LSSVM predictive control based on improved free search algorithm for nonlinear systems

TIAN ZHONGDA, LI SHUJIANG, WANG YANHONG, ZHANG CHAO
College of Information Science and Engineering, Shenyang University of Technology, Shenyang 110870, CHINA
Corresponding author: Tian Zhong-da,Email:tianzhongda@126.com

Abstract: In order to improve control performance of nonlinear systems, a predictive control method based on improved free search algorithm and least square support vector machine was proposed. This predictive control method utilized least square support vector machine to estimate the nonlinear system model and forecast the output value. The output error is reduced through output feedback and error correction. The rolling optimization of control values are obtained through an improved free search algorithm. This predictive control method can be used to design effective controllers for nonlinear systems with unknown mathematical models. Through the simulation experiment of single variable and multivariable nonlinear systems, the simulation results shown that the predictive control method has an excellent adaptive ability and robustness.

Key-words: nonlinear systems; predictive control; improved free search algorithm; least square support vector machine

1 Introduction

The predictive control use model prediction, on-line rolling optimization and feedback correction strategy, it has good control effect, strong adaptability and robustness characteristics. The essence of predictive control is to optimize the system behaviours according to the prediction of the future state of the system[1]. At present, the predictive control has a lot of practical applications[2-4].

But there are still certain difficulties for the predictive control of nonlinear systems, the prediction accuracy of the predictive model which plays an important role [5]. The precision of the prediction model limits the application of predictive control in nonlinear systems. The predictive control problem of nonlinear systems can be transformed into predictive control of linear systems, the method include Hammerstein model, Wiener model, Volterra model or neural network[6-8], etc. However, Hammerstein model, Wiener model and Volterra model can only be used for some specific process. The neural network has disadvantages include network topology is difficult to determine, convergence speed is slow, easily falling into local minimum.

Support vector machine (SVM) overcome the disadvantages of neural network, and has very strong generalization ability[9-10]. Based on SVM, least square support vector machine (LSSVM) reduce the complexity of SVM algorithm, and it is a good tool for the modeling and control method for nonlinear systems[11-13].

In the practical process, the mathematical model of nonlinear systems is difficult to be obtained, so it is very necessary to research predictive control under condition of mathematical model of the control system is not known. Because the prediction model is nonlinear, the calculation of control values is a nonlinear constrained optimization problem. When the control value sequence is very long, the optimization problem is complicated, time-consuming is longer, can not meet the real-time requirement of the system, so the length of control value sequence in each sampling moment should be as short as possible.

In order to solve the nonlinear predictive control problem, this paper proposed an LSSVM predictive control based on improved free search algorithm for nonlinear systems. LSSVM is used to establish the prediction model of nonlinear systems, and then an improved free search (IFS) algorithm which can be applied to rolling optimization in nonlinear system is proposed. The optimized goal of the nonlinear systems is obtained through output feedback deviation correction, the IFS algorithm is used to determine the future control values by online real-time rolling optimization. The simulation results show that this predictive method can better track the reference trajectory and has good robustness to disturbances.

2 LSSVM predictive model

LSSVM through a nonlinear mapping $\varphi(\cdot)$, the sample space is mapped into a high-dimensional or even infinite dimensional feature space[14]. In this feature space, there is

$$y(x) = \boldsymbol{w}\varphi(x) + b \tag{1}$$

wherein $\boldsymbol{w}$ is the weight coefficient vector, b is the constant bias. Optimal $\boldsymbol{w}$ and b can be obtained by minimizing the objective function.

$$\min_{w,b,e} J(\boldsymbol{w}, e) = \frac{1}{2}\boldsymbol{w}^T\boldsymbol{w} + \frac{1}{2}\gamma\sum_{k=1}^{N} e_k^{\ 2} \tag{2}$$

Lagrange function is established for solving constrained optimization problems mentioned above:

$$L(\boldsymbol{w},b,\boldsymbol{e},\boldsymbol{a}) = J(\boldsymbol{w},\boldsymbol{e}) - \sum_{i=1}^{N} a_i(\boldsymbol{w}^T\Phi(x_i) + b + e_i - y_i) \tag{3}$$

Wherein a_i is a Lagrange multiplier, the Lagrange function is used for solving extreme value, and the above optimization problem is transformed into solving the linear equations.

