Methodology and experience of primary design of a transonic axial compressor

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Abstract: - The work presents the main provisions underlying the program for axial compressors calculation and design. The calculation of head losses and grids deflection capacity is based on the formulas of A. Komarov. The model contains empirical coefficients, values of which are selected during verification of the program based on the tests results of multistage compressors and compressor stages. The main equations and algorithm for pressures and velocities calculation under the radial equilibrium condition are presented. The use of computer programs based on these models in the design of a 4-stage gas turbine engine compressor of moderate power with a total pressure ratio of 3.2 and a given velocity is shown. For the first compressor stage, two variants with different flow coefficients were compared. Variant #1 is designed with the classic recommendation to approach the same mechanical gas energy at the exit of the stage along the radius. Variant #2 is designed for a smaller flow coefficient, but in order to ensure radial equilibrium, it was necessary to introduce a significant unevenness of mechanical energy supply along the radius. Due to the lower kinetic energy, variant 2 has a 1.9% higher stage efficiency. Despite the fact that the loss coefficients of the blade devices are lower for variant #1. The question remains as to how much the unavoidable mixing losses of variant #2 will reduce its efficiency in the process of gas mechanical energy equalizing.

Key-Words: - Mathematical simulation, axial compressor, impeller, guiding device, input guiding device, efficiency, loss coefficient, theoretical head, velocity triangle, bushing ratio


Legend
$c$ - absolute velocity (flow rate in fixed coordinate system);
$c_p$ - heat capacity at constant pressure;
$\bar{c} = c / u, \bar{w} = w / u$ - velocities (as well as their components) related to the circumferential velocity at the considered radius;
$h_T$ - Euler head;
$h_{lo}$ - loss of head;
k - isentropic coefficient;
m - power exponent;
p - pressure;
r - radius;
$\bar{F} = r / r_{calc}$ - ratio of the current blade radius to the calculated radius;
T - temperature;
u - blade velocity;
w - relative velocity (flow rate in a rotating coordinate system);
$\beta$ - angle between the relative velocity and the circumferential direction;
$\varphi = c/u = w/u$ - flow coefficient;
$\eta$ - efficiency;
$\pi$ - pressure ratio;
$\rho$ - density;
$\zeta$ - loss of head coefficient in grid;
v - hub-tip ratio;
$\omega$ - angular velocity of rotor spinning;
$\psi_T = (w_{u1} - w_{u2}) / u$ - loading factor;
$\Omega = 1 - (c_{u1} + c_{u2}) / 2u$ - kinematic degree of reactivity.

Abbreviations
GD - guiding device
Im - impeller

Subindexes
1, 2, 3 - flow parameters at inlet and outlet of the impeller bucket, at inlet to the guiding device;
ad - adiabatic;
b - hub;
ex - external;
GD - guiding device grid;
pol - polytrophic;  
calc - parameters on the calculated radius;  
des – design parameters  
BW – impeller bucket system;  
u – velocity projection on circumferential direction;  
z – velocity projection on axial direction.

Superscript
* - refers to total parameters.

1 Introduction
Axial compressors are the most powerful and productive machines among other compressor types. They are the main part of aircraft engine gas turbines, power gas turbines [1], gas industry compressor units [2]. Axial compressors play a significant role in the modern metallurgical, chemical industry and, especially, in the energy sector [3]. A specific field of application is the deep supercharging of steam generators of marine steam turbine engines, installations for breaking ice in icebreakers, etc. [4]. There is also a clear trend towards introducing axial compressors to new areas; for example, there are interesting proposals for using axial compressors in the gas and oil industry [4], [5], where volumetric flow rates are lower and initial pressures are significantly higher than in traditional application fields. The work [6] shows the prospects for increasing the rotation velocity of GPU drive engines. When implementing this development direction, the use of axial blowers will become rational.

There are huge prospects for increasing the production of axial compressors as part of power gas turbines, since modern thermal power plants use steam-gas units, where gas turbines are the main source of mechanical energy. According to the estimation presented in the work [1], in the near future, 3000 billion US dollars should be spent on the production of power gas turbines, and another 1000 billion US dollars on gas turbines of aircraft engines. As part of gas turbine engines, axial compressors take up at least half the size and cost of the engine, and their power is 30-50% higher than the power on the engine shaft. For the Russian Federation, the development problems of gas industry compressor equipment with a gas turbine drive, the total capacity of which is now about 40 million kW and it is constantly increasing [2].

All of the above shows the importance of the problem of optimal and fast design of axial compressors for various purposes. To solve this problem, both specialized engineering programs and CFD calculations are used.

