligned} p_1-p_2 &= \frac{1}{2m}\left(w-u+C_p+C_m-\frac{\gamma_0 m}{\gamma_1}\right) \\ &\quad -\frac{1}{2m}\left(w+C_p+C_m-\frac{\gamma_0 m}{\gamma_1}-\frac{\gamma_2 um}{\gamma_1}\right) \\ &= \frac{1}{2m}\left(-u+\frac{\gamma_2 um}{\gamma_1}\right)<0 \quad ;\gamma_1<0 \end{aligned}$$

The proof is complete. ■

Remarks: The statement $n_1<n_2$ indicates that we need more buses when using the second model scheme. It is absolutely plausible since it is equvalent to the statement "the higher the number of passengers we have, the higher the number of buses we need". Consequently, we have $\Psi_{m_1}<\Psi_{m_2}$ which means manufacturer has more profit when using the second model scheme. On the other hand, the statement $p_1<p_2$ indicates that before subsidy is given, ticket price for the second model is higher than the first model. But after subsidy is given, $p_1>p_2^*$ which means that ticket price for the second model is cheaper than the first model.

# 4 Case Study

## 4.1 Cummulative failure data

We use 55 buses' first failure data which are collected from operator's maintenance workshop. Using graphical trend test and Mil-Hdbk 189 we find that the data follow NHPP process as shown in Fig. 2.

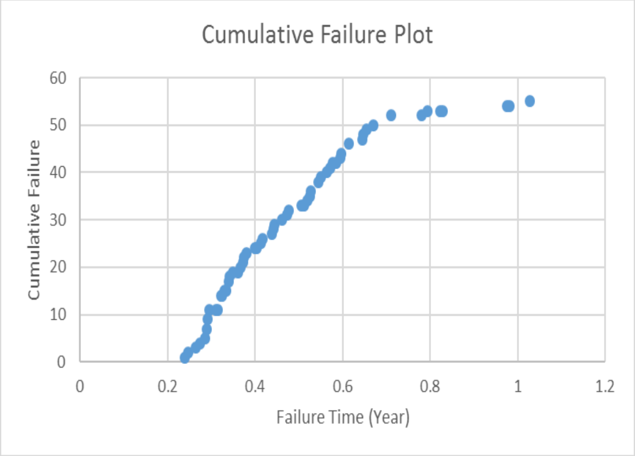



Fig. 2. Cumulative failure plot of bus. Plot looks convex which show that bus is deteriorating over time and follow NHPP process [6].

Using Mil-Hdbk 189 test with hypotheses $H_0$: HPP and $H_1$: NHPP we have $\beta=1.183897$ so $\frac{2k}{\beta}>\chi^2_{\frac{\alpha}{2},\upsilon}$ and $H_0$ is rejected ($k$ : number of data, $\alpha=0.05$, and $\upsilon=k-1$). Next, we fit the data to Weibull $(t;\alpha,\beta,\gamma)$ distribution with special value $\beta=2$ (see Fig. 3.a for the probability plot and Fig. 3.b for the histogram). We apply MLE to find the estimation parameter values. By using Microsoft Excel spreadsheet, the other estimation parameter values are $\alpha=0.355$ and $\gamma=0.151$.

The resulting three-parameter Weibull failure intensity function is $\lambda(t)=\frac{f(t)}{1-F(t)}=\frac{\beta}{\alpha}\left(\frac{t-\gamma}{\alpha}\right)^{\beta-1}$, which after PM take place, the failure intensity function becomes

$$\lambda(t)=\lambda_0(v(t))=\frac{\beta}{\alpha}\left(\frac{(i\delta\tau+t-i\tau)-\gamma}{\alpha}\right)^{\beta-1}.$$

Substituting all the parameter values, we have

$$\lambda_0(v(t))=15{,}87(i\delta\tau+t-i\tau-0{,}151). \tag{21}$$

Hence, the total cost of CM (5) for $K$-years period becomes

$$C_m = \frac{C_f K}{(0{,}355)^2}\left(\frac{K(\delta N+1)}{N+1} - 0{,}302\right). \qquad (22)$$

We also obtain Proposition 2 characterizing the degree of repair. It shows that the value of $\delta = 0$ which indicates that the operator should perform a perfect repair every PM action.

**Proposition 2.**

The Degree of repair and the number of PMs that maximize the operator's profit is $\delta = 0$ and $N = \sqrt{\frac{K\left(C_f K - \alpha^2 b\right)}{\alpha^2 a}} - 1$, where $N$ is a positive integer and $C_f K - \alpha^2 b > 0$.