According to Mercer conditions, the presence of mapping function $\varphi(\cdot)$ and kernel function $K(\cdot)$ satisfy the following equation:

$$K\left(x_i, x_j\right) = \varphi(x_i)^T\varphi(x_j) \tag{4}$$

Nonlinear mapping ability of LSSVM prediction model is determined by the kernel function. Kernel function is used for LSSVM prediction model samples mapping from input space to feature space. Therefore, different kernel function has different learning ability and generalization ability. The most widely used is radial basis function (RBF) kernel, which applicable for low-dimensional, high-dimensional, small sample, large sample, etc., so this paper chooses RBF as the kernel function.

$$K(x_i, x_j) = \exp\left(\frac{-\left\|x_i - x_j\right\|^2}{2\sigma^2}\right) \tag{5}$$

Wherein, σ^2 is the width of RBF kernel function. The predictive value of LSSVM model is:

$$y = \sum_{i=1}^{N} \alpha_i K(x_i, x_j) + b \tag{6}$$

3 Improved free search algorithm

3.1 Standard free search algorithm

Free search (FS) algorithm was proposed in 2005, it is a new optimization method based on the population optimization[15]. The FS algorithm has good global search capability, has been widely used in many optimization problems[16-18]. The FS algorithm includes initialization, searching and end condition judgment. The algorithm is described as follows: m is the individual's number, $j,(1 \le j \le m)$ is the i_{th} individual. $k,(1 \le k \le m)$ is pheromone markers coordinates point. n is the number of variables in the target function. $i,(1 \le i \le n)$ is space dimension, T is the search step, $t,(1 \le t \le T)$ is the current step, $R_{ji},(R_{ji} \in [R_{\min}, R_{\max}])$ is the i_{th} individual's search range in the i_{th} variable spatial neighborhood, G is variable spatial neighborhood.

Free search algorithm's population initialization generally adopts the following strategy:

$$x_{0ji} = X_{\min i} + (X_{\max i} - X_{\min i}) \cdot random_{ji}(0,1) \tag{7}$$

x_{0ji} is the animal's initial position component. $X_{\min i}$ and $X_{\max i}$ is the boundary of search space, $random_{ji}(0,1)$ is a random number between 0 and 1. In the search processes, the individual animal update the position as following:

$$x_{tji} = x_{0ji} - \Delta x_{tji} + 2\Delta x_{tji} random_{tji}(0,1) \tag{8}$$

Wherein, x_{tji} is the updated animal individual position component. In the free search algorithm model, individual moves a search step comprises T steps. The individual is move a small step in multidimensional space, its purpose is to find better solutions to the objective function. The modification strategy Δx_{tji} can be expressed as:

$$\Delta x_{tji} = R_{ji}(X_{\max i} - X_{\min i}) random_{tji}(0,1) \tag{9}$$

In individual search process, the target function symbol's regulation is:

$$\begin{aligned} f_{tj} &= f(x_{tji}), \\ f_j &= \max(f_{tj}). \end{aligned} \tag{10}$$

The update pheromone defined as follows:

$$PH_j = f_j / \max(f_j) \tag{11}$$

Wherein, $\max(f_j)$ is the j_{th} optimal values of individual. Then updating the sensitivity SE_j as following:

$$SE_j = SE_{\min} + \Delta SE_j \quad (12)$$

$$\Delta SE_j = (SE_{\max} - SE_{\min}) random_j(0,1) \quad (13)$$

Wherein, $SE_{\min} = PH_{\min}$, $SE_{\max} = PH_{\max}$.

After this search round is end, the next starting point of the search is determined.

$$x'_{0ji} = \begin{cases} x'_{0ji},\ (P_k < S_j), \\ x_{0ji},\ (P_k \geq S_j). \end{cases} \quad (14)$$

Free search algorithm end conditions can be described as follows:

(1) The objective function, which achieve the global optimal solution: $f_{\max} \geq f_{opt}$

(2) The search algebra g reaches the termination algebra $G: g \geq G$

(3) One of the above conditions are met.

