Novel Criterion for Synchronization Stability of Complex Dynamical Networks with Coupling Lags

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Abstract: In the paper, a general complex dynamical network with coupling lags is introduced. Then the problem of synchronization stability analysis for the complex dynamical network is further discussed. By use of linear matrix inequalities (LMI), a novel Lyapunov functional is constructed. And then we have obtained a new general stability criterion of synchronization state in the complex dynamical networks. In the end, numerical simulations are provided to verify the effectiveness and feasibility of the developed theorem.

Key–Words: Complex dynamical network, synchronization stability analysis, linear matrix inequalities, coupling lags, asymptotic stability.

1 Introduction

Complex dynamical networks have become a focal research topic in recent years, and have drawn more and more attention from many fields of science and engineering [1-6]. The main reason is that most practical systems can be modeled by all kinds of complex networks, which are usually referred to as structures that consist of nodes or vertices connected by links or edges. Examples of such complex networks can be found everywhere in our daily lives, such as the Internet, the World Wide Web, food webs, electric power grids, cellular and metabolic networks. These properties of complex dynamical networks have widely been studied. The synchronization motion of its dynamical elements is one of the most important dynamical properties of complex networks. Much work has been researched on synchronization stability analysis for complex networks in the literature [7-11].

Recently, Wang and Chen [12] introduced a uniform dynamical network model and investigated its synchronization and control. Lu and Chen [13] provided a systematic review on the framework to analyze synchronization in complex networks of coupled systems with a focus on the situation of directed graphs. Li and Chen [14] further extended the uniform dynamical network model to include coupling delays among the network nodes and studied its synchronization. L$\ddot{u}$ and Chen [15] studied the synchronization of time-varying complex dynamical networks in which the inner-couplings are time-varying. Zhou *et al.* [16] studied the adaptive synchronization of uncertain complex dynamical network. Zhang *et al.* [17] studied the synchronization of a general complex dynamical network with delayed nodes with adaptive feedback control. A robust tracking control problem of a class of dynamical complex networks was presented through a distributed adaptive approach [18]. The dynamics of networks with delayed coupling have been extensively studied in recent years [19-25]. Tang *et al.* [26] discussed a general complex dynamical network with time-varying delays. Based on the theory of asymptotic stability of linear time-delay systems, two new general stability criteria were proposed. It was researched that sampled-data exponential synchronization of complex dynamical networks with time-varying coupling delay and uncertain sampling[27]. By combining the time-dependent Lyapunov functional approach and convex combination technique, they raised a novel criterion to ensure the exponential stability of the error dynamics.

It is worth noting that most of the existing results on complex networks are concerned with a diagonal inner-coupling matrix, and little progress has been made toward a non-diagonal inner-coupling matrix from complex networks with time coupling delays. The synchronization stability analysis for complex networks, as one of the foremost problems, also reserves largely unsolved and challenging, which stimulates this study.

In the paper, we discuss the problem of synchronization stability analysis for the complex dynamical network with coupling lags. Taking advantage of linear matrix inequality, then a new general stability criterion has been proposed. Finally, two numerical examples are provided to show the effectiveness of the proposed theorem. The rest of the brief is organized as follows. In Section II, a general complex dynamical network is introduced. The stability criterion of the synchronization state is derived in Section III. Then numerical examples are given in Section IV, and conclusion is presented in Section V.

Notation: The notation used throughout the paper is fairly standard. Let R^n denotes the n-dimensional Euclidean space over the reals with the norm $\|\cdot\|$. For any $u=(u_i)_{1\le i\le n}$,$v=(v_i)_{1\le i\le n}$ and $u,v\in R^n$, we define the scalar product of the vectors u and v as:$\langle u,v\rangle=\sum_{i=1}^n u_iv_i$. Let $R=(-\infty,+\infty)$,$R_+=[0,+\infty)$, $R_+^*=(0,+\infty)$, $R_+^{*n}=\{v=(v_i)_{1\le i\le n}\in R^n, v_i\in R_+^*, \forall i=1,2,...,n\}$. Let $\lambda(M)$ denotes the set of eigenvalues of the matrix M, M' its transpose and M^{-1} its inverse.We define$|M|=(|m_{ij}|)_{1\le i,j\le n}$ if $M=(m_{ij})_{1\le i,j\le n}$. Let $C_n=C([-\tau,0],R^n)$ be the Banach space of continuous functions mapping the interval with the topology of uniform convergence. For a given $\phi\in C_n$,we define $\|\phi\|=sup_{-\tau\le\theta\le 0}\|\phi(\theta)\|$, $\phi(\theta)\in R^n$. We define the function $sgn(\cdot)$ and $M^*=(m_{ij})_{1\le i\le n}$ are

