Robust Fault-Tolerant Tracking Control Scheme for Nonlinear Nonaffine-in-control Systems Against Actuator Faults

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Abstract: In this paper a theoretical framework of robust fault-tolerant control (FTC) for a class of nonlinear nonaffine-in-control systems is developed. In the framework, an observer-like auxiliary system is designed via adaptive and sliding mode techniques, which is only required to ensure that the output of the auxiliary system asymptotic tracking plant output. Based on the auxiliary system, a reconfigurable fault-tolerant tracking controller is proposed for nonaffine-in-control systems. In this paper, the focus is on the accommodation of the actuator faults and resulting disturbances. It is shown that the proposed FTC design results in asymptotic convergence of the tracking error to zero. Finally, the proposed approach is verified using a three-degree-of-freedom simulation of a typical fighter aircraft and the significantly improved system response demonstrates the practical potential of the theoretic results obtained.

Key–Words: Fault tolerant control, Adaptive sliding mode observer, Non-affine nonlinear systems, Actuator faults, Disturbances.

1 Introduction

The increasing demands on system performance will consequently increase the possibility of system failures. Faults may occur in any locations and dramatically change the system behaviour resulting in degradation or even instability. To improve control system reliability and stability, fault-tolerant control (FTC) for dynamic systems has become an attractive topic and have received considerable attention during the past two decades. A fault tolerant controller is a controller able to satisfy control specifications both in nominal operation and after the occurrence of a fault. FTC can be mainly classified into two types: passive and active [1]. In passive approach, the same controller is used throughout the normal case as well as the fault case [2]. An active FTC system compensates for the effect of fault by synthesizing a new control strategy based on online accommodation [3-9]. Generally speaking, the active approach is less conservative than the passive one, which has increasingly been the main methodology in designing FTC systems [10].

Fig. 1 (A) shows the block diagram of traditional active FTC. It can be seen that there are two major difficulties in traditional robust active FTC for nonlinear nonaffine-in-control systems. One is the robust estimation of fault parameters. Obviously, *how to draw accurate fault information from uncertain sys-*

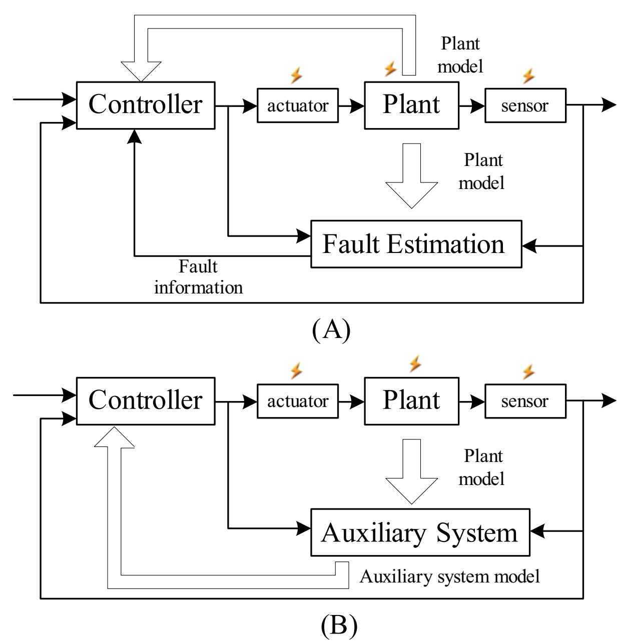

Figure 1: (A) Block diagram of traditional active FTC. (B) Block diagram of Proposed robust FTC.

tems with disturbances is not an easy task. The other is reconfigurable controller design. Traditional methods include linearization of the nonlinear plant model around one or multiple operating points, inverse system method et al. However, in some flight nonaffine-in-control systems, operating points dependent on the current flight regime. Using a fixed linear controller or finite linear controllers may result in an unacceptable response and even in instability of the closed-loop system [11], and it is generally difficult to prescribe a technique to actually obtain such an inverse. Recently, some new nonaffine controller design methods have been developed in [12-15]. Although [12] proposed a PI-like fault tolerant controller design method for a class SISO nonaffine-in-control systems. For more general multivariable nonlinear systems, there is a clear need for the development of systematic FTC design techniques.