3.2 Improved free search algorithm

Standard free search algorithm is put forward to find the maximum objective function as the optimized object. The prediction model in this paper needs to make the error between the predicted value and the actual value is smaller . Thus the optimization object is to obtain minimum value. This paper will change the standard free search algorithm pheromone updating algorithm modified as following equations to achieve the minimum value:

$$\begin{aligned} f_{tj} &= f(x_{tji}) \\ f_j &= \min(f_{tj}) \end{aligned} \quad (15)$$

$$PH_j = \min(f_j)/f_j \quad (16)$$

Search radius R_{ji} is an important parameter in free search algorithm. R_{ji} decide the quality of search process. It is a constant in standard free search algorithm. If the search radius big, the individual search range is wide, and it required a longer time, and it has low convergence accuracy. If the search radius is too small, it is easy to fall into local optimum. Therefore this paper uses the following dynamic search radius method. In the process of optimization search, radius decreases. Its initial value is $R_j(0)=1$. While the step of search increases, the radius reduced.

$$R_j(t) = \begin{cases} R_{\min} + R_j(t-1)\times\lambda & R_j(t) \geq R_{\min} \\ R_{\min} & R_j(t) < R_{\min} \end{cases} \quad (17)$$

The sensitivity parameters also have a great influence on the free search algorithm's performance. The appropriate reduction on sensitivity can increase random in the individual's search neighborhood and enhance the search ability of the algorithms. So this article modifies the sensitivity as follows.

$$SE_j = SE_{\min} + \Delta SE_j \quad (18)$$

$$\Delta SE_j = (SE_{\max} - SE_{\min}) \times random_j(0,1) \times \delta \quad (19)$$

Above all, in this paper, the steps of LSSVM predictive control based on improved free search algorithm for nonlinear systems can be described as follows:

Step1 Initialization

1.1 Set search initial value: population size M, maximum algebra G, search step T, the range of parameters to be optimized.

1.2 Uniformly generating initial population according to Equation (7).

1.3 Initialize search: according to the initial value, we obtain the initial pheromone. Release the initial pheromone. Get the initial search results.

Step 2 Searching

2.1 According to the Equation (18) and Equation (19), calculate the sensitivity.

2.2 According to the Equation (8) and Equation (9), determine the new starting point.

2.3 Select the appropriate objective function, calculate the fitness value.

2.4 According to Equation (15) and Equation (16) , calculate pheromone, according to Equation (7), release pheromones, we obtain the search results.

2.5 select and retain the best individual.

2.6 adjust the search radius according to Equation (17).

Step 3 Termination condition judgment

Judge the termination condition, if it satisfied, then output optimal results.

4 LSSVM predictive control based on IFS algorithm

Considering a following constrained discrete-time multivariable nonlinear system with m dimensional input and n dimensional output[10].

$$\begin{cases} y(k+d) = f(y(k+d-1),\cdots,y(k+d-p),u(k),\cdots,u(k-q+1), \\ s.t.\ \Delta u_{j\min} \leq \Delta u_j \leq \Delta u_{j\max}, u_{j\min} \leq u_j \leq u_{j\max}, j=1,\cdots,m \end{cases}$$

(20)

Wherein, $u(k)$ is input, $y(k)$ is output at moment k, d is system delay, p is order of input, q is order of output, the constraints condition limit variation range of control variables and control increment.

For a given sample data set $\{x_k, y_{k+d}\}$, $x_k = (y(k+d-1),\cdots,y(k+d-p),u(k),\cdots,u(k-q+1))$, the predicted output at next moment through LSSVM can be expressed as following

$$\hat{y}_{k+d+1} = \sum_{k=1}^{N} a_k K(x_k, x_{k+1}) + b \qquad (21)$$

4.1 The design of the predictive controller

For a control system with delay as d, output is $y(k+d)$ when system input is $u(k)$. The system LSSVM model predicted output $y(k+d)$ can be obtained by current moment control variable $u(k)$, past input and output values. If input control variable to be optimized is $u(k+1)$, then system predicted output will be $\hat{y}(k+d+1)$, the error between system actual output and predicted output is:

$$e(k+d) = y(k+d) - \hat{y}(k+d) \qquad (22)$$

The correction is obtained by deviation correction:

$$y_p(k+d+1) = \hat{y}(k+d+1) + e(k+d) \qquad (23)$$

For a control system with m dimensional input and n dimensional output, the design of controller is find the minimum value of following fitness function through IFS optimization algorithm.

$$F(\cdot) = \sum_{i=1}^{P}[y_{ri}(k+i) - y_{pi}(k+i)]^2 + \sum_{j=1}^{M}\lambda_j[u_j(k+j+1) - u_j(k+j)]^2 \qquad (24)$$

The IFS algorithm is used for find a group of optimal control vector $(u(k), u(k+1),\cdots,u(k+M-1))$ that make fitness function has minimum value, and the first vector $u(k)$ will be acted on the controlled object.