The problem of engineering programs is their accuracy and reliability, which imposes high requirements on the underlying mathematical models and approximating dependencies.

The problem of CFD calculations is the correct formulation and execution of calculations, which guarantees the receipt of physically justified and reliable results.

The current study shows the work on the design of a low-pressure transonic axial compressor using an engineering program developed by SPbPU employees. The main equations and algorithm for pressures and velocities calculation under the radial equilibrium condition are presented. The use of computer programs based on these models in the design of a 4-stage gas turbine engine (GTE) compressor of moderate power with a total pressure ratio of 3.2 and a given velocity is shown.

2 Research methods
CFD calculations and optimization are successfully applied to stator elements of the flow part of centrifugal compressors [1], [8], [9], [10], [11], [12]. CFD simulation of centrifugal compressors in general is still a problem [13]– [15]. In design practice, engineering models and methods play a crucial role [16]–[18].

When calculating axial turbomachines, CFD methods are more successful, in particular, works [19]–[21] present a positive experience of CFD calculations in relation to axial turbines. However, in case of axial compressors, the calculation results may not be completely correct. For example, specialists of the Leading Research Center "Digital design and modeling (Smart Design)" analyzed the flow structure in one of the axial compressors of GTE. Figure 1 shows the total and static pressures in the section "1" between the inlet guiding device and the impeller.

Fig. 1. Static and total pressure in section "1" between the EGD and Im of the axial stage according to CFD calculation.
The inlet guiding device creates a negative spin of the flow at the hub and a positive one at the shroud. This is a rational decision. Negative spin at the hub helps to bring the necessary mechanical energy to the flow at a circumferential velocity, which in this case is 2.5 times less than at the periphery. A positive spin of the flow at the shroud reduces the relative velocity.

But the calculated dependency $p_1(r)$ is incorrect. Regardless of the direction of rotation, the spin of the flow forms a radially directed centrifugal force, balanced by a pressure gradient: $\partial p / \partial r = \rho \omega^2 / r$. That is, the static pressure should increase along the radius regardless of how other flow parameters change.

When calculating the circumferential velocity components, the numerical iterative calculation was inaccurate. The algebraic sum of the absolute and relative circumferential velocity components must be equal to the portable, i.e. the circumferential velocity: $\vec{z}_c + \vec{v}_c = \vec{u} = \omega \vec{r}$. Shown in Fig. 2 values of the circumferential velocity components according to the CFD calculation are in total less than the circumferential velocity by 10-12% at different radii along the height of the blade. This inaccuracy is undesirable in engineering calculations.

![Fig. 2. Velocities and velocity components in section "2" at the output from the Im according to CFD calculation](image)

A number of authors present the results of calculations and optimization of axial compressors using CFD methods [22]–[25]. In our opinion, when creating a flow part, the first stage – initial design - should be carried out using engineering models and programs. This will bring subsequent CFD calculations closer to the optimal solution and help to control the results.

The compressor group of the “Technological processes modeling and power equipment design” laboratory of the CSTI SPbPU uses computer programs based on mathematical models of head loss and grids deflection capacity of A. Komarov [26] in its design practice. Models have been verified, and computer programs have proven their effectiveness in research and design practice [27]. The article demonstrates the possibilities of primary design on a particular example of a transonic GTE compressor.

### 3 Literature review

The complex nature of the flow and the lack of adequate computer technology until the 70s of the last century led to the empirical nature of the study of axial stages and compressors. The adequacy of the flow in the elementary annular grid of the blade rows of Im or GD to the flow in an equivalent flat grid significantly simplified obtaining the desired results. The main information used in the design practice of AC blade devices is obtained as a result of elementary flat grids testing in aerodynamic tunnels. The technique of these experiments is much simpler than, for example, when testing model stages of axial or centrifugal compressors.

The principles of AC blade devices profiling based on testing of flat grids in the Russian literature were described in [28], in more detail - in [29]. A later publication [30] summarizing the results of research by Western specialists actually sets out the same approach. The most complete statement of these principles is contained in the monograph [31], published in 2003. For axial fans, the results of extensive research and design methods are presented in the monograph [32]. Recommendations for the design of industrial AC are contained in the technical materials [33] and others. A number of specific design issues have been solved by domestic researchers and described in a number of publications, for example, in [34].