**Proof:**

$$\Psi_{d_1} = R_{d_1} - \left(w - u + C_p + C_m\right) n_1$$

$$= R_{d_1} - \left(w - u + Na + \frac{NbK(1-\delta)}{(N+1)} + \frac{C_f K}{\alpha^2}\left(\frac{K(\delta N+1)}{N+1} - 2\gamma\right)\right) n_1$$

Hence,

$$\frac{\partial \Psi_{d_1}}{\partial N} = -\left(\left(a + \frac{bK(1-\delta)}{(N+1)^2}\right) + \frac{C_f K}{\alpha^2}\left(\frac{K(\delta-1)}{(N+1)^2}\right)\right) n_1 = 0$$

$$a + \left(\frac{2bK(1-\delta)}{2(N+1)^2}\right) = \frac{C_f K}{\alpha^2}\left(\frac{K(1-\delta)}{(N+1)^2}\right)$$

$$a(N+1)^2 = \frac{C_f K^2 (1-\delta)}{\alpha^2} - bK(1-\delta)$$

$$(N+1)^2 = \frac{K(1-\delta)\left(C_f K - \alpha^2 b\right)}{\alpha^2 a}$$

$$N = \sqrt{\frac{K(1-\delta)\left(C_f K - \alpha^2 b\right)}{\alpha^2 a}} - 1 \;;\; C_f K - \alpha^2 b > 0$$

On the other hand,

$$\frac{\partial \Psi_{d_1}}{\partial \delta} = -\left(\left(-\frac{NbK}{(N+1)}\right) + \left(\frac{C_f K^2 N}{\alpha^2 (N+1)}\right)\right) n_1$$

$$= -\left(\frac{NK(C_f K - \alpha^2 b)}{\alpha^2 (N+1)}\right) n_1$$

Because $C_f K - \alpha^2 b > 0$ then $\frac{\partial \Psi_{d_1}}{\partial \delta} < 0$ (decreasing) thus $\delta = 0$ will maximize operator's profit, so that

$$N = \sqrt{\frac{K\left(C_f K - \alpha^2 b\right)}{\alpha^2 a}} - 1.$$

This completes the proof. ■

Remark: This proposition indicates that for every PM action, a perfect repair is the best option for the operator. The perfect repair can effectively prevent more failures of the buses so that less money is needed for repairing.

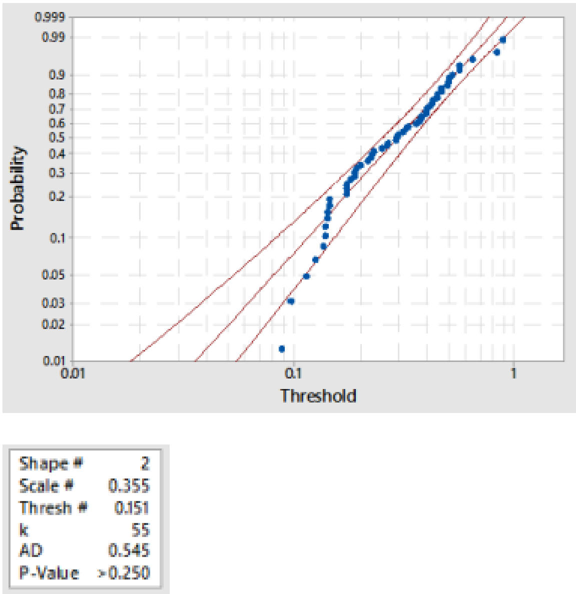



Fig. 3. (a) Probability plot of data for 3-parameter Weibull with significance level 5%.

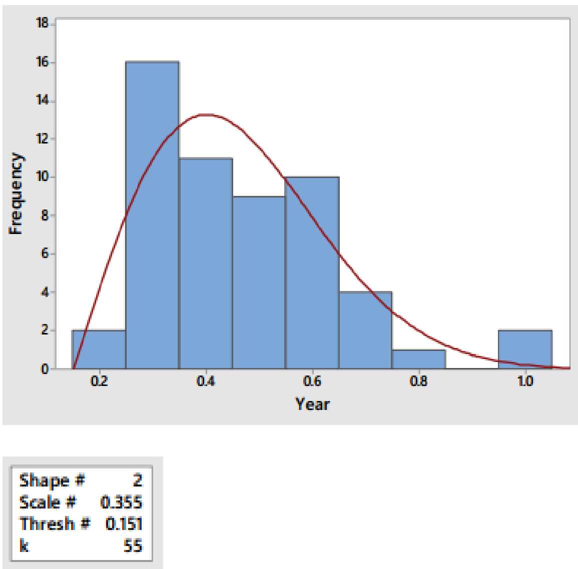



Fig. 3. (b) Histogram with fit the 3-parameter

Weibull distribution

### 4.2 Subsidy model comparison

In this section we consider that failure rate distribution is given by the Weibull distribution $(\alpha,\beta,\gamma)$ in equation (21). Let the parameter values of demand function be $\gamma_0 = 2{,}600{,}000$, $\gamma_1 = -200$ and $\gamma_2 = 0.0006$. These parameters tell us that the maximum passenger is 2,600,000 persons every year. While, for every increasing ticket price as much as IDR 1,000, there will a decrease of 200,000 of passengers. The parameter $\gamma_2 = 0.0006$ tell us that every subsidy as much as IDR 100,000,000, there will be an increase of 60,000 of passengers. Furthermore, let the bus capacity for one year be $m = 54{,}000$ passengers. Other parameter values are given as in Table 1 with the resulting profit shown in Fig. 4. Note that in Fig. 4, D1 and D2 denote the Operator's profit for 1st and 2nd model, while M1 and M2 denote Manufacturer's profit for 1st and 2nd model.