LSSVM predictive control based on IFS algorithm for nonlinear systems as shown in Figure 1.

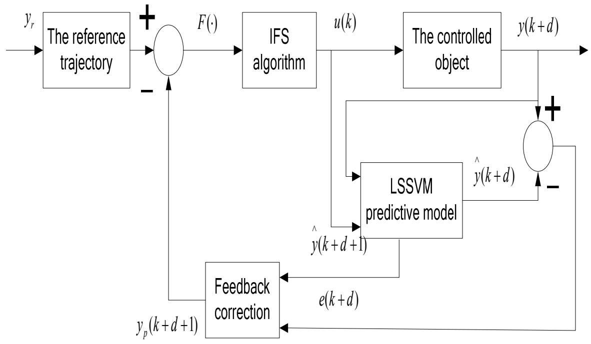

Fig.1 LSSVM predictive control model based on IFS algorithm

4.2 The algorithm stability analysis

Assuming F_k is the objective function of k-th optimization, by the Equation (24) shows $F_k \geq 0$. If $k \to \infty$, the system is stable when $F_k = F_\infty = 0$. When $F_\infty = 0$, by Equation (19) know $y_{pi}(\infty) = y_{ri}(\infty)$, that is system output is equal to the desired output. It is only need to prove $F_\infty = 0$ when $k \to \infty$, then the system is asymptotically stable.

Using reduction to absurdity, assuming $k \to \infty$ there is $F_\infty \neq 0$, it is can be seen F_k is monotone decreasing function by IFS algorithm, at time k, there is:

$$\Delta F_k = F_{k+1} - F_k < 0, F_\infty \leq F_k \leq F_0$$

F_0 is object fitness function value at initial time and it is a constant. Obviously F_k is a bounded closed set, from the Weierstrass theorem known ΔF_k have maximum value. Let maximum value of ΔF_k is $\Delta F_{\max}$, because $F_\infty \neq 0$ and object fitness function is non-negative, there is $\Delta F_{\max} \neq 0$, F_k is transformed into the following form:

$$J_k = J_0 + \sum_{t=0}^{k-1}\Delta J_t \leq J_0 - k\Delta J_{\max}$$

Because of $-\Delta F_{\max} < 0$, the above equation indicate that $F_\infty \to -\infty$ when $k \to \infty$, obviously, it is conflict with the objective fitness function is non-negative, therefore it is $F_\infty = 0$.

From the above proof process shows the control system of this paper is asymptotically stable.

4.3 The steps of algorithm

The steps of LSSVM predictive control model based on IFS algorithm in this paper can be described as follows:

Step1 The parameters of LSSVM and IFS algorithm initialization;

Step2 Applying the input excitation signal to nonlinear systems, get the input and output data samples, LSSVM model is trained using training samples, LSSVM prediction model of nonlinear systems is established;

Step3 LSSVM model is tested by using the testing samples, repeatedly testing and modifying the prediction parameters, until test error meet the requirements;

Step4 The determined control value $u(k)$ at moment k, system output is $y(k+d)$, system predicted output is $\hat{y}(k+d)$, the optimal control vector $(u(k+1), u(k+2), \cdots, u(k+M))$ as the individuals of IFS algorithm, the system predicted output $\hat{y}(k+d+1)$ will be obtained by LSSVM predictive model, fitness function $F(\cdot)$ is calculated;

Step5 The control value sequence $(u(k+1), u(k+2), \cdots, u(k+M))$ will be optimized by IFS algorithm, the optimal control value will be obtained;

Step6 The optimal control value $u(k+1)$ at next moment will be act on nonlinear systems, return to Step4, until simulation is end.

5 Simulation

5.1 IFS algorithm simulation

In order to verify the performance of IFS algorithm, Sphere function and Rosenbrock function is chosen as test function.

Sphere function:

$$f_1 = \sum_{i=1}^{n} x_i^2 \tag{25}$$

Rosenbrock function:

$$f_2 = \sum_{i=1}^{n-1} [100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2] \tag{26}$$

The optimization objective is to find the minimum value of f_1 and f_2, the range of testing function parameters, algorithm parameters and best fitness after optimization is shown in Table 1. For comparison, Table 1 also presents the standard FS algorithm optimization results.