The works [35], [36] present the development of a software package for the optimization of size and form of axial compressors. The developed mathematical model takes into account various components of head losses in axial compressors and allows optimizing their dimensions, formulas of impeller and guiding device vanes, and provides predicted gas-dynamic characteristics of the compressor. A one-dimensional calculation on the center current line during the initial design and a two-dimensional approach with the calculation of flow parameters and velocity triangles on several current lines along the height of the blade are used. This allows to achieve the optimal shape of the blades in its height and achieve a non-incidence flow. Calculation of head losses is carried out according to the Lieblein formulas. The compressor can be designed with a constant hub diameter, a constant outside diameter, or a constant center current line diameter. It is also possible to set the linear law of outer diameter change. The program was identified
based on experimental data for a multistage axial compressor obtained at NASA. The program was applied to optimize several axial compressors, and the results of calculations for engineering models were verified by means of CFD calculations in the ANSYS CFX program. The verification results confirmed the high efficiency of engineering methods for axial compressors calculation and design.

The work [37] presents an improved model for transonic axial compressors calculation and design based on two-dimensional calculations. The model was verified based on experimental studies of a transonic axial stage and a three-stage P&W 3S1 compressor. The calculation results were also compared with the CFD calculation of these objects. Verification has shown high efficiency and accuracy of calculations using the engineering program.

The work [38] presents the concept of the through flow method. In recent decades, the through flow method has become one of the most important tools for designers - a compromise between the accuracy of the flow field representation and the corresponding computational cost. This publication presents the results of the development of a new through flow model that solves the Euler equations. The solver is based on a numerical finite volume method with second-order accuracy. To verify the accuracy and reliability of the developed model, two tests were performed – the calculation of the NASA-67 Rotor in the design mode, and the simulation of the characteristics of a high-speed transonic compressor with a nominal velocity of 68%. The possibility of non-calculation mode simulation using the developed method is proved. When calculating the NASA-67 Rotor, the overall results are satisfactory. Despite the discrepancies, which, as the analysis shows, are mainly caused by errors in the forecast of deviation angle and losses, the calculated results are considered to be well consistent.

The work [39] presents a program for optimizing the impeller blades of axial compressor stages based on a proprietary CFD program. The authors created an automatic algorithm for computational grid rebuilding and an optimizer that allows varying the geometry of the blade surfaces on the current lines to achieve a given pressure distribution on the blades. Using this method for two NASA model stages has improved the efficiency of the impellers. At the same time, the program has a number of limitations, in particular, the authors do not provide information about the preliminary verification of the CFD program, which allows to judge the calculations reliability. Also, the practical application of the program is limited by the fact that the designer must set the pressures distribution on the blade surfaces, which can cause difficulties during new compressors design. The application of the method has not been shown in relation to the guiding devices of axial compressors.

The work [22] presents the design process of a five-stage axial compressor. For this, proprietary 3D engineering analysis programs and the commercial CFD package NUMECA are used. Blade profiles in the engineering program are described by Bezier curves. During the optimization process, the shape of blade midline varies. The obtained coordinates are transmitted to the commercial CFD package for further refining analysis.

The work [40] presents a program for preliminary evaluation of the layout and form of axial turbomachines of transport GTE. The program is designed to analyze a variety of options for the flow part and select the optimal number of compressor stages, the meridional form of the flow part, etc. The authors note that the use of pre-design programs is an integral step in the creation of new GTE. They allow to start detailed optimization of the axial compressor blade shape at the next design step for efficient compressor configuration and save time when designing a new machine.

The work [41] presents a program for preliminary design of an axial multistage compressor. The calculation and optimization in it is carried out for the center current line along the height of the compressor blades. The underlying mathematical model allows also to design supersonic compressors. However, the loss calculation is performed only for direct shock waves and the calculation algorithm does not take into account changes in flow parameters after shock wave. This slightly reduces the functionality of this program.

The work [42] presents a mathematical model for calculating the characteristics of multistage axial compressors. It is an extended one-dimensional model for calculating the flow parameters on the center current line along the blade height, which the authors themselves call 1.5D. The authors present the results of verification of their model for two axial compressors. The accuracy of gas dynamic characteristics simulation at various velocities shows sufficient accuracy for engineering practice.

In [43], [44] the code of the center line of an axial compressor operating in non-calculating modes is described for specialized engineering software that is developed for the analysis of the entire gas turbine engine. A generalized methodology for losses simulation on the midline of the axial compressor has been developed. The generalized methodology, according to the authors, will allow to accurately
evaluate the characteristics of axial compressors in non-calculating modes at transonic flow.

The development and implementation of digital design methods for axial compressors is developing dynamically in all countries. CFD calculations are an integral part of these methods. They are carried out both for individual elements of the flow part for the purpose of finalizing their size and shape, and for the compressor as a whole. The large time spent on calculation shows that even with the modern development of computer technology, CFD calculations cannot fully replace one-dimensional and two-dimensional engineering programs for turbomachines calculation and design. CFD calculations also need engineering models to correctly set initial approximations when calculating gas dynamic characteristics. A separate problem is the quality verification of CFD calculations. A number of researchers only use CFD calculations to optimize the form of axial compressor stages. An example of such work is the publication [45].