In summary, a novel robust FTC framework is proposed in this paper, which is shown as Fig. 1 (B). In the framework, an auxiliary system is designed, which does not need the accurate fault information for uncertain nonlinear systems. In doing so, a thorny problem can be avoided, that is *robustness and accuracy of fault estimation*. Then, using dynamic approximation technique [16-17], the nonaffine nonlinear control method the reconfigurable controller is designed based on the dynamic model of auxiliary system. The rest of this paper is organized as follows. In Section II, a brief of actuator faults is described for non-affine nonlinear systems. In Section III, main results are given, which include auxiliary system design, reconfigurable controller design, and stability analysis of tracking close-loop system. Finally, the proposed approach is tested using a three-degree-of-freedom (3-DOF) unmanned aerial vehicle (UAV) point mass model.

2 Actuator Fault Modeling

Consider the following non-affine nonlinear system

$$\dot{x} = f(x,u) + d(t) \tag{1}$$

where $x \in \mathbb{R}^r$ is the state vector, $u \in \mathbb{R}^m$ is the input vector, and $d \in \mathbb{R}^r$ is unknown but bounded external disturbance vector, respectively. $f(\cdot)$ is the nonlinear functions. Let $\Omega_x \in \mathbb{R}^p$ be compact set defined by $\Omega_x \triangleq \{x||x| \leqslant b_x\}$ where $b_x > 0$ is a positive constant.

To formulate the FTC problem, the fault model must be established. According to the fault type for flight-control system established in [24], the fault type considered in this study is the loss of actuator effectiveness. We use $u_i^F(t)$ to describe the control signal sent from the ith actuator [3,4] as follows:

$$\begin{aligned} &u_i^F(t) = \sigma_i u_i(t), \quad \sigma_i \in [\underline{\sigma}_i, \overline{\sigma}_i] \\ &0 < \underline{\sigma}_i \leq 1, \quad \overline{\sigma}_i \geq 1 \quad, i = 1, 2, \cdots\cdots, m \end{aligned} \tag{2}$$

where σ_i is an unknown constant, and which is called as lose of effectiveness (LOE) factor. $\underline{\sigma}_i$ and $\overline{\sigma}_i$ represent the known lower and upper bounds of σ_i, respectively. When $\underline{\sigma}_i = \overline{\sigma}_i = 1$, meaning that the ith actuator $u_i(t)$ is in fault free case.

The control input in faut case can be described by:

$$u_1^F(t) = [u_1^F(t), u_2^F(t), \cdots\cdots, u_m^F(t)]^T = \boldsymbol{\sigma} u(t)$$

with $\boldsymbol{\sigma} = \mathrm{diag}[\sigma_1, \cdots, \sigma_m]$.

Hence, the non-affine nonlinear systems (1) with actuator faults (2) can be transformed into

$$\dot{x} = f(x, \sigma u) + d(t) \tag{3}$$

and the failure model of (3) can be expressed as the following general formula

$$\dot{x} = f(x, u, \sigma) + d(t) \tag{4}$$

where $\boldsymbol{\sigma} = [\sigma_1, \cdots, \sigma_m]^T$. The preceding model is subject to the following Assumption 1.

Assumption 1: $f(x,u,\sigma)$ is C^1 for all x, u, σ and is a smooth function with respect to state x, control input u and LOE factor σ. And the norm of the control signal $u(t)$ is uniformly bounded by constants $\beta > 0$, i.e., $\|u\| \leq \beta$.

The desired dynamics of the nonlinear systems (1) is now chosen in the form

$$\dot{x}_m = A_m x_m + B_m r \tag{5}$$

where $x_m \in \mathbb{R}^r$ denotes the state of the reference model. $A_m$is asymptotically stable. $r \in \mathbb{R}^n$ denotes a vector of bounded piecewise continuous reference inputs.

FTC Objective: Design a FTC input $u(t)$ such that $\|x(t) - x_m(t)\| \leq \epsilon$ for all time despite the effect of the disturbance $\omega(t)$, actuator faults.

3 Main Results

For uncertain nonaffine-in-control systems, two parts must be developed, which are auxiliary system and reconfigurable tracking controller. And some stability analysis is given for the closed-loop fault tolerant tracking system.