Table 1 Simulation parameters and optimization results

Parameters	Best fitness	
$x_i \in [-50\ 50]$, $n=4$, $\lambda=0.9$, $\delta=0.95$,	FS	2.3211
$R_j(0)=1$, $M=50$, $G=50$, $T=20$	IFS	1.0738
$x_i \in [-2.048\ 2.048]$, $n=6$, $\lambda=0.9$,	FS	6.6546
$\delta=0.95$, $R_j(0)=1$, $M=20$, $G=50$, $T=20$	IFS	4.7782

Figure 2 is the fitness curve of function f_1, and Figure 3 is the fitness curve of function f_2. The results from Table 1, Figure 2 and Figure3 can be seen the IFS algorithm is better than standard FS algorithm in the accuracy of convergence, convergence speed and optimization results.

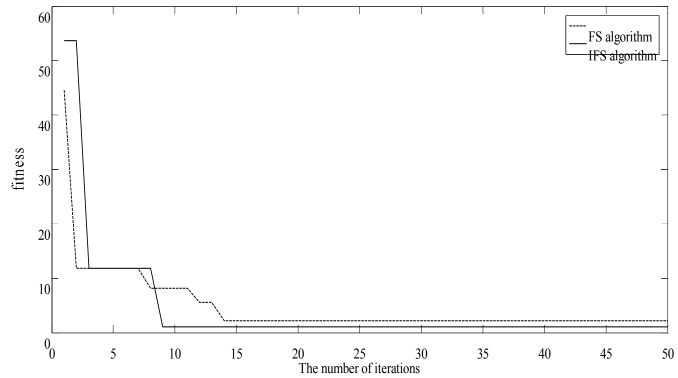

Fig.2 Fitness curve of function 1

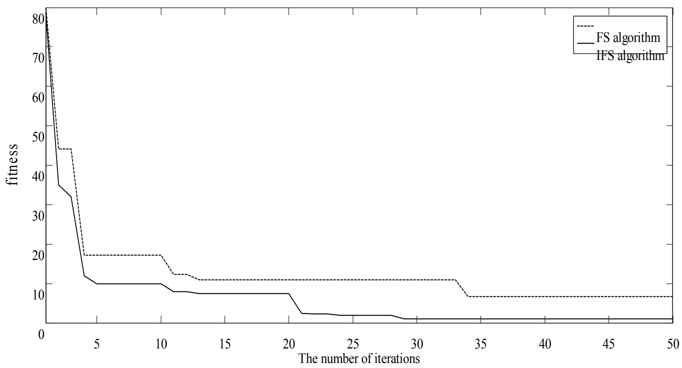

Fig.3 Fitness curve of function 2

5.2 SISO nonlinear systems simulation

Considering the nonlinear object as follows:

$$y(k) = \frac{0.5y(k-1)y(k-2)}{1+y(k-1)^2+y(k-2)^2} + 0.3\cos(0.5y(k-1)+y(k-2))) + 1.2u(k-1)$$

(27)

The predictive model is established through LSSVM algorithm, input signal is white noise with amplitude from -1 to 1, the former 250 group data for training, the latter 250 group data for testing, off-line modeling for LSSVM model, the parameters of LSSVM are obtained by cross validation method, modeling parameters are γ is 7.22, σ^2 is 218.92. Figure 4 is the actual value compared with the predictive value of the test set, Figure 5 is predictive error comparison of the test set. From Figure 4 and 5 can be seen that the LSSVM model has a good predictive effect for nonlinear object.