In work [46], a three-stage axial compressor with an inlet guiding device operating with a transonic flow pattern is calculated. The influence of accounting for leaks through labyrinth seals on compressor characteristics is studied.

The large amount of time spent both on creating calculation grids of the required quality and on obtaining converged solutions shows that the use of CFD methods alone for calculating the gas dynamic characteristics of turbomachines and their optimization is inefficient, and the resource is expensive.

In general, the review of publications shows that engineering methods for axial compressors and compressor stages calculation and optimization are widely used. For some of them there is not enough verification based on the results of experimental data, others are limited only to preliminary design, without subsequent optimization of the blade elements over the entire height.

4 Research results

Divisions of the Leading Research Center "Digital design and modeling (Smart Design)", the Center of the National Technological Initiative "New Production Technologies" of the SPbPU have a great positive experience in research and design of turbomachines, in particular, axial and centrifugal compressors, using engineering mathematical models and Computational Fluid Dynamics (CFD) programs.

4.1 Design principles for axial compressors

Axial compressors are part of the most powerful gas turbines, as they are able to provide the highest air consumption at an acceptable size. High air velocities create the main problem of obtaining compressor high efficiency. Figure 3 shows a simplified diagram of the AC stage – so-called homogeneous stage in which an inviscid incompressible gas moves. In a homogeneous stage, the gas moves along cylindrical surfaces. At the inlet section "1" and at the outlet section "2" on each cylindrical surface, the flow rate components are the same.

Fig. 3. a) The scheme of a homogeneous stage; b) unfolded flat pattern of the annular blade grids of the Im and GD; c) velocity triangles at rex radius; d) velocity triangles at rb radius

The loss of efficiency in the stage is the ratio of the lost head \( h_w \) in the impeller and guiding device to the Euler head \( h_T \) (the main equation of turbomachines) [47]:

\[
\Delta \eta = \frac{h_{im} + h_{gd}}{h_T} = \xi = \frac{u_i^2}{2(c_{in} - c_{ut})u} + \phi = \frac{c_{it}^2}{2(c_{in} - c_{ut})u}.
\]

This is the same ratio in dimensionless form [47]:

\[
\Delta \eta = 0,5\xi \left( \frac{1-c_{ut}}{c_{in}-c_{ut}} \right) + \phi = \frac{c_{it}^2}{c_{in} - c_{ut}} + \phi.
\]

The higher the flow rate \( \phi = c_{it} / u \), the more turbine power the compressor can provide, but the loss of efficiency is greater, all other things being
equal. However, with smaller $\varphi$, the angle of blades installation decreases, which increases the loss coefficient according to known mathematical models [26], [48], [49], [50]. If the mathematical models are reliable, finding of the maximum efficiency for this design parameter is not difficult. But the additional condition is the need for a rational organization of the flow in all axially symmetric surfaces.

In accordance with the basic equation of turbomachines, the mechanical operation of the engine is transmitted to the gas by creating a circumferential component of the velocity $C_u$. The centrifugal force from the gas rotation is balanced by a pressure gradient that increases along the radius [29]:

$$\frac{\partial p}{\partial r} = \rho \frac{C_u^2}{r}. \quad (3)$$

Expressing static pressure in terms of total pressure and dynamic head, we write the radial equilibrium equation as follows [29]:

$$\frac{C_u^2}{r} = \frac{1}{\rho} \frac{\partial p}{\partial r} + C_u \frac{\partial u}{\partial r} + c_* \frac{\partial c_*}{\partial r}. \quad (4)$$

Equation (4) shows that the flow coefficient we are analyzing $\varphi = c_*/u = f(r)$ can be chosen arbitrarily, but the dependencies $c_* = f(r)$ and $c_u = f(r)$, that satisfy equation (4) must correspond to this law.

At the entrance to the first stage of the compressor in the inviscid core, the flow is potential, vortex-free. The total mechanical energy of gas particles is the same as the sum of the potential pressure energy and the kinetic motion energy: $p^* = \text{const}$. If a theoretical pressure variable by height $p_i(r) = \text{const}$ of the blades is applied to the potential flow at the input to the Im $h_i(r) = \text{var}$, then at the output from the Im the potentiality of the flow is violated. The mechanical energy of the gas varies according to the height of the blades; $p_i(r) = \text{var}$. The subsequent alignment of mechanical energy occurs due to friction between particles moving at different velocities. These are so-called mixing losses, which are not present in equations (1, 2).

