- pressure difference between the layer and the oil well.

In the presence of solid particles bed rock permeability is reduced, especially when some part of the pores is blocked.

The phenomenon of solid particles penetration is prevented by the use of fluids with particles sized in such a way so as to block the pores without going inside the bed rock (still a several millimeters depth flat-cake is formed inside). If the particles are soluble, then they can be removed. For instance, calcium carbonate particles are dissolved with hydrochloric acid, while the polymers are degraded by hypochlorite or enzymes etc.

Regardless of the type of the mud used for the opening of productive layers, it is desirable that its filter speed should be as small as possible, and the clogging flat-cake to be as thin as possible.

But the "ideal" solution for all of the above issues can be the under-balanced drilling.

When the pressure of the oil well is smaller than the pressure of the crossed bed rocks pores, there are no pressure forces to push the liquid phase and solid particles into the pores.

The forces produced by the difference in chemical potential between mud and layer and by gravity within higher inclinations oil wells generally have less significant effects. The result is an increase in oil wells productivity, at least in the initial phase of the operation.

Moreover, when discovering a productive area and the oil well is under-balanced drilled, gas or oil penetrate into the oil well and reach the surface. Through adequate monitoring of the mud that comes out of the oil well, the presence of hydrocarbon bearing zones can be detected without the need to utilize core and detritus analysis, geophysical logging or layers testing. If the oil well is left to debit, through an appropriate production system at the surface, the oil well productivity can also be quantitatively assessed. Furthermore, the pressure inside the pores and the layer permeability right during the drilling process may be assessed.

At the same time, by reducing the pressure on the oil well's foot, the conditions of bed rocks displacement and detritus disposal are improving. As a result, the advancing speed of the drilling rig will increase. When drilling with air or other gases, the advancing speed of the drilling rig may increase by several times. Consequently, the pressure on the drilling rig can be reduced and the oil wells deviation trend will be reduced.

For the same reasons, the drilling rig working time increases. Under these conditions, as a final result, total costs of operating the oil wells are reduced. Therefore, a model needs to be built in order to assess oil output and thus determine the optimal oil extraction process that matches the managerial purposes of our analysis.

3 Hydro-geological wells productive capacity

To establish hydro-geological wells productive capacity Q (that is the extracted water flow output) it is necessary to know the coefficient of permeability k (determination may be made: using the Darcy equation methodology; with some empirical formulas; when disposing of a group of wells - by pumping from a reference well etc.).

Figure 1 shows the case when the production layer's couch and roof are impermeable, the hydrostatic level coincides with the lower roof, and the well penetrates the entire length of the productive layer.

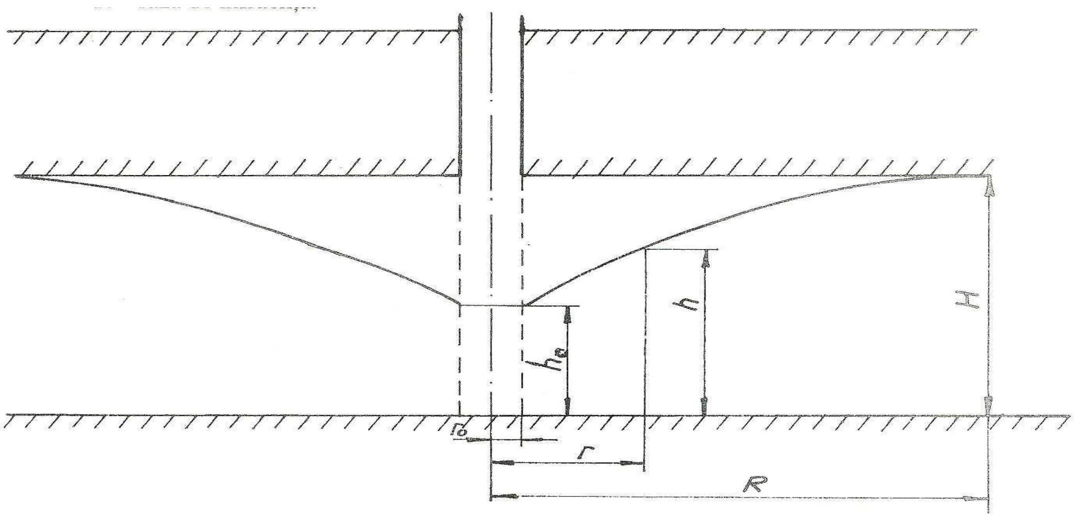

Fig. 1 Perfect oil well with free level layer

Notations in the figure are:

H is the thickness of the productive layer (coincides therefore with the hydrostatic level);

h_o – hydrodynamic level of well's hole;

r_o – radius of well's hole;

R – radius of influence.