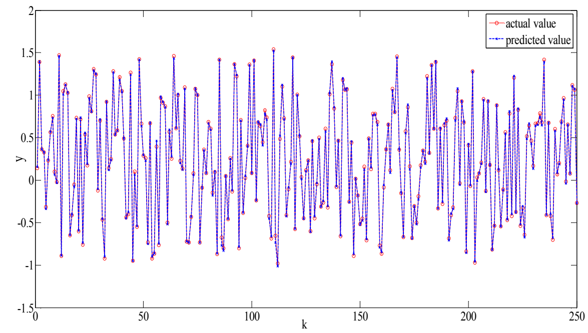

Fig.4 The comparison between predicted and actual value

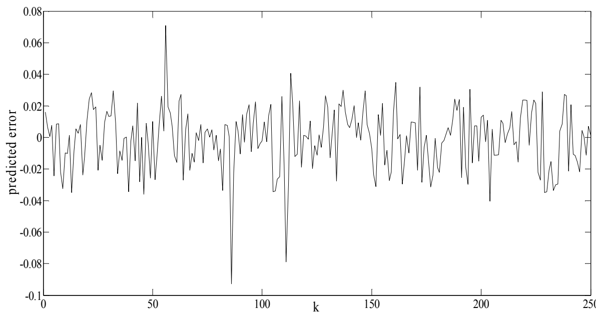

Fig.5 The predictive error curve

The given reference signal is a square wave signal $sign(\sin(0.2\pi k t_s))$, the simulation steps $t_s = 0.1$, simulations parameters are: $\lambda = 0.9$, $\delta = 0.95$, $R_j(0) = 1$, $M = 5$, $G = 10$, $T = 5$, $u(k) \in [-3,3]$, the predictive length $P = 5$, the control length $M = 3$. In order to compare the control effect, compared with the particle swarm optimization (PSO) optimized LSSVM predictive control method in literature [10]. Figure 6 is the output tracking curve of reference signal for nonlinear systems. In order to verify anti-interference ability of method in this paper, a step signal with amplitude of 0.5 is added as interference in the 12s, Figure 7 is the output tracking curve of reference signal for nonlinear systems under interference. From Figure 6 and Figure 7 can be seen for the SISO system, this predictive method in this paper has a good tracking ability for the input reference signal. At the same time, the method can also track input reference signal under interference, it has good robustness for the disturbance.

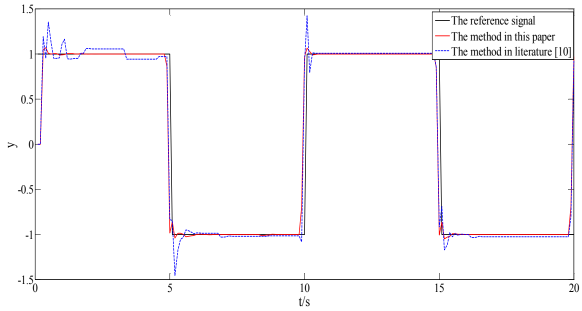

Fig.6 The output tracking curve of reference signal

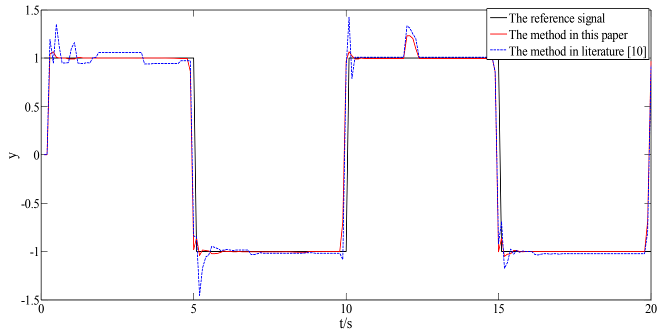

Fig.7 The output tracking curve of reference signal under interference

5.3 MIMO nonlinear systems simulation

The following two input and two output nonlinear systems as the simulation object:

$$y_1(k+1) = \frac{2}{1+y_1^2(k)} + 0.6y_1(k) + u_1(k-1) + 0.2u_2(k-2),$$
$$y_2(k+1) = \frac{2}{1+y_2^2(k)} + 0.5y_2(k) + 0.3u_1(k-1) + u_2(k-1),$$
$$s.t.\ -5 \le u_1, u_2 \le 5. \tag{28}$$

The input signal is 1000 groups white noise with amplitude from -1 to 1, the former 800 groups for training, the latter 200 groups for testing, two nonlinear systems are modeling with off-line, the modeling parameters are: γ is 1.23 and σ^2 is 67.14 for y_1, γ is 1.96 and σ^2 is 42.38 for y_2. Figure 8 is comparison between predicted and actual value of y_1, Figure 9 is comparison between predicted and actual value of y_2, Figure 10 is predicted error of y_1 and y_2. From these figures can be seen, LSSVM model has good predictive ability for MIMO nonlinear systems.