3.1 Auxiliary System Design

From Assumption 1, the first-order Taylor expansion of the nonlinear function $f(x,u,\sigma)$ with respect to σ around the neighborhood $\hat{\sigma}$ can result in

$$f(x,u,\sigma) = f(x,u,\hat{\sigma}) + g_1(x,u,\hat{\sigma})(\sigma-\hat{\sigma}) + \xi(t) \tag{6}$$

where

$$g_1(x,u,\hat{\sigma}) = \left.\frac{\partial f(x,u,\sigma)}{\partial \sigma}\right|_{\sigma=\hat{\sigma}}$$

and

$$\xi(t) = \sum_{i=2}^{\infty} \left.\frac{\partial^i f(x,u,\sigma)}{\partial \sigma^i}\right|_{\sigma=\hat{\sigma}} (\sigma-\hat{\sigma})^i$$

Then, the (4) can be rewritten as

$$\dot{x} = f_1(x,u,\hat{\sigma}) + g_1(x,u,\hat{\sigma})\sigma + \upsilon(t) \tag{7}$$

where

$$f_1(x,u,\hat{\sigma}) = f(x,u,\hat{\sigma}) - g_1(x,u,\hat{\sigma})\hat{\sigma}$$
$$\upsilon(t) = \xi(t) + d(t)$$

Notice that $\upsilon(t)$ is unknown but bounded, that is to say there exists an unknown constant $\overline{\upsilon}$ such that $\|\upsilon(t)\| \leq \overline{\upsilon}$. Let $\tilde{x} = \hat{x} - x$. Under Assumption 1, we can design a following auxiliary system for (7).

$$\dot{\hat{x}} = A(\hat{x}-x) + f_1(x,u,\hat{\sigma}) + g_1(x,u,\hat{\sigma})\hat{\sigma} + \nu(t) \tag{8}$$

where $\hat{\boldsymbol{\sigma}} = [\hat{\sigma}_1, \cdots, \hat{\sigma}_m]^T$ denotes the estimation of actuator efficiency factor, which is obtained as

$$\dot{\hat{\boldsymbol{\sigma}}} = \mathrm{Proj}_{[\underline{\sigma}_i,\overline{\sigma}_i]}\{-2\gamma_1 g_1^T(x,u,\hat{\sigma})P\tilde{x}\} \tag{9}$$

where $\gamma_1 > 0$, and $P = P^T > 0$ is a solution to the Lyapunov matrix equation $A^TP + PA = -Q$, where $Q = Q^T > 0$. (In the preceding equations $\mathrm{Proj}\{\cdot\}$ denotes the projection operator [19] the role of which is to project the estimates $\hat{\sigma}_i$ to the intervals $[\underline{\sigma}_i, \overline{\sigma}_i]$). And $\nu(t)$ is defined as

$$\nu(t) = \begin{cases} -\frac{P\tilde{x}}{\|P\tilde{x}\|}m(t), & \text{if } \|P\tilde{x}\| \neq 0 \\ 0, & \text{otherwise} \end{cases} \tag{10}$$

In (10), $m(t)$ is given by the following update law

$$\dot{m}(t) = G\tilde{x}^T\tilde{x}, \quad m(0) > 0 \tag{11}$$

where G is a design constant that can be used to regulate the increasing rate of $m(t)$ and thus the rate of the state estimation error converging to zero. Larger G means that the state estimation error converges to zero faster.

3.1.1 Stability analysis

We have the following results regarding to the stability of the resulting state estimation error dynamics resulting from the proposed auxiliary system.

Let $\tilde{\sigma} = \hat{\sigma} - \sigma$. By using the above auxiliary system, the resulting state estimation error dynamics is

$$\dot{\tilde{x}} = A\tilde{x} + g_1(x,u,\hat{\sigma})\tilde{\sigma} + \nu(t) - \upsilon(t) \tag{12}$$

Theorem 1: Under Assumptions 1, if we apply the auxiliary system given by (8) – (11) to system (4), then state estimation error dynamics given by (12) is globally asymptotically stable, that is, for any initial conditions $\tilde{x}(0)$ and $m(0)$, we have $\lim_{t\to\infty}\tilde{x}(t) = 0$, $\tilde{\sigma}(t)$ and $m(t)$ is bounded.