To avoid these losses, the design methods based on the homogeneous stage theory [28], [29] adopt the principle of preserving the flow potentiality $h_i(r) = \text{const}$, which $p^*(r) = \text{const}$ means when considering an ideal flow. In this case the equilibrium condition appears in a particular form [29]:

$$\frac{c_u}{r} + \frac{\partial c_*}{\partial r} + c_* \frac{\partial c_*}{\partial r} = 0. \quad (4a)$$

The principle of radial equilibrium and the flow potentiality in a homogeneous stage will be maintained if, for example, taking the law of change $c_i(r)$, we calculate the dependence $c_i(r)$ from equation (4a) - or vice versa.

The simplest way to construct a spatial flow is obtained if we take the constancy of the expenditure component of the velocity along the radius [29]:

$$c_i(r) = \text{const}, \quad c_u, \quad r = \text{const}. \quad (5)$$

The analytical solution has a dependence $c_i(r)$ if we accept the change in the circumferential velocity component according to the law $c_u, \quad r^m = \text{const}$, where the exponent lies within the range of $+1 ... -1$ [29]. For example, the flow rate for the height of the blade at the input to the Im changes by law [29]:

$$\frac{c_u}{c_*} = \left[\frac{1 + mr - 1 - \frac{1 - \varphi}{\varphi_{des}}}{mr - 1 + \frac{1 - \varphi}{\varphi_{des}}}\right]^{\frac{m - 1}{m + 1}} \frac{1 - \varphi}{\varphi_{des}} \frac{1 - \varphi}{\varphi_{des}} \left[1 - \left(1 - \frac{1 - \varphi}{\varphi_{des}}\right)\left(1 - \frac{1 - \varphi}{\varphi_{des}}\right)\right]. \quad (6)$$

In the equation, the "des" index marks the parameters on the cylindrical surface on which the design parameters are selected: the flow coefficient, loading factor, and the degree of reactivity. Obviously, if $m \neq 1$, then the flow rate decreases from the hub to the shroud. If a small value of the flow coefficient is chosen to reduce the kinetic energy at the calculated radius $\varphi_{des} = c_* c_* / u_{des}$, then the flow velocity may become negative at the shroud, which is unacceptable. This limits the possibility of increasing efficiency by taking small values $\varphi_{des}$.

Computer programs based on the flow organization principle described above and mathematical models from [26] work well in relation to compressors with subsonic gas movement [52]–[56]. At high velocities, it is impossible to neglect the pressure losses in the blade devices, the compressibility, and the fact that the flow moves not on cylindrical, but on conical surfaces of the current. Equations that determine the flow kinematics and gas parameters by the radius should be solved numerically.

4.2 Main provisions of the calculation algorithm

Fig. 4 shows the meridional cross section of the stage in the classical quasi-three-dimensional calculation. Pressures, temperatures, and velocities are calculated in the middle of the gaps between the blades. First, on the calculated radius, according to the design parameters, the flow coefficient, the loading factor, the degree of reactivity, the triangles of velocity, pressure and temperature are calculated. Then, on a number of axisymmetric current surfaces within the height of the blade, velocity triangles are calculated using the equations of the homogeneous stage theory.
When solving the problem numerically, the principle approach is the same – design parameters are selected and implemented on the calculated radius, then the calculation is performed on all axisymmetric surfaces. The primary projects presented below follow this pattern (for example Im, the obvious kinematic and thermodynamic calculations are omitted). By the height the blades it is divided into 20 sections with the same mass flow rate.

The number and chord of the blades, the meridional dimensions, and the rpm are chosen based on generally accepted considerations. The calculated axisymmetric surface is the hub surface. Design parameters – mass flow rate, meridional velocity \(c_{m1}\), flow spin \(c_{m1}\), deceleration of the meridional flow in the Im \(c_{m2}/c_{m1}\), the output angle of the relative velocity \(\beta_{3b}\).

Accordingly, the Euler head at the hub surface [27]:

\[ h_{3b} = (u_{2b} - c_{m2} \cot \beta_{2b})u_{2b} - c_{m1}u_{1b} \]  
(7)

Lost head [47]:

\[ h_{b} = c_{p} \cdot 0.5w_{21b}, \]  
(8)

where the loss coefficient is calculated by mathematical models of A. Komarov [26] with empirical coefficients from [27], [57], [58].

