Volatility Forecast Combination of Brazilian Selic interest rate and exchange rate by means of Principal Component Analysis

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Abstract: - Using volatility models for macroeconomic variables can provide more efficient results than models which estimate the average of the process. In this context, the purpose of this research was to evaluate the efficiency of individual models and combination models in forecasting the Brazilian SELIC interest rate and exchange rate between January, 1974 and June, 2012 and from January, 1980 to May, 2012, respectively. The analysis of the series confirmed the presence of volatility in those periods, where Brazil's economic scenario, marked by both internal and external crises, was decisive for such variance. For this purpose, joint modeling was used for the average (ARIMA) and variance (ARCH, GARCH, EGARCH, TARCH) of the process. The results showed that, in general, the performance measures considered (MAPE, MSE, and U-THEIL) are better for forecast combinations. In addition, forecast combinations by PCA using different kinds of weighting were not conclusive for the kinds of weighting used. This shows that when forecast combination by the PCA method is performed, the best alternative is to use more than one type of PCA in order to obtain the best results.

Key-Words: - time series, volatility models, selic interest rate, exchange rate, forecast combination, pca method.

1 Introduction

In a country's economic scenario, the behavior of macroeconomic variables, such as interest rate and exchange rate, can influence other variables and change the economy's performance. The interest rate, for example, plays a significant role in the economy by influencing economic agents, and exchange rate fluctuations directly influence country's imports and exports, thus affecting the balance of trade.

The assessment and measurement of variance within macroeconomic variables is not a simple task, because these variables are closely associated with other variables that depend not only on the Brazilian scenario, but also on the international scenario. However, many studies are currently being conducted to find models where estimates can reflect reality more accurately. These models started

being developed in the 1980 to measure, analyze and forecast the volatility intrinsic to the financial and/or macroeconomic series to reduce the risks of previous investment while trying to increase the expected return, as opposed to previous econometric models that focused on the first condition only.

Thus, measuring the variance of economic series allows decision-makers to obtain better results, reducing risks inherent in financial transactions. Although volatility models are widely applicable to economic series, their use is not restricted; several economic variables use this kind of modeling because they have volatile properties in the series.

The use of models that capture the volatility of the series are important because there are time series whose behavior typically features volatility clustering, i.e., periods in which the variable shows great fluctuations for a long period of time followed

by moments of relative calm. This justifies the use of such models, which enables research on the process that generates the series, producing consistent results because of the stability of the series over time. The macroeconomic series used in this research are the SELIC interest rate and the exchange rate. These series were chosen because they have a strong influence on economic activity and change companies willingness to invest.

The economic literature provides many volatility forecasting models with the aim of forecasting variance more accurately. The most prominent volatility models are known as the ARCH family models. According to Engle [7], Bollerslev [1], Nelson [15], Zakoian [26] and Bueno [2], volatility models have been developed over time, creating terms and parameters that did not exist in their predecessors. Thus, volatility modeling gradually improved and acquired properties of volatile series that had not been captured by early models. In this way, each volatility model has preserved its own properties and contains the parameters that can determine conditional variance (volatility).

According to Rausser and Oliveira [19], Libby and Blashfield [14], Makridakis and Winkler [16] and Werner [25], it is known that forecast combination produces better results than individual models when the level of a time series is forecasted. Thus, the research hypothesis of this study is to verify whether or not forecast combination is also better when applied to volatility models. It also investigates whether or not the use of principal component analysis (PCA) as a way of weighting the different models produces efficient results in the composition of the weights of the combination, as well as in dimensionality reduction.

The objective of this study was to make the forecast combination of the exchange rate and the SELIC interest rate by using the models ARCH, GARCH, EGARCH, and TARCH, using   the weights obtained by principal component analysis as the weighting factor, and verifying which method is the best to perform forecast combination, compared with others methods of combination.

This paper is organized as follows: Section 2 describes volatility models; Section 3 presents performance criteria; Section 4 on describes the methods for forecast combination used in this study; Section 5 contains the analysis and discussion; and Section 6 makes the final considerations, which are followed by the references.