Proof: Consider the Lyapunov function

$$V_1(t) = \tilde{x}^TP\tilde{x} + \frac{1}{2\gamma_1}\tilde{\sigma}^T\tilde{\sigma}$$

The derivative of $V_1(t)$ along the trajectory of the augmented state-error dynamics (12) can be written as

$$\begin{aligned}\dot{V}_1(t) =& \dot{\tilde{x}}^TP\tilde{x} + \tilde{x}^TP\dot{\tilde{x}} + \frac{1}{\gamma_1}\tilde{\sigma}^T\dot{\tilde{\sigma}} \\ =& -\tilde{x}^TQ\tilde{x} + \tilde{\sigma}^T(2g_1^TP\tilde{x} + \frac{1}{\gamma_1}\dot{\tilde{\sigma}}) \\ &+ 2\tilde{x}^TP(\nu-\upsilon)\end{aligned} \tag{13}$$

Substituting the adaptive update law (9) and sliding mode term (10) into (13) yields the following equation

$$\begin{aligned}\dot{V}_1(t) =& -\tilde{x}^TQ\tilde{x} + 2\tilde{x}^TP(\nu-\upsilon) \\ =& -\tilde{x}^TQ\tilde{x} - 2\|P\tilde{x}\|m(t) - 2\tilde{x}^TP\upsilon \\ \leq& -\tilde{x}^TQ\tilde{x} - 2\|P\tilde{x}\|m(t) - 2\|P\tilde{x}\|\overline{\upsilon} \\ =& -\tilde{x}^TQ\tilde{x} - 2\|P\tilde{x}\|[m(t) - \overline{\upsilon}]\end{aligned} \tag{14}$$

Now, we prove the $m(t)$ is bounded via reductio ad absurdum. First, we assume that $m(t)$ is unbounded. From the definition (10), we can obtain the $m(t)$ is positive and increasing (i.e. $\lim_{t\to\infty} m(t) = \infty$). Hence, there exists a time T, we have $m(t) \geq T$ for all $t > T$. So we have

$$\dot{V}_1(t) \leq -\tilde{x}^TQ\tilde{x} \tag{15}$$

Because $Q > 0$, so $V_1(t)$ is bounded and decreasing. And we can get

$$\begin{aligned}\lambda_{\min}(Q)\int_T^t \tilde{x}(\tau)^T\tilde{x}(\tau)d\tau \leq& \int_T^t \tilde{x}(\tau)^TQ\tilde{x}(\tau)d\tau \\ \leq& V_1(\tilde{x}(T)) - V_1(\tilde{x}(t))\end{aligned} \tag{16}$$

Hence we can obtain $\int_T^\infty \tilde{x}(\tau)^T \tilde{x}(\tau) d\tau$ is bounded, which also means that $m(t) = \int_0^t G\tilde{x}^T(\tau)\tilde{x}(\tau)d(\tau) + m(0)$ is bounded. This leads to a contradiction to the assumption that $m(t)$ is not bounded. Therefore, we have proved that $m(t)$ has to be bounded and there exists a positive constant $\bar{m}$ such that $\lim_{t\to\infty} m(t) = \bar{m}$.

Because $m(t)$ is bounded. We can obtained $\tilde{x}(t)$ is bounded too. This together with (12) implies that $\dot{\tilde{x}}(t)$ is bounded. By using Barbalat lemma [25], the boundness of $m(t)$, the boundness of $\int_0^\infty \tilde{x}(\tau)^T \tilde{x}(\tau) d\tau$ implies $\lim_{t\to\infty} \tilde{x}(t) = 0$. This together with the inequality in (14) implies that $\lim_{t\to\infty} \dot{V}(t) = 0$, which in turn implies that $\tilde{\sigma}(t)$ is bounded. The proof is complete.

Remark 1: In auxiliary system (8), sliding mode term $\nu(t)$ is modified as

$$\nu(t) = -\frac{P\tilde{x}}{\|P\tilde{x}\| + \delta} m(t)$$

where $\delta = \delta_0 + \delta_1 \|\tilde{x}\|$, and δ_0, δ_1 are two positive constants.

Remark 2: The novelty of the auxiliary system lies in the adaptive update law (9) and the sliding mode gain $m(t)$, which is introduced to deal with unknown bounded $\overline{\upsilon}$.

3.2 Controller Design and Stability Analysis

The problem of controlling the plants characterized by models that are non-affine in the control input vector is a thorny one, especially for the tracking control. So far, concentrated research has been conducted for the controller design only for affine nonlinear systems.