$$sgn(\vartheta)=\begin{cases}1 & \vartheta\in R_+^*\\ -1 & -\vartheta\in R_+^*\\ 0 & \vartheta=0\end{cases}, m_{ij}^*=\begin{cases}m_{ij} & \text{if } i=j\\ |m_{ij}| & \text{if } i\ne j\end{cases}$$

2 Problem formulation

In what follows, we consider a general complex dynamical network consisting of N identical linearly and diffusively coupled nodes, with each node being a n-dimensional dynamical system and introduce a complex delayed dynamical network model.

$$\begin{cases}\dot{x}_i=f(x_i)+c\sum_{j=1}^N C_{ij}G(t)x_j(t-\tau), i=1,2,...,N\\ x(t)=\Phi(t), t\in[-h,0]\end{cases} \quad (1)$$

where $x_i=(x_{i1},x_{i2},,x_{in})^T\in R^n$ is a state vector representing the state variables of node i, $G(t)=(g_{ij}(t))_{n\times n}\in R^n$ is a coupling link matrix between node i and node $j(i\ne j)$ for all $1\le i,j\le N$ at time t, the constant $c>0$ is the coupling strength, $C(t)=(C_{ij}(t))_{N\times N}$is the coupling configuration matrix representing topological structure of the network at time t, in which $C_{ij}(t)$ is defined as follows: if there is a connection from node i to node $j(i\ne j)$,then $C_{ij}=C_{ji}=1$, otherwise $C_{ij}=C_{ji}=1(i\ne j)$, $\tau(\tau>0)$ is the time delay and the diagonal elements of matrix $C(t)$ are defined by

$$C_{ii}(t)=-\sum_{j=1,j\ne i}^N C_{ij}, i=1,2,...,N \quad (2)$$

The time delay, τ, is a constant that satisfies

$$0\le\tau\le h$$

where $h>0$ is a constant. The initial condition, $\Phi(t)$, is a continuous and differentiable vector-valued function of $t\in[-h,0]$.

There have been various definitions of synchronization in the literature [14, 15]. Here a rigorous mathematical definition of complete synchronization for delayed dynamical networks is introduced as follows.

Let $x_i(t,X_0)(i=1,2,,N)$ be a solution of the nonautonomous dynamical network

$$\dot{x}_i=f(x_i)+g_i(t,x_1(t-\tau),...,x_N(t-\tau)), i=1,2,...,N \quad (3)$$

where $X_0=((x_1^0)^T,(x_2^0)^T,...,(x_N^0)^T)^T\in R^{nN}$, $f: D\to R^n$ and $g_i: D\times\cdots D\to R^n$ are continuous differentiable with $D\subseteq R^N$, and for all t. If there is a nonempty open subset $E\subseteq D$, with $x_i^0\in E(i=1,2,...,N)$, such that $x_i(t,X_0)\in D$ for all $t\ge 0$,$i=1,2,...,N$, and

$$\lim_{t\to\infty}\|x_i(t,X_0)-s(t,X_0)\|=0, 1\le i\le N \quad (4)$$

where $s(t,x_0)$ is a solution of the system $\dot{x}=f(x)$ with $x_0\in D$, then the dynamical network (3) is said to realize synchronization and $E\times E...\times E$ is called the region of synchrony for network (3). Moreover, $X(t,X_0)=(x_1^T(t,X_0),...,x_N^T(t,X_0))$ is called the synchronous solution of network (3), if $x_i(t,X_0)=x_j(t,X_0)$ for all $t\ge 0$ and $1\le i,j\le N$.

Hereafter, the delayed dynamical network (1) is said to achieve (asymptotical) synchronization if

$$x_1(t)=x_2(t)=...=x_N(t)=s(t), \quad \text{as} \quad t\to\infty \quad (5)$$

where $s(t)\in R^n$ is a solution of an isolate node, namely, $\dot{s}(t)=f(s(t))$.