2 Volatility Models

Many mathematical models are used for researching the volatility of financial series, and such models assume that conditional heteroscedasticity exists, that is, non constant variance in the residuals from linear models or from non autocorrelated series.

2.1 ARCH Model

In the literature, models are commonly used to capture intrinsic volatility in financial time series because they allow capturing the behavior of variance in the series, which reveals the risk of making decisions towards these variables as well as the return of such variables, usually the price of assets, interest rates, exchange rates, among others. The - ARCH (p) – model, proposed by Engle in 1982 [7], seeks to estimate volatility by using the variance of previous errors. According to Souza, Souza, and Menezes [23], this kind of model emerges because the series depends on the residuals of the estimation of the process average. Thus, variance is observed to emerge from the volatility that exists between the periods of the residuals.

$$\varepsilon_t = \sqrt{\sigma_t^2} a_t \qquad (1)$$

Where a_t errors are independent and identically distributed with mean 0 and variance 1.

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \cdot \varepsilon_{t-i}^2 = \omega + \alpha(B)\varepsilon_t^2 \qquad (2)$$

Where p corresponds to the model order; α_i is the autoregressive component of the quadratic residuals of the model (ARCH parameter); ε_t^2 correspond to non-serially autocorrelated residuals.

This model must satisfy the assumption that the conditional variance must be positive; thus, the parameter must satisfy the following assumptions: $\omega > 0$ and $\alpha_i > 0$ and $\sum_{i=1}^{p} \alpha_i < 1$ so that the assumption of stationarity is satisfied [3].

2.2 GARCH Model

Bollerslev [1] has proposed the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model, where the author included the past variance of the series into the ARCH model. This way, the model captures the effects of the quadratic errors and of the variance itself at the past instants. Thus, it is possible to obtain a more

economic model, free from the estimation problems of the ARCH model, as shown in equation 4:

$$\varepsilon_t = \sqrt{\sigma_t^2 a_t} \qquad (3)$$

Which can be rewritten as:

$$\varepsilon_t = \omega + \sum_{i=1}^{q} \alpha_i . \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_i . \sigma_{t-j}^2 \qquad (4)$$

Where $\omega > 0$, and $\beta \geq 0$ ensuring that $a_t \geq 0$. Being able to be rewritten as:

$$\sigma_t^2 = \omega + \alpha(B)\varepsilon_t^2 + \beta(B)\sigma_t^2 \qquad (5)$$

2.3 EGARCH Model

Nelson [15] has proposed an extension of the GARCH model, the Exponential GARCH (EGARCH), which captures the asymmetric effects of random shocks suffered by the series. These shocks tend to influence the variance of the series, and there is a difference of impact of these shocks compared to their impact signal. However, negative shocks are usually known to cause greater impact on volatility [24]. For this reason, this model can verify if positive or negative shocks influence volatility with the same weight.

$$\varepsilon_t = \sqrt{\sigma_t^2 a_t} \qquad (6)$$

Where Nelson [15] uses the logarithm (ln) of variance σ_t^2 described in equation 6, thus modifying, the formulation of the model, besides including the term that captures the asymmetry of the series.

$$\ln\sigma_t^2 = \omega + \sum_{j-1}^{p} \beta_j \ln\sigma_{t-j}^2 + \sum_{i=1}^{q} \alpha_i \left| \frac{\varepsilon_{t-i}}{\sigma_{t-i}} - E\left(\frac{\varepsilon_{t-i}}{\sigma_{t-i}}\right) \right|$$
$$+ \sum_{k=1}^{r} \gamma_k \frac{\varepsilon_{t-i}}{\sigma_{t-i}} \qquad (7)$$

Where εt i.i.d. $\sim \mathcal{N}(0.1)$.

The parameter γ adjusts the asymmetry of the effects, evidencing that negative and positive shocks produce a different impact on the volatility of the series. If the effect of leverage is observed, with negative shocks (bad news) causing a greater impact on volatility than positive ones (good news), the parameter must be: $\gamma < 0$.