According to Darcy's equation, to any point on the piezometric curve (for radius r and level h), the penetration speed is:

$$v = k\frac{dh}{dr} \quad (4)$$

It then results the continuity equation:

$$Q = vA \quad (5)$$

where:

$$A = 2\pi rh \quad (6)$$

thus leading to:

$$Q = k\frac{dh}{dr}2\pi rh \quad (7)$$

and by separating variables:

$$Q\frac{dr}{r} = 2\pi khdh \quad (8)$$

After integration, when the limits are r_o and R for r, and h_o and H respectively for h, Q is reached:

$$Q = \frac{\pi k\left(H^2 - h^2\right)}{\ln\frac{R}{r_o}} \quad (9)$$

which is the hydro-geological well's productive capacity we were seeking.

4 Models for the production of oil deposits

To estimate the production of crude oil from deposits, analytical models have been developed corresponding to different geometric configurations. Some models consider oil as an incompressible fluid, and others (more complex) take account of its compressibility. Finally, the motions to be modeled shall be either stationary - independent of time -, or non-stationary.

The following incompressible liquid stationary motions are considered, characterized by the condition: dp / dt = 0, at any point in the motion range and at any time t. Fundamental equations solutions obtained for these motions can be used either to qualitatively study certain aspects of non-stationary motions, or to study non-stationary motions through methods that assimilate them with a succession of stationary states.

4.1 Dimensional motion in a homogeneous porous environment

Let's consider a homogeneous and isotropic porous environment, of rectangular shape (figure 2), with waterproof sides and permeable bases (perpendicular to the axis *Ox)*. Through this porous environment incompressible fluid filters between pressure p_c and p_s ($p_c > p_s$) constant in time (for the motion to be stationary), at the volume flow rate *Q*.

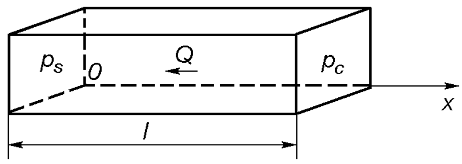

Fig. 2 One-dimension motion range of a liquid through a homogeneous porous environment

Fundamental equations of the motion are: linear filtration equation (equation of Darcy), continuity equation and state equation, which in terms of a single component of filtration velocity and incompressibility of the liquid may be reduced to the following relations:

$$v = v_x = -\frac{k}{\mu}\frac{\mathrm{d}p}{\mathrm{d}x}, \quad \frac{\mathrm{d}}{\mathrm{d}x}(\rho v_x) = 0, \quad \rho = \rho_0 = \mathrm{const} \quad (10)$$

Replacing from (10) the first and the third equation into the second, we obtain the differential equation of motion:

$$\frac{\mathrm{d}^2 p}{\mathrm{d}x^2} = 0, \quad (11)$$

whose solution:

$$p = ax + b, \quad (12)$$

is associated with the boundary conditions:

$$\begin{cases} \text{la } x = 0,\ p = p_s, \\ \text{la } x = l,\ p = p_c. \end{cases} \quad (13)$$

Replacing the solution at (12) to the boundary conditions (13) we obtain the equality:

$$p_s = b, \quad p_c = al + b,$$

leading to the equations of integration constants:

$$a = \frac{p_c - p_s}{l}, \quad b = p_s$$

while pressure variation law (12) becomes:

$$p = p_s + \frac{p_c - p_s}{l} x \ . \qquad (14)$$

Introducing derivative dp / dx obtained from the equation (14) into the first relation (10) then the filtration rate equation is:

$$v = -\frac{k}{\mu}\frac{p_c - p_s}{l} \ , \qquad (15)$$

which replaced into the macroscopic continuity equation:

$$Q = A|v| , \qquad (16)$$

where A is the cross-sectional surface area of the porous environment, gives the volume flow equation we were seeking:

$$Q = \frac{Ak(p_c - p_s)}{\mu l} \ . \qquad (17)$$

4.2 Two-dimensional motion in a homogeneous porous environment

4.2.1 Flat radial motion

Let us consider an oil well that crosses the entire thickness of the productive wall, receiving fluid through its natural wall, and call it *the perfect oil well from a hydrodynamic point of view*. If the productive layer is horizontal with a constant thickness h, and the oil well produces the constant pressure p_c, at a constant pressure p_s of a coaxial cylindrical zone with the outer border of radius r_c, (figure 3), then the radial motion is flat radial stationary, generally speaking.

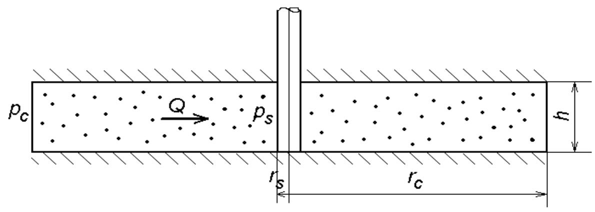

Fig. 3 Configuration of the flat radial motion of a fluid through a homogeneous porous environment

Notation used in figure 3 has the following meanings:

- r_s – oil well radius;
- r_c – supply radius contour (border);
- p_s – oil well dynamic pressure at a certain depth (measured from the average depth of perforated interval),
- p_c - static pressure of productive layer, called the supply pressure on the perimeter.

Providing the stationary character of the motion generated by the oil well, the condition that the pressure p_c is constant is satisfied if through the outer border radius r_c, which must be permeable and it is named *supply border*, a quantity of fluid equal to that produced by the oil well enters into the drainage area of the oil well.