**Table 1.** Nominal value of parameters for simulation purposes

| Parameter | $C_f$ | $a$ | $b$ | $w$ | $C_r$ |
|---|---|---|---|---|---|
| IDR Currency | 600,000 | 500,000 | 300,000 | 750,000,000 | 500,000,000 |

The table shows that the cost of every CM is IDR 600,000 and the cost for minimal repair PM is IDR 500,000. We see that manufacturer profit for every bus is $w - C_r = \text{IDR } 250{,}000{,}000$. A direct computation from the formula in Proposition 2 gives $\delta = 0$ and $N = \sqrt{\dfrac{K\left(C_f K\theta - 2b\right)}{2a}} - 1$ which maximize the profit. Figure 3 shows the resulting profit of the operator and the manufacturer for $K = 3, 5,$ and 10 years.

## 5 Conclusion

We have studied two models of the government subsidy for a transportation operator. This subsidy aims to increase people's interest in using a bus for their daily transportation mode so that traffic congestion reduces, which eventually could reduce the air pollutions in big cities. In the first model, subsidy is purported to purchase bus from manufacturer. While, the second model subsidy is aimed to reduce ticket price for passengers. Upon analyzing the models, we reached the following conclusion:

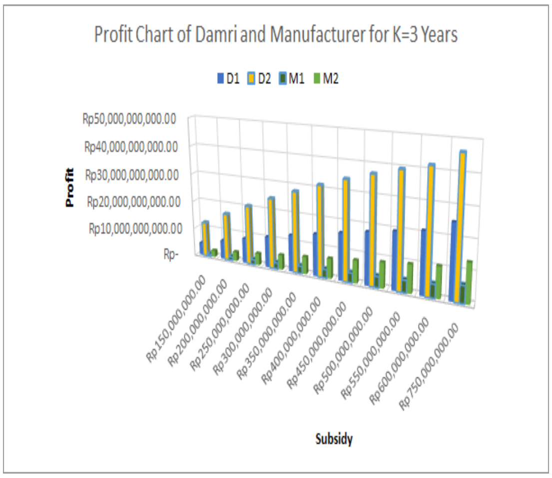
a) Preventive Maintenance (PM) could reduce failure rate and makes bus' operating time longer.

b) To maximize profit, the operator has to do a perfect repair for every PM action.

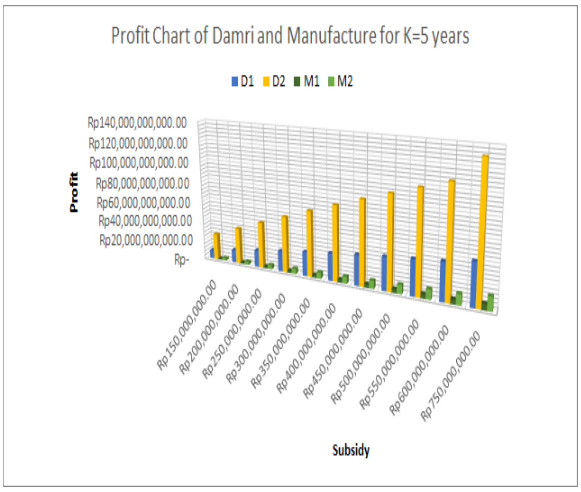
c) The number of passengers in second model is always higher than the first model, which implies the second model needs more buses than the first model. Thus, the second model will give the manufacturer earns more profit.

d) Numerical examples shows that the second model also gives a higher profit to the operator.

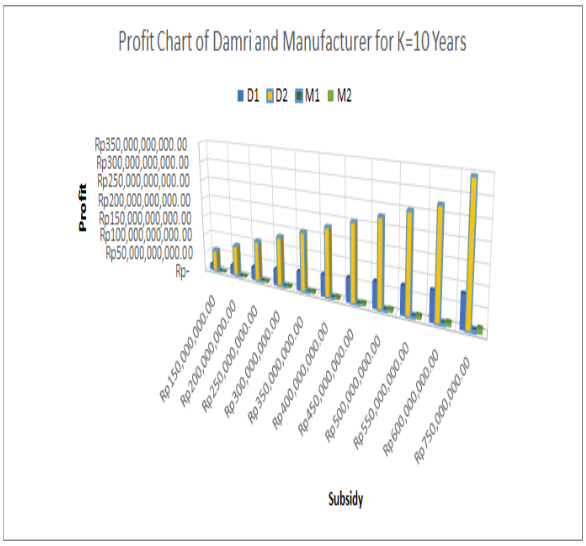
This paper restricted to non-cooperative solution game theory. A further research can apply cooperative solution to the model. This is currently under investigation.




(a)




(b)




(c)

**Fig. 4.** Profit chart of Operator and Manufacturer for several periods.

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