2.4 TARCH Model

The asymmetries in volatility can be captured by the model developed by [26]. This model is another variant of the GARCH model, referred to as TARCH by some researchers. In the financial markets, there are periods of price downfalls which are often followed by periods of intense volatility, while in periods of price increases, volatility is not so intense. This effect is designated as leverage. The conditional variance of the TARCH (Threshold Autoregressive Conditional Heteroscedasticity) model (p, q) can be defined by:

$$\varepsilon_t = \sqrt{\sigma_t^2 \varepsilon_t} \qquad (8)$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_1 \varepsilon_{t-i}^2 + \cdots + \sum_{j=1}^{s} \alpha_j \sigma_{t-j}^2$$
$$+ \gamma \varepsilon_{t-i}^2 d_{t-i} \qquad (9)$$

Where $d_t = \begin{cases} 1, & \text{se } \varepsilon_t < 0 \ (\text{"Bad news"}) \\ 0, & \text{se } \varepsilon_t > 0 \ (\text{Good news}) \end{cases}$

If $\gamma = 0$, there is no asymmetry in the conditional variance. Negative market forecasts ($\varepsilon_t < 0$), such as an abrupt dollar downfall or political instability, have an impact of $(\alpha + \gamma)$ while positive information ($\varepsilon_t > 0$), for example, intense demand for an asset, has an impact α. To confirm the leverage effect for this model, $\gamma > 0$.

The use of these models favors the behavior analysis of the time series, jointly estimating the average and the process variance, enabling distinct models to capture different behaviors, since the models are not excluding; they are able to complement each other.

3 Evaluation Criteria

This section describes the methodological steps as a way to test the research hypothesis, which is the use of principal component analysis (PCA) as a way to weight the forecast combination, and achieve the

research objective, which is to perform the forecast combination for the volatility of the series of interest rate and SELIC rate, in order to obtain the better results than the models previously used.

First, the series was estimated by the linear models of the ARIMA general class; later, a series of residuals is obtained, with such series presenting the characteristics of white noise. Volatility is then estimated by the models of the ARCH family. This way, the ARIMA-ARCH mixed model is obtained.

After estimating various models for the series, the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC) penalization criteria are used to help to choose the best model, using the maximized value of the Likelihood Function for the estimated model (L), the number of parameters (n) and the size of the sample (T) [23].

$$AIC = -2\ln(L) + 2n \qquad (10)$$

$$BIC = -2\ln(L) + n\ln(T) \qquad (11)$$

As described by Diebold and Lopez [6], there are other evaluation criteria that help to evaluate the performance of the forecasting models. These indicators evaluate precision capacity. Among the accuracy evaluation measures available in the literature, the expressions 12, 13, and 14 will be used; they respectively correspond to: mean absolute percentage error (MAPE), mean squared error (MSE) and U-Theil statistics.

$$MAPE = \frac{1}{n}\sum_{i=1}^{n}\frac{\left|(Z_i - \hat{Z}_i)\right|}{Z_i} \qquad (12)$$

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(Z_i - \hat{Z}_i)^2 \qquad (13)$$

$$U - Theil = \frac{\sqrt{\frac{1}{n}\sum_{t=1}^{n}(\hat{Z}_i - Z_i)}}{\sqrt{\frac{1}{n}\sum_{t=1}^{n}(\hat{Z}_i)^2} + \sqrt{\frac{1}{n}\sum_{t=1}^{n}(Z_i)^2}} \qquad (14)$$

Where n corresponds to the number of standard forecasts carried out, Z_i represents the real value at instant i, and $\hat{Z}_i$ represents the value forecast at instant i.

4 Forecast Combination Techniques

To carry out the forecast combination of the interest rate and exchange rate, the ARCH, GARCH, EGARCH and TARCH individual models were used to obtain the best estimates and consequently the best values forecasted by each model, following the MAPE, MSE, and U-Theil criteria. To compose the proposed combinations, principal component analysis (PCA), Simple Average (SA) and the Ordinary Least Squares (OLS) method will be used.

The first forecast combination technique is PCA, first proposed by Pearson [18] and later by Hotelling [13]. Other researchers, such as Morrison [17], Seber [22], Reinsel [20], and Jackson [11];[12], showed that this method enables the reduction of the data set to be analyzed, for example, when the data are composed by a great number of interrelated variables.