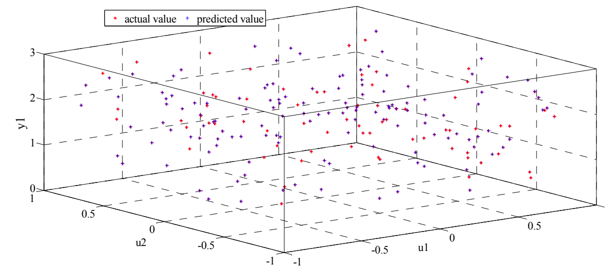

Fig.8 The comparison between predicted and actual value of y_1

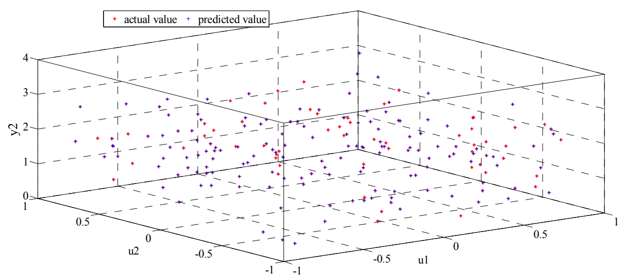

Fig.9 The comparison between predicted and actual value of y_2

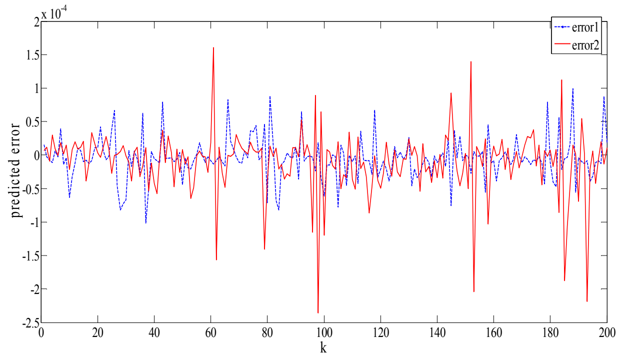

Fig.10 The predictive error curve

The given reference signal is a square wave signal $sign(\sin(0.2\pi k t_s))$, the simulation steps $t_s = 0.1$, the simulation parameters are: $\lambda = 0.95$, $\delta = 0.95$, $R_j(0) = 1$, $M = 10$, $G = 15$, $T = 5$, $u(k) \in [-5, 5]$, the predictive length $P = 4$, the control length $M = 3$. In order to com pare the control effect, compared with the particle s warm optimization (PSO) opti mized LSSVM predictive control method in literature [10]. Figure 11 is output tracking curve of reference signal for y_1, Figure 12 is output tracking curve of reference signal for y_2.

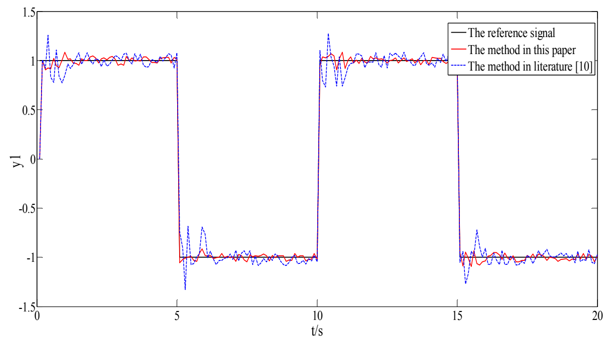

Fig.11 The output tracking curve of reference signal for y_1

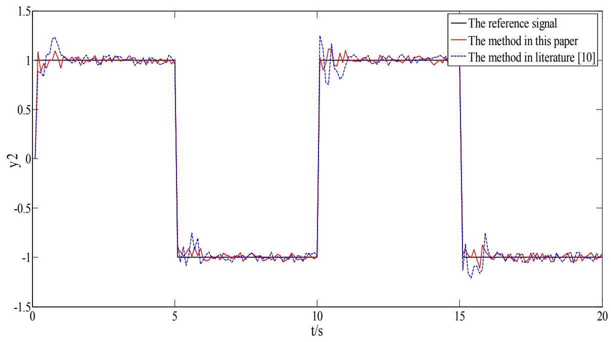

Fig.12 The output tracking curve of reference signal for y_2

From above figures can be seen, for MIMO nonlinear systems, the predictive contr ol method in this paper can gi ve the a ppropriate control value, make nonlinear sy stems can better tracking reference trajectory, therefore, the predictive controller is also applicable to the MIMO nonlinear systems.