For later use, we will need the following lemmas.

Lemma 1 *Suppose that the irreducible matrix C satisfies Eq. (2), then:*

(1) 0 is an eigenvalue of C with multiplicity 1associated with the eigenvector $(1,1,..,1)^T$;

(2) All the other eigenvalues of C are less than 0 and $\lambda_i(i=1,2,...,N)$ can be ordered as follows: $0=\lambda_i\ge\lambda_i\ge...\lambda_N$.

Lemma 1 implies that the irreducible matrix C has a 0 eigenvalue of multiplicity 1.

Lemma 2 *[12] If C satisfies Lemma 1, then there exists a unitary matrix,$\Phi(\phi_1, \phi_2, .., \phi_N)$, such that*

$$C^T\phi_k = \lambda_k\phi_k, k = 1, 2, ..., N$$

where $\lambda_i, i = 1, 2, ..., N$, are the eigenvalues of C.

Lemma 3 *[14] Consider the delayed dynamical network (1). Let*

$$0 = \lambda_1 \geq \lambda_2 \geq ...\lambda_N \tag{6}$$

be the eigenvalues of the outer-coupling matrix C. If the following $N-1$ of n-dimensional delayed differential equations are asymptotically stable about their zero solutions:

$$\dot{w} = J(t)w(t)+c\lambda_i G(t)w(t-\tau), i = 2, 3, ..., N \tag{7}$$

where $J(t) = f'(s(t)) \in R^{n\times n}$ is the Jacobian of $f(x(t))$ at $s(t)$, then the synchronized states (5) are asymptotically stable for the dynamical network (1).

3 Main results

In this section, we will derive the main results about the stability analysis of synchronization for the complex delayed dynamical network (1). The inner-coupling matrix $G(t)$ is often regarded as a diagonal matrix, we assume that it is an ordinary constant matrix in the paper and $J(t)$ of system (7) is a constant matrix. For simplicity, we let $G(t) = G$ and $J(t) = J$. Based on the above-mentioned assumptions and definitions, we can obtain the following theorem.

Theorem 4 *For the dynamical system (7), if there exists $Y = (y_{ij})_{1\leq i,j\leq n} = -A - \tau B$ such that*

(1) $y_{ii} > 0, i = 1, 2, ..., n$ and $y_{ij} \leq 0$, for $i \neq j$, $i, j = 1, 2, ..., n$

(2) Successive principal minors of Y are positive, that is,

$$det \begin{pmatrix} y_{11} & y_{12} & \cdots & y_{1i} \\ \cdots & \cdots & \cdots & \cdots \\ y_{i1} & y_{i2} & \cdots & y_{ii} \end{pmatrix} > 0, i = 1, 2, ..., n$$

where $A = (J + c\lambda_i G)^*$ and $B = |c\lambda_i||GJ| + (c\lambda_i)^2|G^2|$, then zero solution of system (7) is asymptotically stable.

That is, the synchronized states (5) are asymptotically stable for the dynamical network (1).

Proof: Suppose that $w(t)$ is continuously differentiable when $t \geq 0$, by using the Newton–Leibniz formula, we can get

$$w(t-\tau) = w(t) - \int_{t-\tau}^{t} \dot{w}(s)ds \tag{8}$$

Substituting Eq. (7) into Eq. (8), we achieve

$$w(t-\tau) = w(t)-J\int_{t-\tau}^{t} w(s)ds-c\lambda_i G\int_{t-\tau}^{t} w(s-\tau)ds \tag{9}$$

Then systems (7) can be rewritten

$$\begin{aligned}\dot{w}(t-\tau) &= (J + c\lambda_i G)w(t) - c\lambda_i GJ\int_{t-\tau}^{t} w(s)ds \\ &- (c\lambda_i)^2G^2\int_{t-\tau}^{t} w(s-\tau)ds\end{aligned} \tag{10}$$