When the set of original variables are transformed into a new set that keeps, at most, the variance of the data set, reduction of information takes place. This new set of transformed variables is called Principal Components (PC), which are independent and non-correlated, thus favoring analysis, especially when many variables have to be analyzed [21].

Each principal component is represented by equation 15:

$$CP_j = a_{1j}X_1 + a_{2j}X_2 + \cdots + a_{pj}X_{p1} = a_jX \qquad (15)$$

Where the vector of constants that must keep the normality condition.

Forecast combination by the PCA method will be carried out by choosing the best individual models out of the four different models - ARCH, GARCH, EGARCH, TARCH. PCA will be performed with the forecasts within the samples of each model previously mentioned. This analysis will allow the construction of weights for each model in the principal components in addition to the weighting of eigenvalues.

The principal component analysis allows the use of more than one weighting factor, which can provide the weights given by the factor loading or factor loadings based on the correlation of the variables - which in this study are models of the ARCH family, or the Variable Contributions weight or contribution of the variables also based on the correlations of the models in each PC. Besides these considerations, weighting will also be used to explain each component as performed by Casarin et al [4]. In addition, verification can take place through the correlation matrix between the PCs and the estimates of the models to find out which model has

greater representation in each component, thus forming the combination based on the strength of expression of each model.

The formulation of the CPs principal components is described in equation 16.

$$CP_j = a_{1j} * X_1 + a_{2j} * X_2 + a_{3j} * X_3 + a_{4j} * X_4 \tag{16}$$

Where is the a_{ij} the factor loadings or the contribution of the models, and X_i is the variables that form the linear combination. Besides, it should be noted that the components to be used in the present research will be the significant ones, that is, the ones whose eigenvalues are bigger than one $(\lambda_j > 1)$.
The forecast combination by PCA will be given by three different weightings to verify if any of the weightings provides a better forecast result according to the performance indicators. The weights of the first combination by PCA will consider the values given by the matrix of factor loadings shown in equation 17. The second PCA is given by weighting the contribution of models in the components represented by equation 18. The third and final PCA combination will be given considering the weight of the explanation of each component according to equation 19.

$$PCA_1 = r_{11} * X_1 + r_{21} * X_2 + ra_{31} * X_3 + r_{41} * X_4 \tag{17}$$

$$PCA_2 = a_{12} * X_1 + a_{22} * X_2 + a_{32} * X_3 + a_{42} * X_4 \tag{18}$$

$$PCA_3 = \lambda_1 * (X_1 + X_2 + X_3 + X_4) + \cdots \\ +\lambda_p * (X_1 + X_2 + X_3 + X_4) \tag{19}$$

Considering that X_1, X_2, ..., X_n are the variables, represented by the ARCH family models; r_{ij} correspond to the factorial load or the correlation between the PCs and the forecasts of the models used; a_{ij} correspond to the contribution of each one of the models in the formation of the PC, that is, the weight that each model has on the formation of the PC, and λ_p represents the eigenvalues referring to the main component and that represents the variance corresponding to the explanation of each component. In this study, eigenvalues bigger than one were selected.
The second combination technique to be used was proposed by Gupta and Wilton in [9], and it is based

on the arithmetic mean of the forecast models as shown below by expression 20.

$$F_c = \frac{ARCH + GARCH + EGARCH + TARCH}{4} \tag{20}$$

Where F_c is the forecast combination, $F_1, F_2, \ldots, F_n$ correspond to the forecasts of the individual models and n is the number of individual models used in the combination. According to Makridakis and Winkler [16], using a combination of arithmetic average is better than using the forecast of a bad model. This method is one of the most frequently used in the literature for the sake of convenience and because of the good results it generates compared to individual analyses. The third technique is the estimation by Ordinary Least Squares (OLS) method, having as regressors the forecasts of individual models and as a dependent variable, the differenced series that gave rise to each model, where it is known that each regressor will have a weight factor to be estimated by considering the time horizon used in the forecast.

$$F_c = \alpha_0 + \beta_1 \, ARCH + \beta_2 \, GARCH + \beta_3 \, EGARCH \\ +\beta_4 \, TARCH \tag{21}$$