Given the axial symmetrical character of the motion, cylindrical coordinates for flat motion are reduced to polar coordinates r, q. Under these conditions, microscopic continuity equation associated with state equation of incompressible liquid is reduced to:

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}(r v_r) = 0 \ , \qquad (18)$$

where the radial velocity, which is the only component of the velocity filter, is given by Darcy's law:

$$v = v_r = -\frac{k}{\mu}\frac{\mathrm{d}p}{\mathrm{d}r} \ . \qquad (19)$$

From relations (18) and (19) follows the differential motion equation:

$$\frac{\mathrm{d}}{\mathrm{d}r}\left(r \frac{\mathrm{d}p}{\mathrm{d}r} \right) = 0 \ , \qquad (20)$$

that is successively integrated until leading to the solution:

$$p = a \ln r + b, \qquad (21)$$

which is the pressure variation law and is associated with boundary conditions (as shown in figure 3):

$$\begin{cases} \text{for } r = r_s \ , \ p = p_s \ , \\ \text{for } r = r_c \ , \ p = p_c \ . \end{cases} \qquad (22)$$

By replacing the conditions (22) into (21) we obtain the equation for the two constants of integration:

$$a = \frac{p_c - p_s}{\ln\frac{r_c}{r_s}} , \ b = p_s - \frac{p_c - p_s}{\ln\frac{r_c}{r_s}} \ln r_s = p_c - \frac{p_c - p_s}{\ln\frac{r_c}{r_s}} \ln r_c$$

and pressure variation law (21) becomes:

$$p = p_s + \frac{p_c - p_s}{\ln\frac{r_c}{r_s}} \ln\frac{r}{r_s} = p_c - \frac{p_c - p_s}{\ln\frac{r_c}{r_s}} \ln\frac{r_c}{r} \ . \qquad (23)$$

If the pressure derivative dp / dr is replaced in the Darcy equation, the filtration velocity equation is established:

$$v = -\frac{k}{\mu}\frac{p_c - p_s}{\ln\frac{r_c}{r_s}}\frac{1}{r} \ , \qquad (24)$$

which introduced into the macroscopic continuity equation (16), where the normal section at filtration velocity at any point has the form of a cylinder of any radius r , the area of this section being:

$$A = 2\pi r h , \qquad (25)$$

leads to the equation of the incompressible fluid flow volume of the oil well:

$$Q = \frac{2\pi k h (p_c - p_s)}{\mu b_t \ln \frac{r_c}{r_s}} . \qquad (26)$$

In the previous equation we have considered an oil well. For the oil flow to be expressed at the surface conditions (at the pressure in the storage tank that is practically equal to atmospheric pressure), in the denominator of equation (26) the oil volume factor b_t was introduced, being defined as the amount of oil at deposit conditions that corresponds to the oil unity volume under surface conditions. Given that, at the deposit pressure, the oil contains a significant amount of dissolved gases, which are separated in the liquid phase prior to its tank storage, volume factor of the oil has a value higher than the unity.

Figure 4 shows the variation graphics of filtration speed and pressure as related to the radius. It is noted that the maximum values of gradients pressure and velocity are recorded in the vicinity of the oil well, which highlights the importance of this area to the oil well production flow.

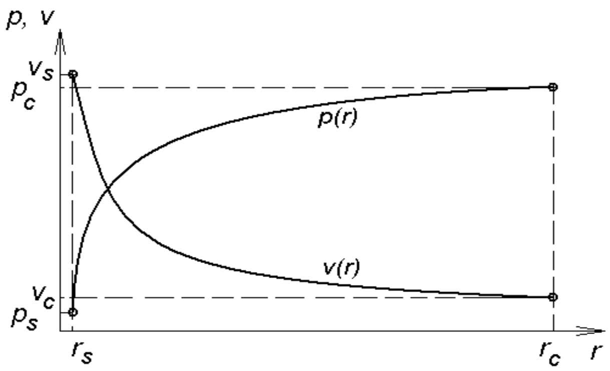

Fig. 4 Graphic of filtration speed and pressure as related to the radius, according to the stationary flat radial motion of a liquid

The ratio between the oil well flow and differential pressure of production is called *the oil well productivity index* and has the expression:

$$I_p = \frac{Q}{p_c - p_s} = \frac{2\pi k h}{\mu b_t \ln \frac{r_c}{r_s}} . \qquad (27)$$

Specific productivity index is the ratio between I_p and the thickness h of the productive layer, ie:

$$I_{ps} = \frac{I_p}{h} = \frac{Q}{h(p_c - p_s)} = \frac{2\pi k}{\mu b_t \ln \frac{r_c}{r_s}} , \qquad (28)$$

while the capacity:

$$C = k h \qquad (29)$$

is called *the collector oil layer production capacity*. The parameters I_p and I_{ps} directly characterize the performance of the oil extraction well.

From equations (26) and (27) we see that productivity index is the slope Q $(p_c$ - $p_s)$, and is called *the indicating diagram* (figure 5). In the area of validity, for Darcy's equation upper limited by the critical point C, the indicating diagram is the straight line described by the equation (27), while in nonlinear filtering the indicating diagram is a curve with decreasing slope.