Let $v \in R^n$ with components $v_i > 0 (i = 1, 2, ..., n)$ and let us consider the radially unbound Lyapunov functional given by

$$U(t) = U_1(t) + U_2(t) + U_3(t) + U_4(t) \tag{11}$$

where

$$U_1(t) =< |w(t)|, v > \tag{12}$$

$$U_2(t) = |c\lambda_i| < |GJ|\int_{-\tau}^{0}\int_{t+s}^{t} |w(\theta)|d\theta ds, v > \tag{13}$$

$$U_3(t) = (c\lambda_i)^2 < |G^2|\int_{-\tau}^{0}\int_{t+s}^{t} |w(\theta-\tau)|d\theta ds, v > \tag{14}$$

and

$$U_4(t) = \tau(c\lambda_i)^2 < |G^2|\int_{t-\tau}^{t} |w(\theta)|d\theta, v > \tag{15}$$

Then it is obvious that

$$U(t) < \infty, t > 0 \tag{16}$$

The right Dini derivative of U along the solution of Equation (10) gives

$$\begin{aligned}D^+U(t)|_{(10)} &= D^+U_1(t)|_{(10)} + D^+U_2(t)|_{(10)} \\ &+ D^+U_3(t)|_{(10)} + D^+U_4(t)|_{(10)}\end{aligned} \tag{17}$$

We have

$$\begin{aligned}D^+U_1(t)|_{(10)} &=< \frac{d^+|w(t)|}{dt^+}, v > \\ &< D_w(t)\frac{d^+w(t)}{dt^+}, v >\end{aligned} \tag{18}$$

where $D_w(t) = \text{diag}\{sgn(w_1), sgn(w_2), ..., sgn(w_n)\}$. Then we can obtain

$$\begin{aligned} D^+U_1(t)|_{(10)} =& < D_w(t)(J + c\lambda_i G)w(t), v > \\ &- < D_w(t)(c\lambda_i GJ \int_{t-\tau}^{t} w(s)ds), v > \\ &- < D_w(t)(c\lambda_i)^2 G^2 \int_{t-\tau}^{t} w(s-\tau)ds, v > \end{aligned} \tag{19}$$

Next, by overvaluing$D^+U_1(t)|_{(10)}$, we can get

$$\begin{aligned} D^+U_1(t)|_{(10)} \le& < (J + c\lambda_i G)^* |w(t)|, v > \\ &+ < |c\lambda_i||GJ| \int_{t-\tau}^{t} |w(s)|ds, v > \\ &+ < (c\lambda_i)^2 |G^2| \int_{t-\tau}^{t} |w(s-\tau)|ds, v > \end{aligned} \tag{20}$$

Similarly, we have

$$\begin{aligned} D^+U_2(t)|_{(10)} =& |c\lambda_i| < |GJ|(\tau|w(t)|), v > \\ &- |c\lambda_i| < |GJ|(\int_{t-\tau}^{t} |w(\theta)|d\theta), v > \end{aligned} \tag{21}$$

$$\begin{aligned} D^+U_3(t)|_{(10)} =& (c\lambda_i)^2 < |G^2|(\tau|w(t-\tau)|), v > \\ &- (c\lambda_i)^2 < |G^2|(\int_{t-\tau}^{t} |w(\theta-\tau)|d\theta), v > \end{aligned} \tag{22}$$

and

$$D^+U_4(t)|_{(10)} = (c\lambda_i)^2 < \tau|G^2|(|w(t)| - |w(t-\tau)|), v > \tag{23}$$

From Eqs. (20)-(23) and Eq. (11), we obtain

$$D^+U(t)|_{(10)} \le < -Y|w(t)|, v > \tag{24}$$

where $Y = -A - \tau B$, $A = (J + c\lambda_i G)^*$ and $B = |c\lambda_i||GJ| + (c\lambda_i)^2|G^2|$.

If Y satisfies the condition (1) and the condition (2), we can find a vector$\rho \in R_+^*$[20], i.e with components $\rho_k \in R_+^*$ satisfying the relation $Y_1 v = \rho$, $\forall v \in R_+^*$, here $< -Y|w(t)|, v > = < -Y_1 v, |w(t)| >$. So, we have

$$< -Y|w(t)|, v > = < -\rho, |w(t)| > \tag{25}$$

In the end, we can get

$$D^+U(t)|_{(10)} < -\sum_{k=1}^{n} \rho_k |w_k(t)| < 0 \tag{26}$$

Then, it follows that zero solution of system (7) is asymptotically stable. Form Lemma 3, we know that the synchronized states (5) are asymptotically stable for the dynamical network (1).The proof is completed. □

4 Numerical simulation

In the previous sections, we discuss a general complex dynamical network.Then a new general stability criterion of synchronization state have been proposed. The above synchronization conditions can be applied to networks with different topologies and different sizes. In order to illustrate the main results of the above theoretical analysis, we consider a lower-dimensional network model with five nodes, in which each node is a simple three-dimensional stable linear system described in the reference [14].

$$\begin{cases} \dot{x}_1 = -x_1 \\ \dot{x}_2 = -2x_2 \\ \dot{x}_3 = -3x_3 \end{cases}$$

which is asymptotically stable at $s(t) = 0$, and its Jacobian is $J(t) = \text{diag}\{-1, -2, -3\}$.