Define $F(x, u, \hat{\sigma}) = f_1(x, u, \hat{\sigma}) + g_1(x, u, \hat{\sigma})\hat{\sigma}$. Then, the Taylor expansion of the nonlinear function $F(x, u, \hat{\sigma})$ with respect to $u(t)$ around the neighborhood $u_n(t)$ can result in

$$F(x, u, \hat{\sigma}) = F(x, u_n, \hat{\sigma}) + F_d(x, u_n, \hat{\sigma})(u - u_n) + O(\cdot) \tag{17}$$

where

$$F_d(x, u_n, \hat{\sigma}) = \left.\frac{\partial F(x, u, \hat{\sigma})}{\partial u}\right|_{u=u_n}$$

and

$$O(\cdot) = \sum_{i=2}^{\infty} \left.\frac{\partial^i F(x, u, \hat{\sigma})}{\partial u^i}\right|_{u=u_n} (u - u_n)^i$$

If we let $F_n(x, u_n, \hat{\sigma}) = F(x, u_n, \hat{\sigma}) - F_d(x, u_n, \hat{\sigma})u_n$, so we can rewrite (8) as

$$\dot{\tilde{x}} = A\tilde{x} + F_n(x, u_n, \hat{\sigma}) + F_d(x, u_n, \hat{\sigma})u + O(\cdot) + \nu(t) \tag{18}$$

From (17), it can be seen that if we let $\lim \|u - u_n\| = 0$, then $\lim \|O(\cdot)\| = 0$. We consider lag property of the filtering as [16-17]

$$\dot{u}_n = -\zeta u_n + \zeta u \tag{19}$$

Then $\lim_{\zeta\to\infty} u_n = u$. So use the above filter (19), it can be ensure that $\lim_{\zeta\to\infty} \|O(\cdot)\| = 0$.

From above analysis, the observer (8) can be described as an affine system with time-varying parameters by following

$$\begin{aligned} &\dot{u}_n = -\zeta u_n + \zeta u \\ &\dot{\tilde{x}} = A\tilde{x} + F_n(x, u_n, \hat{\sigma}) + F_d(x, u_n, \hat{\sigma})u + \nu(t) + O \end{aligned} \tag{20}$$

Remak 3: Here, $\zeta \to \infty$ is only a rigorous expression for mathematics meanings, in general, $\zeta \in [5, 50]$. The filter (19) is not unique. The filtering u_n can be completely replaced by other filtering equation, such as higher-order differentiator etc. In actual processes, the dynamics of filter (19) also can be chosen as same as actuator dynamics.

For the given bounded piecewise continuous reference inputs r. Define the estimated tracking errors as $\hat{e} = \hat{x} - x_m$. So the control law can be chosen with (19) as

$$\begin{aligned} &\dot{u}_n = -\zeta u_n + \zeta u \\ &u = -F_d^{-1}(x, u_n, \hat{\sigma}) \left[A\tilde{x} + F_n(x, u_n, \hat{\sigma})\right. \\ &\qquad \left. + \nu(t) + K\hat{e} - A_m x_m - B_m r\right] \end{aligned} \tag{21}$$

It can be seen that we can designed the gain matrix K by following Riccati equation:

$$K^T P_1 + P_1 K = -Q_1 \tag{22}$$

where $P_1 = P_1^T > 0$ and $Q_1 = Q_1^T > 0$.

Remark 4: Strictly speaking, when the dimension of control inputs is not equal to that of state variables, the inverse matrix of $F_d(x, u_n, \hat{\sigma})$ is in the nonexistence. Thus, in this study, we adopt the generalized matrix inverse of $F_d(x, u_n, \hat{\sigma})$, denoted as $F_d^{-1}(x, u_n, \hat{\sigma})$, which also meets the condition $F_d^{-1}(x, u_n, \hat{\sigma})F_d(x, u_n, \hat{\sigma}) = I$.

Theorem 2: Consider the actuator faulty system (4), under the auxiliary system (8)-(11) and control law (21). Let $e = x - x_m$, we have $\lim_{\zeta\to\infty} \hat{e}(t) = 0$ and $\lim_{\zeta\to\infty} e(t) = 0$.