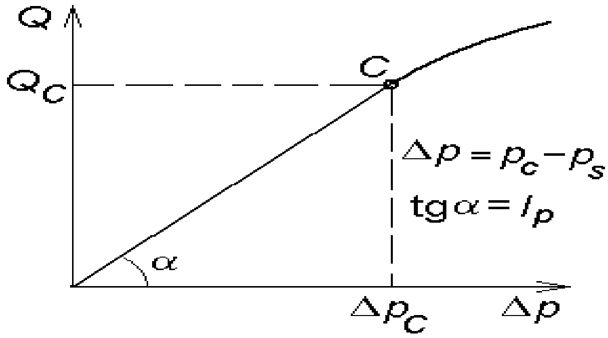

Fig. 5 Graphic of volume flow according to the differential pressure under flat radial motion of a stationary liquid

One of the main purposes of oil extraction technology is to increase productivity index of each well to the maximum possible values from an economical point of view. This process is known as *stimulation of wells productivity* and can be done through various ways, which can be deduced from the analysis of how each factor in relation (27) should be amended to increase the productivity index.

Increasing the permeability k through hydraulic acidizing or cracking and thermal methods for viscosity lowering (as cyclic steam injection in the productive layer) are the main ways to boost productivity of oil wells.

By stimulation, oil recovery period is reduced as effect of increasing the extraction rate, but this does not necessarily imply obtaining an increase of the final recovery factor as well.

Methods to increase the final factor involve enhancing the natural energy of a deposit.

Thus, while displacement (washing) crude oil with steam, which is a way of increasing the ultimate recovery factor, leads to an increase of the total energy of the system, most of the heat supplied to the deposit in the cyclic steam injection (which is a method of stimulating oil wells productivity) is lost during production as a result of thermal conduction towards impermeable layers from the collector's roof and couch, and during heat transport to the surface at the same time with heated fluids produced by the oil well.

4.2.2 Motion generated by an oil well eccentrically located in a circular contour supply deposit

Let us consider the motion generated by an oil well under the conditions mentioned in the previous paragraph, the only difference being that the oil well is placed eccentrically, at a distance *d* from the center of the circular contour supply of radius r_c.

To study this motion, the method of complex variable functions associated to flat potential motion can be used. In this regard it is noted that if the function is defined as:

$$\varphi = -\frac{k}{\mu} p \ , \qquad (30)$$

velocity components presented according to Darcy's law as follows:

$$v_x = -\frac{k}{\mu}\frac{\partial p}{\partial x} \ , \ v_y = -\frac{k}{\mu}\frac{\partial p}{\partial y} \ , \qquad (31)$$

take the form:

$$v_x = \frac{\partial \varphi}{\partial x} \ , \ v_y = \frac{\partial \varphi}{\partial y} \ , \qquad (32)$$

showing that the two-dimensional motion of single-phase incompressible fluids in porous homogeneous environments behaves like a potential motion with speed potential *j*. Neglecting the oil well radius r_s related to radius r_c of the supply contour, the oil well can be assimilated to a linear distribution of negative sources.

The complex potential of the motion generated by the oil well in the plane *xOy* in figure 6 is given by:

$$f_1(z) = -\frac{Qb_t}{2\pi h}\ln(z+\delta) \ , \qquad (33)$$

where $z = x + iy$, *Qy and* Qb_t are expressed as oil flow at oil deposit conditions (corresponding to flow *Q* at surface conditions), while complex number z_1 which defines the position of the oil well to the axis system is expressed as:

$$z_1 = x + \delta + i\,y = z + \delta$$

The oil well was considered as a negative flat source, S_1, because it absorbs fluid from the motion, while intensity (flow) flat source is the ratio between flow rate and length *h* of the linear source.

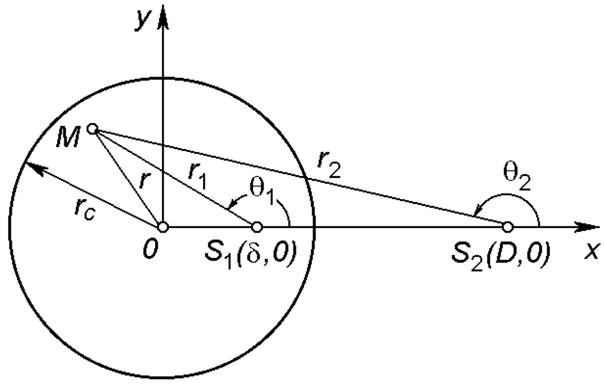

Fig. 6 The two flat sources equivalent to the motion generated by an eccentrically located oil well

For the extension of the motion (which can nly take place within the circle of radius $r_{c)}$ throughout the plane *xOy*, a positive source S_2 (considered as an injection well, with intensity, or volume flow, + *Q)* is introduced, symmetric to the circle in point S_2, located from the field motion center at a distance:

$$D = r_c^2 / \delta \, . \qquad (34)$$

Thus, the motion generated by the eccentric oil well of an oil deposit can be studied using the model of flat potential motion generated by two sources of opposite sign.