Where α_0 corresponds to the linear coefficient of the regression; β_n is the angular coefficient, the one which will give the weight to each model (regressor);
Using the forecast combination by the techniques mentioned above aims to produce better results than those achieved by using individual models. In addition, it can also show which combination method provides better forecast performance, considering the series under study.
Soon after volatility was modeled with ARCH models, the principal component analysis will be used for the combination of forecasts formulating three different kinds of combination by PCA, considering the weight factors: factor loading, the contribution of models and eigenvalues. These will be evaluated jointly to verify which kind of weighting works best. Furthermore, the combinations by PCA will be compared to combinations by OLS and Arithmetic Mean, and also the individual models. These comparisons will be made by the following performance criteria: MAPE, MSE and U-THEIL. The combined forecasts are expected to have better results than individual forecasts. In addition, it is believed that the combination by PCA will provide better results than the others for all criteria because of the differenced method to obtain the weights.

5 Results and Discussion

In this research, two macroeconomic series were used: the SELIC interest rate series from January, 1974 to June, 2012 and the exchange rate series from January, 1980 to May, 2012. They are both relevant in the Brazilian macroeconomic scenario; they influence other economic variables as well as decisions for investments by Brazilian companies.

5.1 Analysis and Modeling of the Time Series

The first step of this research was to perform a visual analysis of the series over time. Figure 1 shows that, apparently, the series are not stationary at level. Also, autocorrelation was observed between observations by means of CAF and PCAF, where the coefficients slowly decrease as the number of lags increases, indicating that there is a strong dependence on the data. After this step, the following tests were performed: Augmented Dickey-Fuller (ADF), and the test proposed by KWIATKOWSKI, PHILLIPS, SCHMIDT, SHIN (KPSS), where it was confirmed that the series are not stationary at level. Then, the first difference was verified in the series, and tests were subsequently redone showing that both series, at first difference, became stationary.
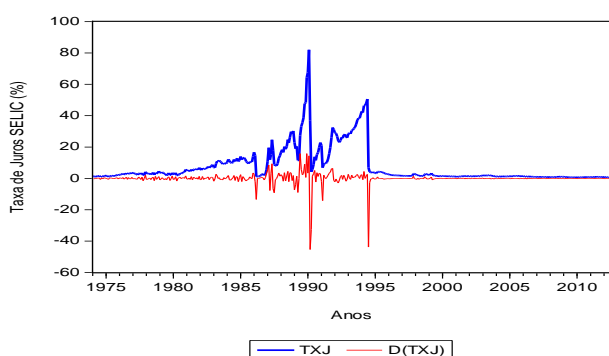

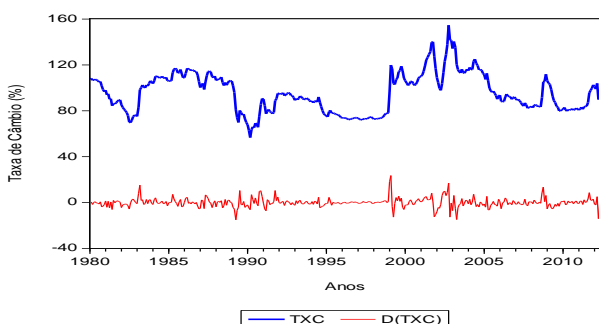Fig. 1 – Behavior of the SELIC interest rate at level and with a simple difference.

Fig. 2 - Behavior of the exchange rate level and with a simple difference.

After modeling of the series, several models were estimated in the search for the most suitable data set, and the selection was made by using the AIC and BIC criteria. Subsequently, the residuals of these models were analyzed and characterized as white noise. Still on the residual analysis, through the Jarque-Bera test, it was found that there are clues to heavy tails (kurtosis> 3) for both modeling. In addition, the residuals do not have a normal distribution with p-value <0.0001. The ARCH-LM test showed that there is heteroscedasticity in the residuals of the quadratic model (p-value <0.0001).