6 Conclusion

This paper studied single v ariable and multivariable nonlinear systems control problem, proposed LSSVM predictive control method based on IFS alg orithm. The LSSVM is used for of nonlinear systems modeling, this m odel only need system input and output data, d on not need t he accurate mathematical model of nonlinear sy stems. IFS algorithm is used for online rolling optimization of control value that meet the system requirements. Comparison of experim ental simulation show that this predictive control method has good control effect, and also has bet ter adaptive abilit y and robustness.

Reference

[1] Sslsbury T, Mhaskar P, Qin S J. Pre dictive control methods to improve energy efficiency and reduce demand in buildings[J]. *Computers and Chemical Engineering*,2013,51:77-85.

[2] Bryds M A , Grochowski M, Gm inski T, Konarczak K,Drewa M. Hierarchic al predictive control of integrated wast ewater treatment systems[J]. *Control Engineering Practice*, 2008, 16(6): 751-767.

[3] Dufour P, Michaud D J, TouréY, Dhurjati P S. A partial dif ferential equation m odel predictive control strategy : application to autoclave composite processing[J]. *Computers & Chemical Engineering*, 2004, 28(4): 545-556.

[4] Silani E, Lovera M. Magnetic spacecraft attitude control: a surve y and som e new results[J]. *Control Engineering Practice*,2005, 13(3): 357-371.

[5] Ge S S, Li G Y, Zhang J, et al. Direct ad aptive control for a cla ss of MIMO n onlinear systems using neural networks[J]. *IEEE Trans. on Automatic Control*,2004,49(11):2001-2006.

[6] Zhang H, Cu i L, Zhang X , et al. Data-driven robust approximate optimal tracking c ontrol for unknown general nonlinear sy stems using adaptive dynamic programming method [J]. *IEEE Transactions on Neural Networks*, 2011, 1(1): 2226–2236.

[7] Zhang R, Xu Z, Huang G , et al. Global convergence of onli ne BP training with dynamic learning rate [J] . *IEEE Transactions on Neural N etworks and Learning Systems*, 2012, 32(2): 330–341.

[8] Sun C Y , Mu C X, Li X M. A weighted LS-SVM approach for the identification of a class of nonli near inverse systems[J]. *Science in China Series F-Information Sciences*, 2009, 52(5): 770–779.

[9] Bai L,Wang Z, Shao Y H, et al. A novel feature selection m ethod for twin support vector machine[J].*Knowledge-Based Systems*,2014(59):1-8.

[10] Sun Z Q, Fox G . Traffic Flow Forecasting Based on Com bination of Multidim ensional Scaling and SVM[J]. *International Journal of Intelligent Transportation Systems Research*,2014,12(1):20-25.

[11] Mu C X., Zhang R M, Sun C Y .LS-SVM predictive control based o n PSO for no nlinear systems[J].*Control Theory & Applications*,2010,27(2):164-168.

[12] Xiao R S,Y ong G X, Luo D Z. Analysis on AUV heading contr ol system based on RS-Chaos-LSSVM[J]. *Sensors and Transducers*,2013, 155(8):162-170.

[13] Xing J,Jiang A A,Wen Z W, et al. A nonlinear optimization technique of tunnel construction based on de and LSSVM[J]. *Mathematical Problems in Engineering*,2013:980154.

[14] Kennedy J，Eberhart R C．Particle Swarm Optimization[C]// *Proc．of IEEE International Conference on Neutral Network* , 1995: 1942-1948.

[15] Kalin P, Guy L. Free search a com parative Analysis [J]. *Information Science*, 2005, 172: 173-193.

[16] Li L L, Zhou Y Q, Xie J.A Free Search Krill Herd Algorithm for Functions Optimization[J]. *Mathematical Problems in Engin eering*, 2014:936374.

[17] Ahmed S. Performance of derivative free search ANN training algorithm with time series and classification problems[J]. *Computional Statistics*, 2013,28(5): 1881-1914.

[18] Li L, Zhang Z H, W ang X Y .Analysis of convergence for free search algorithm in solving complex function opti mization problems[C]//*International Conference on Computer, Informatics, Cybernetics and Applications*,2012:1201-1207.