As a result, the complex potential of source S_2 is expressed as:

$$f_2(z) = \frac{Qb_t}{2\pi h}\ln(z+D) \ , \qquad (35)$$

and the complex potential of the resulting motion is obtained by summing the complex potential of the two sources as follows:

$$f(z) = -\frac{Qb_t}{2\pi h}\ln(z+\delta) + \frac{Qb_t}{2\pi h}\ln(z+D) = \frac{Qb_t}{2\pi h}\ln\frac{z+D}{z+\delta} \ . \qquad (36)$$

Writing that:

$$z_1 = z + \delta = r_1 e^{i\theta_1} \ , \ z_2 = z + D = r_2 e^{i\theta_2} \ ,$$

equation (36) becomes:

$$f(z) = \frac{Q\,b_t}{2\,\pi\,h}\left[\ln\frac{r_2}{r_1} + i(\theta_2 - \theta_1)\right] = \varphi + i\,\psi \ , \qquad (37)$$

and the real part becomes:

$$\varphi = \frac{Qb_t}{2\pi h}\ln\frac{r_2}{r_1} + \varphi_0 \ , \qquad (38)$$

While introduced in (30), gives the pressure equation:

$$p=-\frac{Q\mu b_t}{2\pi k h}\ln\frac{r_2}{r_1}+C\ , \qquad (39)$$

where r_1, r_2 are the bipolar coordinates defined in figure 6, and j_0 is the potential value at speed $r = r_s$.

To express the boundary conditions it is assumed that the point *M* in figure 6 is successively owned by the oil well wall and supply contour.

If *M* belongs to the border of radius r_s, the first boundary condition takes the form:

$$\text{for } r_1 = r_s \ \textit{and} \ r_2 = D-\delta,\quad p = p_s\ . \qquad (40)$$

If *M* is on the outer border of the oil well area (figure 7), based on the relationship of symmetry to the circle (40) it results that triangles OS_1M şi OS_2M are alike.

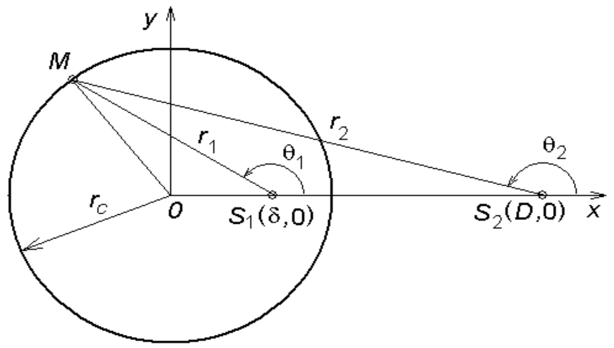

Fig. 7 Illustration of the condition that the point M belongs to the outer border of the eccentric oil well drainage area

We write the equations expressing the two triangles' side proportionality:

$$\frac{r_c}{\delta}=\frac{D}{r_c}=\frac{r_2}{r_1}$$

and the second boundary condition as follows:

$$\text{la}\ \frac{r_2}{r_1}=\frac{r_c}{\delta},\ \ p=p_c\ . \qquad (41)$$

Substituting the boundary conditions (40) and (41) in equation (39), the following equations are obtained:

$$\begin{cases} p_s=-\dfrac{Q\mu b_t}{2\pi k h}\ln\dfrac{r_c^2-\delta^2}{\delta r_s}+C\ , \\ p_c=-\dfrac{Q\mu b_t}{2\pi k h}\ln\dfrac{r_c}{\delta}+C\ , \end{cases} \qquad (42)$$

For constant *C* and oil well flow rate *Q*, from equations (42) results:

$$C=p_c+\frac{p_c-p_s}{\ln\dfrac{r_c^2-\delta^2}{r_c r_s}}\ln\frac{r_c}{\delta}\ , \qquad (43)$$

$$Q=\frac{2\pi k h\left(p_c-p_s\right)}{\mu b_t\ln\dfrac{r_c^2-\delta^2}{r_c r_s}}\ . \qquad (44)$$

Introducing expressions (43) and (44) into (39), pressure variation law in bipolar coordinates becomes:

$$p=p_c-\frac{p_c-p_s}{\ln\dfrac{r_c^2-\delta^2}{r_c r_s}}\ln\frac{\delta}{r_c}\frac{r_2}{r_1}\ . \qquad (45)$$

Returning to Cartesian coordinates, then multiplying by 2 the numerator and denominator from the right ratio of the equation (45) in order to eliminate radicals, we obtain the equation:

$$p=p_c-\frac{p_c-p_s}{2\ln\dfrac{r_c^2-\delta^2}{r_c r_s}}\ln\frac{\delta^2\left[(x+D)^2+y^2\right]}{r_c^2\left[(x+\delta)^2+y^2\right]}\ , \qquad (46)$$

expressing the pressure variation law in Cartesian coordinates.