After this first stage of the modeling of the process average, the joint modeling of the series was started, with addition of the modeling of variance by the models of the ARCH family. Thus, Table 1 shows the competing models of joint modeling of the average and volatility. The competing models shown in Table 1 have their significant parameters with 95% confidence. In addition, all models described have white noise residuals. The models that capture the asymmetry in the volatility shocks are EGARCH and THARCH, these models have the parameter γ nonzero, and confirming the asymmetry of information shocks us.

According to Ferreira, Menezes and Mendes [8], the leverage effect exists when $\gamma <0$ for the models EGARCH and $\gamma> 0$ for the model THARCH. For the models in Table 1, the existence of the leverage effect was observed only for the model THARCH (0,1) for the exchange rate. The other models were asymmetric, but there was no effect of leverage.

5.2 Forecast Combinations and Individual Forecasts

The individual models and the combination forecast methods for the two series used in this study were performed with the following software: Statistica 9.1, Eviews 7.1. As shown in the previous section, the models considered for the forecasts were the volatility models ARCH, GARCH, EGARCH, and TARCH. For forecast combination techniques, PCA was used in three ways: PCA_1, weighting the forecasts by the factorial loads of the selected component, PCA_2, weighting the forecasts by the contribution of the models in the PCs; and PCA3, weighting the forecasts by the explanation percentile of the CPs, the Ordinary Least Squares (OLS) Method and the Simple Average (SA).

The PCA technique enables a linear combination of the ARCH models, of each model within their respective components, and the eigenvalues, which are the contributions that will provide the eigenvalues of each component. Therefore, these

Table 1 – Concurrent models for the series of the SELIC interest rate and exchange rate.

Interest Rate				
Coefficients	ARIMA(0,1,1) ARCH (4,0)	ARIMA(0,1,1) GARCH (2,1)	ARIMA(1,1,1) EGARCH (3,1)	ARIMA(1,1,1) TARCH (2,1)
ϕ_1	-	-	- 0,1107	- 0,3302
θ_1	- 0,2585	- 0,4604	-0,2309	- 0,1897
ω	0,02507	0,01455	-0,0455	0,00311
α_1	0,59880	0,34486	0,1461	0,65288
α_2	0,78108	0,68459	0,3535	-0,07976
α_3	0,15738	-	-0,4806	-
α_4	0,63836	-		-
γ_1	-	-	0,3241	-0,56001
β_1	-	0,42845	0,9974	0,79586
AIC	2,4543	2,4689	2,0754	2,2913
BIC	2,5081	2,5137	2,1472	2,3541
Exchange Rate				
Coefficients	ARIMA(0,1,1) ARCH (1,0)	ARIMA(0,1,1) GARCH (1,1)	ARIMA(0,1,1) EGARCH (1,1)	ARIMA(1,1,0) TARCH (0,1)
ϕ_1	0,40265	-	-	0,29733
θ_1	-	0,3287	0,3202	-
ω	6,45918	4,5302	0,3780	1,3404
α_1	0,68887	0,5194	0,6177	-
α_2	-	-	-	-
α_3	-	-	-	-
γ_1	-	-	0,1323	0,0502
β_1	-	0,2340	0,6652	0,88893
AIC	5,2457	5,2397	5,2265	5,5173
BIC	5,2763	5,2805	5,2776	5,5582

Caption: ϕ_1= Auto-regressive parameter of ARIMA model; θ_1= parameter of mobile mean of ARIMA model; ω = constant of the non-linear model; α_i= parameter that represents the squared errors of the ARIMA model; γ_i= parameter that captures the asymmetry effect of the shocks at volatility; β_i= parameter that represents the past variance; ARIMA = Integrated Auto-regressive and of Mobile Means; EGARCH = GARCH Exponential Model; AIC = Akaike Criterion, BIC = Schwarz Criterion. All parameters have p-value < 0,05.

eigenvalues represent the variance of each component explained. According to Johnson and Wichern [12] and Casarin et al. al [4], the significant components are those whose eigenvalues are greater than one. The eigenvalues, eigenvectors and explained variances are shown in Table 2. Table 3 shows the factor loadings and the contributions of each volatility model in the composition of PCs where the ARIMA models are described in Table 1. As noted in Table 2, significant eigenvalues are those greater than one, so only the first principal components will be used to form the forecast combination because they are significant. The results of these weights are given in Table 3.