Assume that the outer-coupling matrix C and the inner-coupling matrix G are given as follows

$$C = \begin{pmatrix} -2 & 1 & 0 & 0 & 1 \\ 1 & -3 & 1 & 1 & 0 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 1 & 1 & -3 & 1 \\ 1 & 0 & 0 & 1 & -2 \end{pmatrix}$$

$$G = \begin{pmatrix} 1.5267 & 1.2382 & 0 \\ -1.8824 & 0 & 1.0337 \\ 0 & 1.4472 & 0 \end{pmatrix}$$

Obviously, C is an irreducible symmetric matrix. The eigenvalues of C are $\lambda_i = 0, -1.382, -2.382, -3.618, -4.618$. For clearer visions, we take the coupling strength $c = 0.2$ and $\tau = 0.05$.

In terms of Theorem 4, if the condition (1) and the condition (2) are satisfied, then it is inferred that the synchronization of the complex network (1) can be achieved. When $\lambda_i = -1.382, -2.382, -3.618, -4.618$, we can get

$$Y = \begin{pmatrix} 1.4009 & -0.3837 & -0.0049 \\ -0.5573 & 1.9968 & -0.3286 \\ -0.0104 & -0.4400 & 2.9943 \end{pmatrix}$$

$$Y = \begin{pmatrix} 1.6910 & -0.6703 & -0.0145 \\ -0.9742 & 1.9905 & -0.5663 \\ -0.0309 & -0.7584 & 2.9830 \end{pmatrix}$$

$$Y = \begin{pmatrix} 2.0495 & -1.0350 & -0.0335 \\ -1.5054 & 1.9781 & -0.8602 \\ -0.0713 & -1.1519 & 2.9608 \end{pmatrix}$$

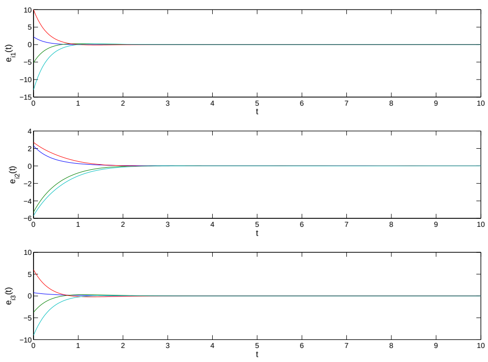

Figure 1: (color online)Synchronization errors for the delayed network with coupling strength $c = 0.2$ and time delay $\tau = 0.05$.

and

$$Y = \begin{pmatrix} 2.3396 & -1.3386 & -0.0546 \\ -1.9481 & 1.9644 & -1.0979 \\ -0.1162 & -1.4703 & 2.9362 \end{pmatrix}$$

, respectively. Obviously, the condition (1) of Theorem 4 is satisfied. At the same time, we observe that eigenvalues of the above four matrices are $d_1 = (1.1287, 2.1280, 3.1353)$, $d_2 = (0.9204, 2.3518, 3.3923)$, $d_3 = (0.5199, 2.6051, 3.8634)$ and $d_4 = (0.1436, 2.7707, 4.3258)$, respectively. Therefore, the condition (2) of Theorem 4 is satisfied and the synchronized states (5) of network (1) are asymptotically stable. In Fig. 1, we plot the curves of the synchronization errors between the states of node i and node $i+1$(that is, $e_{ij}(t) = x_{ij}(t) - x_{i+1,j}(t)$), for $i = 1, 2, 3, 4, j = 1, 2, 3$, with the coupling strength $c = 0.2$ and time delay $\tau = 0.05$.