4.2.3 Motion generated by an oil well in a deposit with linear supply contour

If, under the conditions from the preceding paragraph, we note the oil well finite distance to the contour (r_c – δ) with *d* and admit that the supply border has an infinite curvature, then this border will practically be linear and of infinite length. To extend the motion range from the upper semi plane *xOy* to the entire plane, a fictitious positive source of strength + *Q* is introduced, symmetrical with the supply contour (figure 8).

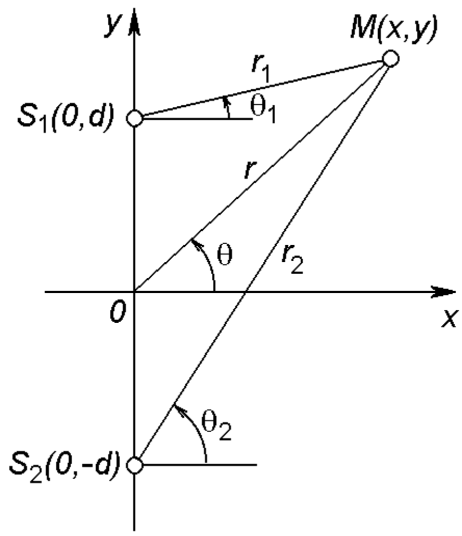

Fig. 8 The two sources system equivalent to motion generated by an oil well located in the vicinity of a supply infinite linear border

Adding the complex potentials of the two sources, expressed as:

$$f_1(z)=-\frac{Qb_t}{2\pi h}\ln(z-id)\,,\quad f_2(z)=\frac{Qb_t}{2\pi h}\ln(z+id)\,,\quad (47)$$

for the complex potential of the resulting motion the following equation is obtained:

$$f(z)=\frac{Qb_t}{2\pi h}\ln\frac{z+id}{z-id}\ . \quad (48)$$

To separate the real part of complex potential (48), we write the complex numbers z_1 and z_2 as follows:

$$z_1=r_1\mathrm{e}^{i\theta_1}\ ,\ z_2=r_2\mathrm{e}^{i\theta_2}$$

and for *f (z)* an expression identical to function (37) is obtained, where r_1 and r_2 are bipolar coordinates defined in figure 8. The speed potential *j* and pressure *p* are expressed by equations (38) and (39).

The boundary conditions associated to equation (39) occurs if point *M* belongs to the oil well border (circle of radius r_s), supply contour *(Ox* axis) respectively, and can be expressed by the following relations:

$$\begin{cases}\text{for } r_1=r_s \text{ and } r_2=2d\ ,\quad p=p_s\ ,\\ \text{for } r_1=r_2\ ,\quad p=p_c\ .\end{cases} \quad (49)$$

By applying these conditions to the pressure equation (49) we obtain the equation system:

$$\begin{cases}p_s=-\frac{Qb_t}{2\pi kh}\ln\frac{2d}{r_s}+C\ ,\\ p_c=C\ ,\end{cases} \quad (50)$$

resulting the equation:

$$\frac{Q\mu b_t}{2\pi kh}=\frac{p_c-p_s}{\ln\frac{2d}{r_s}}\,,$$

that allows expressing the volume flow as follows:

$$Q=\frac{2\pi kh(p_c-p_s)}{\mu b_t\ln\frac{2d}{r_s}}\ . \quad (51)$$

If the second relation in (50) is replaced in equation (39) and then changed to Cartesian coordinates, the variation pressure law is:

$$p=p_c-\frac{p_c-p_s}{2\ln\frac{2d}{r_s}}\ln\frac{x^2+(y+d)^2}{x^2+(y-d)^2}\ . \quad (52)$$

If the supply border has a finite length, equal to 2, and the oil well is symmetrically placed to the ends of the border, the relation (51) takes the form:

$$Q=\frac{2\pi kh(p_c-p_s)}{\mu b_t\ln\left[\frac{2d}{r_s}\left(1+\frac{d^2}{a^2}\right)\right]}\ , \quad (53)$$

showing that if d^2/a^2 is negligible compared to the unity (i.e. oil well is situated at a relatively short distance from a supply contour of relatively high length), supply border behaves as if it had an infinite length.

5 Estimation of oil deposit reserves through production decline method

In order to characterize the rate of decrease in production of an oil deposit, *the declining oil production* term was introduced, which can be defined both as *the actual decline:*

$$D_e=\frac{Q_i-Q}{Q_i}\ , \quad (54)$$

or *the nominal decline:*

$$D=-\frac{\mathrm{d}}{\mathrm{d}t}(\ln Q)=-\frac{1}{Q}\frac{\mathrm{d}Q}{\mathrm{d}t}\ . \quad (55)$$

The actual decline is a step-by-step type function (monthly, quarterly, annual etc.., depending on the time span referred to flow *Q)*, while the nominal decline, which is a continuous function, having a positive value and being defined as the slope of *lnQ = f (t)* in a current point, works much better for a theoretical interpretation.

By using production input data from a large number of oil deposits it is demonstrated that the graphics showing the deposits production decline can be characterized by three types of nominal declining: constant, hyperbolic and harmonic.