Static pressure from the adiabatic process equation [27]:

\[ p_{2(1)} = p_{3} + \frac{c_{g}^{2}}{\rho_{b}^{2}} \left( \frac{T_{0}^{*}}{T_{0}} + \frac{h_{f}}{u_{2(1)}} + \frac{T_{2(1)}}{T_{2(1)}^{*}} \right) \]  
(9)

As the parameter of blades design by the radius it is necessary to specify dependencies \(h_{f} = f(r)\), \(c_{m} = f(r)\). Static pressure on the axisymmetric current surface next to the bushing (1) » [27]:

\[ p^{*}_{2(1)} = \frac{c_{g}^{2}}{2} \left( \frac{T_{0}^{*}}{T_{0}} + \frac{h_{f}}{u_{2(1)}} + \frac{T_{2(1)}}{T_{2(1)}^{*}} \right) + \frac{\Delta p}{\rho_{b}^{2}} \]  
(10)

The circumferential velocity component [27]:

\[ c_{u2(1)} = \frac{h_{f}}{u_{2(1)}} \]  
(11)

Static temperature [27]:

\[ T_{2(1)} = T_{0}^{*} + \frac{p_{2(1)}}{p_{1(1)}} \frac{k-1}{k} \cdot \Delta T_{2(1)} \]  
(12)

Total temperature [27]:

\[ T_{2(1)}^{*} = T_{0}^{*} + \frac{h_{f}}{c_{p}} \]  
(13)

Absolute velocity [27]:

\[ c_{2(1)} = \sqrt{2c_{p}} \left( T_{2(1)}^{*} - T_{2(1)} \right) \]  
(14)

The meridional velocity [27]:

\[ c_{m2(1)} = \sqrt{c_{2(1)}^{2} - c_{g}^{2}} \]  
(15)

The above relations show how, using the design parameters on the hub surface - \(c_{m1}\), \(c_{m2}/c_{m1}\), \(\beta_{2b}\) and the parameters of the spatial flow organization \(h_{f} = f(r)\), \(c_{m} = f(r)\), by means of a numerical solution, to obtain the dependence of the flow rate on the radius \(c_{m} = f(r)\). In contrast to the classical calculation based on the algebraic equations of the homogeneous stage theory, the numerical calculation takes into account the circumstances that the axisymmetric current surfaces are conical, not cylindrical, that the flow is accompanied by pressure losses, and the working medium is a compressible gas.

The practice of calculations shows that with a certain choice of design parameters on the shroud, the flow rate can become unacceptably low. In this case, in equation (14), the absolute velocity at the output from the Im turns out to be less than the circumferential component according to equation (15) (the possibility of a similar situation for a homogeneous stage is shown by equation (6)). It is also possible that the static temperature in equation (12) is greater than the total temperature in equation (13).

These two circumstances greatly limit the range of design parameters and the ability to vary them when searching for a variant with maximum efficiency.

### 4.3 Object of primary design and optimization

As an example, a low-pressure GTE compressor of moderate power with a pressure ratio \(\pi = 3.2\) is selected. At a given flow rate and rpm, the number of stages is 4, the outer diameter of the 1st stage is about...
0.90 m, and the circumferential velocity is about 430 m/s. The type of the solid-state model of one of the compared variants is shown in Fig. 5.

Fig. 5. Solid-state model of a four-stage LPC of gas turbine engine

4.4 Primary design of a four-stage LPC with optimization

In the process of optimal design, the classical principle of approximation to the potentiality of the flow was followed. The dependencies $h_f = f(r)$ for all four impellers were selected so that the total pressure at the stage output was as close as possible to a constant radius $p_i = f(r) \rightarrow \text{const}$. To obtain the maximum efficiency, all design parameters were varied within the limits when, according to equations (12, 13), the total temperature is higher than the static temperature, and the absolute velocity is higher than its circumferential component according to equations (14, 15).

Figure 6 shows the optimized radial dimensions of the control sections and the flow section form.

Fig. 6. Optimized radial dimensions of the control sections and the flow path form of the LPC

The main LPC parameters and its stages are shown in table 1.

Table 1. Characteristic parameters of the flow path of the optimized version of the LPC