Table 2 – Result of the eigenvalues and of the ration of the total variance explained to each macroeconomic series.

Forecast by ARIMA Models			
Series	Eigenvalues (λ)	Variance Explained %	Principal Component
Interest Rate	**3,402528**	**85,06319**	CP$_1$
	0,570326	14,25814	CP$_2$
	0,026974	0,67436	CP$_3$
	0,000172	0,00431	CP$_4$
Exchange Rate	**3,914637**	**97,86591**	CP$_1$
	0,084761	2,11903	CP$_2$
	0,000602	0,01505	CP$_3$
	0,000000	0,00000	CP$_4$
Volatility Forecasts			
Series	Eigenvalues (λ)	Variance Explained %	Principal Component
Interest Rate	**3,087804**	**77,1959**	CP$_1$
	0,749491	18,7377	CP$_2$
	0,117876	2,94689	CP$_3$
	0,044830	1,12075	CP$_4$
Exchange Rate	**2,604995**	**65,12488**	CP$_1$
	0,962788	24,06969	CP$_2$
	0,409008	10,22519	CP$_3$
	0,023209	0,58023	CP$_4$

The formulations of forecast combinations obtained by principal component analysis given by eigenvalues (ACP3) are described in the expressions 25 and 28 for the interest rate and exchange rate, respectively. As for the load factor (PCA1) and contributions of models in the components (PCA2), the results are shown in Table 3, considering both series treated in this research. Next, the expressions 23, 24, 26 and 27 show the formulation of the forecast combinations according to the type of weighting used.

Table 3 - Ponderation given by the factorial load and contribution of the model for the series forecast of the ARIMA models and of the volatilities.

Forecast of ARIMA Models			
		Factorial Load	Contribution Model
	Model	CP$_1$	CP$_1$
Interest Rate	ARCH	-0,953324	0,267103
	GARCH	-0,929216	0,253765
	EGARCH	-0,991325	0,288822
	TARCH	-0,804694	0,190309
Exchange Rate	ARCH	-0,991114	0,250932
	GARCH	-0,998644	0,254759
	EGARCH	-0,998208	0,254537
	TARCH	-0,968824	0,239772
Forecast of Volatility			
Interest Rate	ARCH	-0,886424	0,254468
	GARCH	-0,907016	0,266428
	EGARCH	-0,807141	0,210984
	TARCH	-0,909891	0,268120
Exchange Rate	ARCH	0,964857	0,357371
	GARCH	0,962307	0,355484
	EGARCH	0,740007	0,210215
	TARCH	0,447661	0,076929

Combination of the principal component for the series level (ARIMA modeling)

$$PCA_{1(Tx\ juros)} = -0,8864 * ARCH + (-0,9070) * GARCH + (-0,8071) * EGARCH + (-0,9098) * TARCH \quad (23)$$

$$PCA_{1(Tx\ câmbio)} = -0,9911 * ARCH + (-0,9986) * GARCH + (-0,9982) * EGARCH + (-0,9688) * TARCH \quad (24)$$

$$PCA_{2(Tx\ juros)} = (0,2671) * ARCH + (0,2537) * GARCH + (0,2888) * EGARCH + (0,1903) * TARCH \quad (25)$$

$$PCA_{2(Tx\ câmbio)} = 0,250932 * ARCH + 0,254759 * GARCH + 0,254537 * EGARCH + 0,239772 * TARCH \quad (26)$$

$$PCA_{3(Tx\ juros)} = \frac{3,402528}{4} * (ARCH + GARCH + EGARCH + TARCH) \quad (27)$$

$$PCA_{3(Tx\ câmbio)} = \frac{3,914637}{4} * (ARCH + GARCH + EGARCH + TARCH) \quad (28)$$