5.1 Constant production decline

If the production decline is constant, equation (55) becomes:

$$\ln\frac{Q_i}{Q}=Dt\ , \quad (56)$$

that allows writing the following law for flow variation:

$$Q=Q_i\,\mathrm{e}^{-Dt}\ , \quad (57)$$

Hence, the cumulative oil production will be defined by the relationship:

$$N_p = \int_0^t Q \, \mathrm{d}t \qquad (58)$$

which, based on equation (57), becomes:

$$N_p = \frac{Q_i}{D} \mathrm{e}^{-Dt} \Big|_0^t = \frac{Q_i - Q}{D} . \qquad (59)$$

As the flow exponentially decreases over time, in some works, the constant decline is improperly called as the exponential decline.

The moment to abandon the deposit (that is the time operation length), based on the output of abandonment Q_a and established on economic criteria, results from the equation (56):

$$t_a = \frac{1}{D} \ln \frac{Q_i}{Q_a} . \qquad (60)$$

5.2 Hyperbolic production decline

This type of decline is expressed as:

$$D = c\, Q^n , \qquad (61)$$

where: c is the decline coefficient ,

n - decline exponent.

Replacing the previous relationship in equation (55), the following equality is obtained:

$$-\frac{1}{Q} \frac{\mathrm{d}Q}{\mathrm{d}t} = c\, Q^n , \qquad (62)$$

where, after separation of variables, integration and rearrangement of the terms, the flow equation is obtained:

$$Q = \frac{Q_i}{(1 + n\, D_i\, t)^{1/n}} . \qquad (63)$$

Thus, the decline coefficient c can be expressed in terms of the initial decline D_i and the initial flow rate Q_i as follows:

$$c = D_i / Q_i^n . \qquad (64)$$

As one may see, for $n = 0$, equation (61) corresponds to the constant decline.

Under such conditions, the cumulative oil production is given by equation (59) associated with equation (63). After integration the following equation is obtained:

$$N_p = \frac{Q_i^n}{(1-n) D_i} \left(Q_i^{1-n} - Q^{1-n} \right). \qquad (65)$$

The moment of abandonment results then from equation (65), as follows:

$$t_a = \left[\left(\frac{Q_i}{Q_a} \right)^n - 1 \right] \frac{1}{n D_i} . \qquad (66)$$

5.3 Harmonic production decline

Harmonic decline is the particular case for the corresponding hyperbolic decline when $n = 1$. As a result, relations (61) to (66) become:

$$D = cQ,\ Q = \frac{Q_i}{1 + D_i t},\ c = \frac{D_i}{Q_i},\ t_a = \left(\frac{Q_i}{Q_a} - 1 \right) \frac{1}{D_i}, \qquad (67)$$

and based on equation (58) and the second equality (67), oil cumulative production is expressed as follows:

$$N_p = \frac{Q_i}{D_i} \ln \frac{Q_i}{Q} . \qquad (68)$$

Extrapolation of production decline curves is one of the oldest and most frequently used methods of engineering to oil deposits. It consists in providing flow and cumulative production based on the equation (61), by determining the exponent n that reproduces production data recorded on a conclusive time span.

A new focus in the practice of decline curve analysis was established in 1968 by Slider, by introducing the method of theoretical curves overlap (drawn on transparent paper) against the curves obtained from the oil production database. In 1972, Gentry drew two graphics that can be used for rapid extrapolation of hyperbolic and harmonic type decline curves.

In some cases, due to the effects of deposits' physical characteristics (layers with different permeability opened throughout the same oil well), of reservoir fluid properties and of mechanisms of primary recovery, values greater than the unit were obtained for decline exponent n. Nevertheless, extrapolation of flow-time curves is one of the most effective methods used to estimate the primary resources of crude oil.

Taking into consideration these findings, the following application aims at modeling the oil extraction process.

6 Modelling the oil extraction process

Building the model of an oil extraction process requires writing the mathematical model whereby an input data needs to be identified, as well as the variables that describe this extraction process, as follows:

6.1 Model parameters

a. Input data. Firstly, we have to identify the data to be used in our model:

- n - total number of oil wells in operation in the oil drilling and extraction unit;
- Δ_i - unit flow of oil well i that extracts the oil, $i = 1, 2, ..., n$, <t/h>;
- ε_I - average power absorbed by oil well i, <kw>;
- C_j - tank j storage capacity, $j = 1, 2, \ldots, m$, <t>;
- α - separation coefficient on the first track ($\alpha < 1$);
- β - percentage of petroleum gas ($\beta < 1$);
- α_2 - separation coefficient on the second track ($\alpha_2 = 1 - \alpha$)
- α_1 – degasolination coefficient ($\alpha_1 < 1$, $\alpha_1 = \alpha - \beta$)
- N - number of intervals within the time frame;
- $f_1(k)$ - amount of oil that is delivered to clients on the first track, for each time interval, $K = 0, 1, ..., N-1$, <t>;
- $f_2(k)$ - idem, on the second track, <t>;
- p - scheduled volume of extraction, <t>;
- $f_3(k)$ - amount of petroleum gas to be delivered to clients according to delivery schedules, $<m^{3>;}$
- E - electricity amount allocated to the oil well throughout the time horizon, <kwh>;
- x^s_j - value of safety oil stock in tank C_j;

b) Variables. The second step is to set up the variables that will better describe the oil extraction process:

- U_i(K) - control variable representing the lifetime of the oil well on time interval K, <hours>;
- $X_j(K)$ - state variable showing oil stock in tank C_j at time K, <T>.