<table>
<thead>
<tr>
<th>Parameter</th>
<th>LPC</th>
<th>Stage 1</th>
<th>Stage 2</th>
<th>Stage 3</th>
<th>Stage 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^*$</td>
<td>3.258 94</td>
<td>1.422 71</td>
<td>1.407 62</td>
<td>1.350 86</td>
<td>1.20 503</td>
</tr>
<tr>
<td>$\eta_{ad}^*$</td>
<td>0.864 71</td>
<td>0.876 3</td>
<td>0.885 6</td>
<td>0.884 6</td>
<td>0.87</td>
</tr>
<tr>
<td>$\eta_{pol}^*$</td>
<td>0.885 73</td>
<td>0.882 73</td>
<td>0.890 45</td>
<td>0.889 45</td>
<td>0.87</td>
</tr>
<tr>
<td>$h_f$ J/kg</td>
<td>1343</td>
<td>3494</td>
<td>3759</td>
<td>3671</td>
<td>2508</td>
</tr>
<tr>
<td>$u_{sh}$ m/s</td>
<td>435.2</td>
<td>424.7</td>
<td>416.1</td>
<td>410.0</td>
<td>07</td>
</tr>
<tr>
<td>$c_{z1h}$ m/s</td>
<td>182.1</td>
<td>162.1</td>
<td>134.9</td>
<td>107.98</td>
<td>1</td>
</tr>
<tr>
<td>$c_{z2h}$ m/s</td>
<td>193.5</td>
<td>159.0</td>
<td>135.7</td>
<td>111.89</td>
<td>2</td>
</tr>
<tr>
<td>$c_{z1sh}$ m/s</td>
<td>177.8</td>
<td>159.5</td>
<td>138.0</td>
<td>121.88</td>
<td>3</td>
</tr>
<tr>
<td>$c_{z2sh}$ m/s</td>
<td>122.6</td>
<td>118.5</td>
<td>102.1</td>
<td>95.8</td>
<td>2</td>
</tr>
<tr>
<td>$\beta_{1h}$ deg.</td>
<td>62.88</td>
<td>68.04</td>
<td>59.62</td>
<td>40.6</td>
<td>68</td>
</tr>
<tr>
<td>$\beta_{2h}$ deg.</td>
<td>51.85</td>
<td>52.5</td>
<td>45.2</td>
<td>32.2</td>
<td>5</td>
</tr>
<tr>
<td>$\beta_{1sh}$ deg.</td>
<td>23.29</td>
<td>23.12</td>
<td>20.63</td>
<td>18.0</td>
<td>2</td>
</tr>
<tr>
<td>$\beta_{2sh}$ deg.</td>
<td>34.46</td>
<td>33.25</td>
<td>27.52</td>
<td>22.3</td>
<td>2</td>
</tr>
<tr>
<td>$\beta_{1sh}$ deg.</td>
<td>25.49</td>
<td>23.15</td>
<td>20.72</td>
<td>18.7</td>
<td>2</td>
</tr>
<tr>
<td>$\xi_{bw h}$</td>
<td>0.054</td>
<td>0.04</td>
<td>0.04</td>
<td>0.05</td>
<td>81</td>
</tr>
<tr>
<td>$\xi_{bw sh}$</td>
<td>0.040</td>
<td>0.05</td>
<td>0.05</td>
<td>0.04</td>
<td>64</td>
</tr>
<tr>
<td>$\xi_{gd h}$</td>
<td>0.032</td>
<td>0.06</td>
<td>0.06</td>
<td>0.04</td>
<td>68</td>
</tr>
<tr>
<td>$\xi_{gd sh}$</td>
<td>0.052</td>
<td>0.07</td>
<td>0.07</td>
<td>0.05</td>
<td>6</td>
</tr>
</tbody>
</table>

The efficiency calculation is made according to the mathematical model of A. Komarov [26] with the adjustment of empirical coefficients in the direction of a significant increase in head loss according to the test data of multistage subsonic compressors [52]. The low LPC efficiency indicates the need to check the empirical coefficients, which is the challenge of the future.

4.5 Checking the alternative principle of primary design

At the beginning of the publication, it was pointed out that it is possible to increase efficiency by reducing
the consumption coefficient. The example of optimal design was given, provided that the potentiality of the flow in the control sections is preserved. This condition did not allow making the 1st stage flow coefficient less than 0.4. To study the possibility of designing with lower flow coefficients, the variant of the 1st stage of the LPC was optimally designed with \( h_f = f(r) = \text{var} \). The parameters of the 1st stage from the previous section are compared with the parameters of the analogue stage with a significantly lower flow coefficient - table 2.