Combination of the principal component for the volatility of the series (ARCH modeling)

$$PCA_{1(Tx\ juros)} = -0,9533 * ARCH + (-0,9292) * GARCH + (-0,9913) * EGARCH + (-0,8046) * TARCH \quad (29)$$

$$PCA_{1(Tx\ câmbio)} = (0,9648) * ARCH + (0,9623) * GARCH + (0,7400) * EGARCH + (0,4476) * TARCH \quad (30)$$

$$PCA_{2(Tx\ juros)} = (0,2544) * ARCH + (0,2664) * GARCH + (0,2109) * EGARCH + (0,2681) * TARCH \quad (31)$$

$$PCA_{2(Tx\ câmbio)} = (0,3573) * ARCH + (0,3554) * GARCH + (0,2102) * EGARCH + (0,0769) * TARCH \quad (32)$$

$$PCA_{3(Tx\ juros)} = \frac{3,087804}{4} * (ARCH + GARCH + EGARCH + TARCH) \quad (33)$$

$$PCA_{3(Tx\ câmbio)} = \frac{2,604995}{4} * (ARCH + GARCH + EGARCH + TARCH) \quad (34)$$

This way, the forecast combinations were obtained by the three methods by using the PCAs, simple average of the forecasts and the OLS technique. As the purpose of the research is to ascertain the level of performance of the proposed methods based on weightings (factor loadings, contribution model, eigenvalues), the expressions 12, 13 and 14 were used.

In the above expressions, it can be observed that in the forecast combination by means of the proposed methods, at least one of the results was always better than the individual forecasts, when the MAPE, MSE and U-Theil criteria were used. It is noteworthy that through the combinations, it was plausible to include the characteristics of joint modeling ARIMA-ARCH.

The results for interest rate for the performance measures show that all methods of combination provided better results than the individual models, as stated by Clemen [5]. Among individual models, considering the ARIMA modeling, the EGARCH model would be chosen according to the MAPE and MSE criteria, thus confirming the choice by the AIC and BIC criteria. However, when the combinations for PCA are considered, it was observed that, although all combinations provided good results compared to the individual models PCA$_2$ (weighted-input models), the ARIMA modeling had the best results. As for the forecast of volatility, PCA$_3$ (eigenvalue-weighted), outperformed other PCAs and also other SA and OLS combinations.

For the exchange rate, the performance measures show that all combination methods also gave better results than the individual models. When considering the ARIMA models, the model chosen by the MAPE and MSE criteria would be

THARCH, while the AIC and BIC criteria would choose EGARCH. When considering the combinations of PCA, it is found that PCA3 (eigenvalue-weighted) had the best results for the ARIMA models. Between the performance measures for the volatility forecasts, PCA1 (weighting-factor loading) performed better compared to other PCAs and also the other SM and OLS combinations.

Therefore, it is important to use more than one combination of principal components, considering various weight factors to check which one can better fit the models when performance indicators are considered. As evidenced by the results, the forecast combinations provide better results by allowing the relevant characteristics of each concurrent model to be taken into account, contrary to forecasts by an individual model. Thus, it is feasible to obtain better forecasts for the series under study. This means that this methodology can be applied to any type of time series.

6 Conclusion

This paper sought to evaluate different volatility models for the SELIC interest rate and the Brazilian exchange rate to verify, by means of performance measures, which models would achieve better results. The individual models adopted were ARCH, GARCH, EGARCH and THARCH, plus three forecast combination methods (PCA, SA and OLS), where three different weights were considered for the PCA technique, resulting in five forecast combinations. The MAPE, MSE and U-THEIL criteria were used to evaluate the performance of the models and combinations.

The forecast results show that, in general, the combination techniques produce better results than the individual models for the two series (interest rate and exchange rate). Considering the PCA combination techniques for the interest rate for the ARIMA forecasts, PCA_2 outperformed other PCs, and the OLS combination technique had the worst performance. The best and worst performance for the volatility forecasts were PCA_3 and OLS, respectively. For the exchange rate, the best performance was given by PCA_3 when the ARIMA models are considered, and by PCA1for volatility, while OLS produced the worst measures. Thus, it is relevant to use more than one type of combination, in addition to considering more than one weight factor for the combination by Principal Component Analysis.

A suggestion for future research is the comparison with other forecasting models such as VAR-VEC models, econometric models, among others, comparing them with time-series models, and use other techniques such as moving average combination.

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