6.2 Model equations

The next step is to define the equations that will help building up the model, as follows:

a) State equations. For each variable $X_j(K)$ we write a state equation describing the evolution of oil stock in tank C_j at two consecutive time moments K and K_1. The equations are:

$$X_j(K+1) = X_j(K) + \alpha_j \cdot Q(K) - f_j(K) \quad (69)$$

b) The equation for extraction distribution by oil wells. This equation allocates the amount of extracted oil $Q(K)$ to each oil well, resulting therefore in oil wells extraction schedule:

$$Q(K) = \sum_{i=1}^{n} \Delta_i \cdot U_i(K). \quad (70)$$

6.3 Model restrictions

The final step is to find those mathematical restrictions that may be applied in the oil industry in order to determine the optimal oil extraction process, as follows:

a) The values of control variables $U_i(K)$ and the values of state variables $X_j(K)$ are upper limited:

$$U_i(K) \le Q_i(K) \quad (71)$$

At the same time, the values of crude oil inventories $X_j(K)$ are upper limited by storage capacity C_j:

$$X_j(K) \le C_j \quad (72)$$

b) Delivery terms. The quantities of crude oil in tanks held for supply need to be lower limited to the safety stock level, to ensure a continuous supply of crude oil to beneficiaries:

$$X_j(K) \ge X_j^s \quad (73)$$

Taking into account that oil well gas is delivered directly to consumers for their continuous supply, the following condition must be provided:

$$\beta Q(K) \ge f_3(K) \quad (74)$$

c) Initial conditions. At the moment when $K = 0$, crude oil stocks in the capacities C_j must have the known values X^0_j:

$$X_j(0) = X_j^0 \quad (75)$$

d) Final Terms. Setting a final condition upon the stock is necessary to ensure continuous supply in crude oil to beneficiaries even in the moment following the expiry of the time horizon on which the study is based:

$$X_j(N) \ge X_j^N \quad (76)$$

e) Negativity conditions of the variables. Since the state variables $X_j(K)$ are lower limited by safety stocks and by final terms, negativity conditions will be set only for control variables, and are:

$$U_i(K) \ge 0 \quad (77)$$

f) The condition set for the planned production schedule is:

$$\sum_{K=1}^{N-1} \sum_{i=1}^{n} \Delta_i \cdot U_i(K) \ge P \quad (78)$$

This equation describes the allowable extraction schedule.

6.4 Ranking the extraction schedules

Ranking these extraction schedules can be performed through setting the performance index, which, according to the previously discussed optimization process requirements, we consider to be *minimizing the total energy consumption involved by the extraction process*:

$$\min \sum_{K=0}^{N-1} \sum_{i=1}^{n} \varepsilon_i \cdot U_i(K) \qquad (79)$$

At the same time, ranking these extraction schedules can also be achieved by *maximizing the total quantity of crude oil extracted* throughout the entire process over the complete time horizon, namely:

$$\max \sum_{K=0}^{N-1} \sum_{i=1}^{n} \Delta_i \cdot U_i(K) \qquad (80)$$

And this is the output we were seeking in order to dynamically determine the oil extraction process for an optimal management that matches the purposes stated in the beginning of our analysis.

7 Conclusions

In conclusion, the present paper has aimed to build a dynamic model that can allow for optimal management of the oil extraction process. We initially analyzed the effects of the usual drilling process, showing the oil extraction process adverse impacts on efficiency. We then proposed under-balanced drilling as a modern, yet insufficiently studied, solution to an optimal and efficient management of the drilling process.

Then, we have analyzed the hydro-geological wells productive capacity that led us to better estimate the oil deposit reserves through production decline methods. For studying the rate of decrease in production of an oil deposit, the declining oil production term was introduced, which was defined as both the actual decline and the nominal decline (respectively, constant, hyperbolic and harmonic production decline types).

To estimate the production of crude oil from deposits, analytical models have been developed corresponding to different geometric configurations. Some models consider oil as an incompressible fluid, and others (more complex) take account of its compressibility. Finally, the motions to be modeled were considered either stationary - independent of time -, or non-stationary.

Fundamental equations solutions obtained for these motions can be used either to qualitatively study certain aspects of non-stationary motions, or to study non-stationary motions through methods that assimilate them with a succession of stationary states.

And the end of our analysis, through the use of various mathematical models applied to the conditions and restrictions particular in the oil industry, we managed to determine an extraction policy that dynamically matches the extraction possibilities to beneficiaries' needs while maximizing/minimizing a performance objective function (respectively, the total quantity of crude oil extracted, and minimizing the total energy consumption involved by the extraction process).

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