### Table 2. Comparison of parameters of 1st stage of LPC variants with different flow coefficients

<table>
<thead>
<tr>
<th>Variant No.</th>
<th>1</th>
<th>2</th>
<th>( \text{Var} 1/\text{Var} 2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
<td>( h_f = f(r) = \text{var} )</td>
<td>( h_f = f(r) = \text{var} )</td>
<td>1.438</td>
</tr>
<tr>
<td>( \psi_{1m} = c_{11c}/u_1 )</td>
<td>0.409</td>
<td>0.304</td>
<td>0.992</td>
</tr>
<tr>
<td>( \pi )</td>
<td>1.42207</td>
<td>1.43293</td>
<td>0.978</td>
</tr>
<tr>
<td>( \eta^*_{ad} )</td>
<td>0.87671</td>
<td>0.89598</td>
<td>0.979</td>
</tr>
<tr>
<td>( \eta^*_{pol} )</td>
<td>0.8827</td>
<td>0.90114</td>
<td>0.999</td>
</tr>
<tr>
<td>( h_f ) J/kg</td>
<td>34942.41</td>
<td>34968.61</td>
<td>0.994</td>
</tr>
<tr>
<td>( U_{sh} ) m/s</td>
<td>435.21</td>
<td>437.7</td>
<td>0.985</td>
</tr>
<tr>
<td>( \beta_{2h} ) deg.</td>
<td>62.88</td>
<td>63.85</td>
<td>-</td>
</tr>
<tr>
<td>( \beta_{2h} ) deg.</td>
<td>51.85</td>
<td>53.7</td>
<td>-</td>
</tr>
<tr>
<td>( \beta_{1h} ) deg.</td>
<td>34.46</td>
<td>33.59</td>
<td>-</td>
</tr>
<tr>
<td>( \beta_{1h} ) deg.</td>
<td>25.49</td>
<td>25.16</td>
<td>-</td>
</tr>
<tr>
<td>( \zeta_{bw \ h} )</td>
<td>0.0543</td>
<td>0.0585</td>
<td>0.880</td>
</tr>
<tr>
<td>( \zeta_{bw \ sh} )</td>
<td>0.0409</td>
<td>0.0465</td>
<td>0.776</td>
</tr>
<tr>
<td>( \zeta_{gd \ h} )</td>
<td>0.0326</td>
<td>0.042</td>
<td>1.525</td>
</tr>
<tr>
<td>( \zeta_{gd \ sh} )</td>
<td>0.0523</td>
<td>0.0343</td>
<td>1.438</td>
</tr>
<tr>
<td>( \rho^<em>_{3h}/\rho^</em>_{3h} )</td>
<td>1.0107</td>
<td>1.1267</td>
<td>0.897</td>
</tr>
</tbody>
</table>

## 5 Discussion of the obtained results

The flow kinematics features are illustrated by the velocity triangles in Fig. 7 and graphs of the flow and blade parameters by the radius from the hub to the shroud section.

It is significant that, despite the lower flow coefficient, variant 2 has a relative velocity at the input to the Im at the periphery even greater than variant 1 - Fig. 7b and 7d.

Variant 1 has a much smaller Euler head at the Im shroud, so the flow output angle is even smaller than the input angle - Fig. 8a, 8b, and 8d. The spin of the flow in relative motion is created by reducing the flow velocity - Fig. 7b.

In accordance with the equations of the mathematical model [26], the loss coefficients are greater for variant 2 – Fig. 8e. At the same time, the efficiency loss for variant 1 is less only by 20% of the blade height at the shroud due to the fact that the kinetic energy of the flow is lower in the lower part of the blades for variant 2.
As a result, due to the lower consumption coefficient, variant 2 has the efficiency of more than 1.9%, which is very significant, although the loss coefficients are lower for variant 1.

The problem with variant 2 is the difference in the mechanical energy of the gas by the blade height. In option 1, the mechanical energy of the gas at the stage output is almost the same on all current surfaces. Variant 2 has a 50% higher Euler head on the shroud than the bushing. Approximately the same difference in mechanical energy in the blade height at the stage output. If the loss of mixing - alignment of mechanical energies - is, for example, 10% of the uneven energy, then variant 1 will be more effective.

6 Conclusion
The primary design program applied by the authors is of an engineering nature, considering the problem in a purely simplified formulation. Gas parameters, pressure losses, and velocity vectors are calculated in an axisymmetric setting in the control sections between the blades on a number of axisymmetric current surfaces (quasi-three-dimensional problem). The presented example of optimal design showed, however, that the program is informative and responds adequately to the choice of design parameters. The program, first of all, ensures the radial equilibrium of the gas in the field of centrifugal forces. This equilibrium is possible only with a fairly strict combination of the stage head and flow rate on each of the axisymmetric current surfaces. The example of CFD calculation given in the text that does not meet the radial equilibrium condition shows the need to use programs for the primary design that do not allow such errors. For final CFD optimization,
engineering programs can be used to control the calculation results in control sections.

7 Acknowledgment
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Contribution of individual authors to the creation of a scientific article (ghostwriting policy)

Author Contributions: Please, indicate the role and the contribution of each author:
Example
A.I. Borovkov has organized the researches
Yu.B. Galerkin, O.A. Solovieva, A.A. Drozdov were developed 2D mathematical model
A.F. Rekstin, K.V. Soldatova were designed axial compressor
A.A. Sebelev – CFD-calculations
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