When the coupling strength $c = 0.2$ and time delay $\tau = 0.05$, by using Theorem 2(see Ref. 14), we found that there exist two positive-definite matrices and we know that the synchronized states (5) of network (1) are asymptotically stable as shown in Fig. 2. When the coupling strength $c = 0.5$, we found that there does not exist two positive-definite matrices by use of the Matlab LMI Toolbox. So Theorem 2 fails. Here we give the curves of the synchronization errors in Fig. 3, when the coupling strength $c = 0.5$ and time delay $\tau = 0.05$. However, by using Theorem 4 in our paper, we can verify that the synchronized states (5) of network (1) are asymptotically stable. As shown in Fig. 4, the synchronization errors converge to zero when the coupling strength $c = 0.5$ and time delay $\tau = 0.05$.

When the coupling strength $c = 0.08$ and time delay $\tau = 0.5$ and the two coupling matrix are the

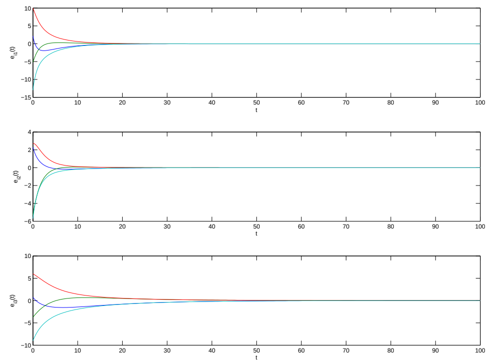

Figure 2: (color online)Synchronization errors for the delayed network with coupling strength $c = 0.2$ and time delay $\tau = 0.05$.

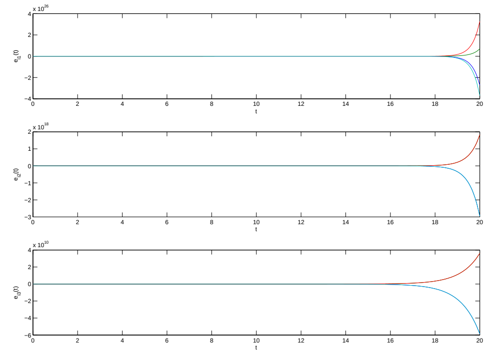

Figure 3: (color online)Synchronization errors for the delayed network with coupling strength $c = 0.5$ and time delay $\tau = 0.05$.

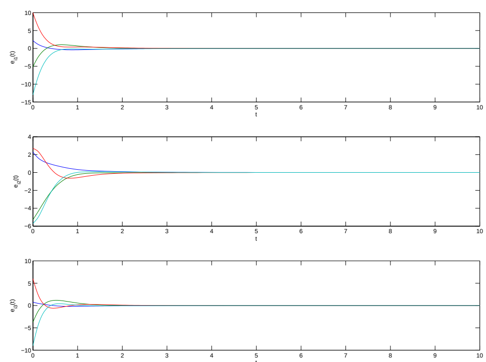

Figure 4: (color online)Synchronization errors for the delayed network with coupling strength $c = 0.5$ and time delay $\tau = 0.05$.

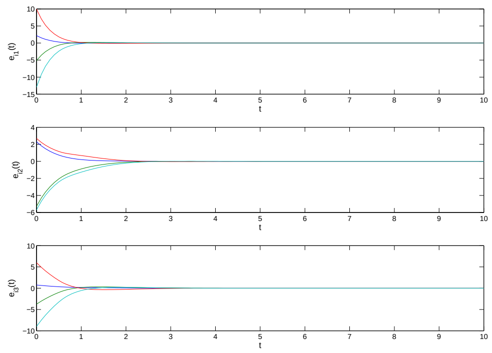

Figure 5: (color online)Synchronization errors for the delayed network with coupling strength $c = 0.08$ and time delay $\tau = 0.5$.

same as the above example, we can also derive the conclusion that the two conditions of Theorem 4 are satisfied. The curves of the synchronization errors are shown in Fig. 5. We see that the synchronization errors converge to zero under the above conditions.

5 Conclusion

In the paper, we introduce a class of time-varying complex delayed dynamical network model. According to the stability theory of the linear time-delay system, we have obtained a new general stability criterion of synchronization state in the complex dynamical networks. By using of constructing the Lyapunov function, the criterion is easy to be verified. And two examples are numerically investigated. All involved numerical simulations are in line with the theoretical analyses.

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