Proof: Substituting (21) into (20) yields

$$\dot{\hat{e}} = K\hat{e} + O(\cdot) \tag{23}$$

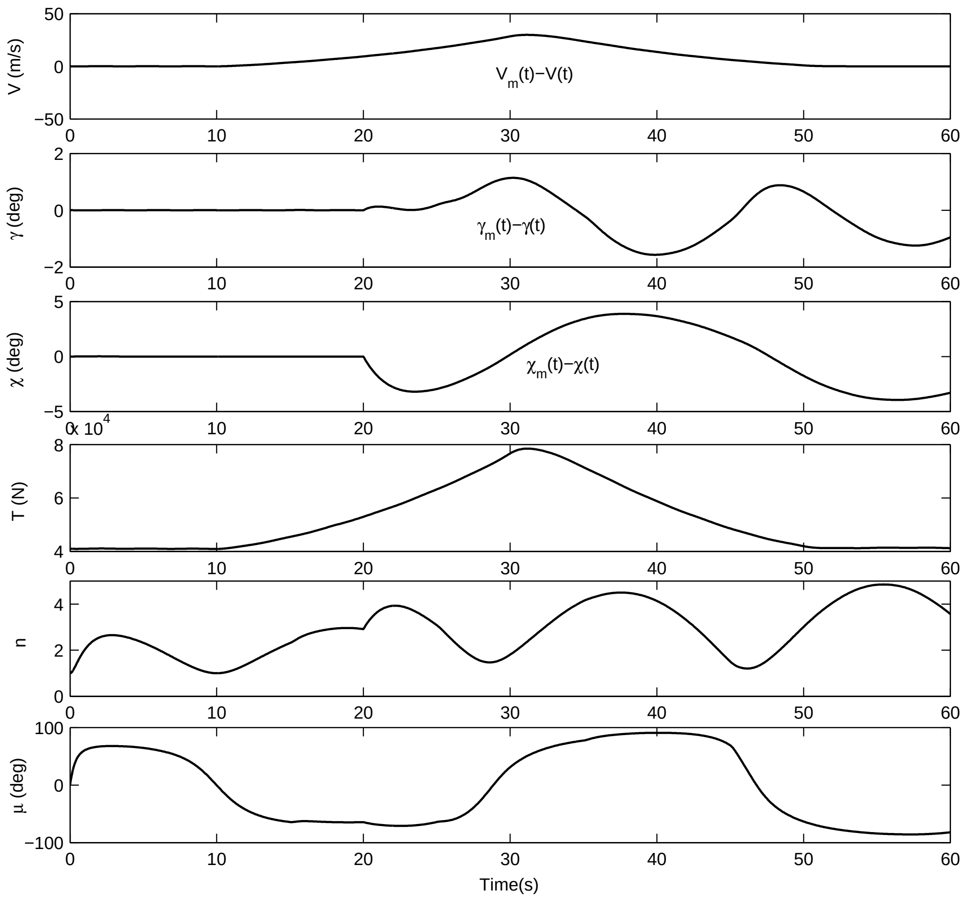

Figure 2: System response without FTC.

Choose the following Lyapunov function:

$$V_2 = \hat{e}^T P_1 \hat{e}$$

The time derivative of V_2 is given by

$$\dot{V}_2 = -\hat{e}^T Q_1 \hat{e} + 2\hat{e}^T P_1 O(\cdot)$$

Under Young inequality $2a^T b \leqslant \varepsilon a^T a + \varepsilon^{-1} b^T b$, we have

$$\dot{V}_2 \leqslant -\left[\frac{\lambda_{\min}(Q_1)}{\lambda_{\max}(P_1)} - \varepsilon_1\right] V + \varepsilon_1^{-1}\lambda_{\max}(P_1)\|O(\cdot)\|^2$$
$$\leqslant -\lambda_1 V_1 + \varphi(t)$$

where

$$\lambda_1 = \frac{\lambda_{\min}(Q_1)}{\lambda_{\max}(P_1)} - \varepsilon_1$$

and

$$\varphi(t) = \sup_{t\to\infty} \left\{\varepsilon_1^{-1}\lambda_{\max}(P_1)\|O(\cdot)\|^2\right\}$$

$\lambda_{\max}(\cdot)$, and $\lambda_{\min}(\cdot)$ are largest and smallest eigenvalues of a matrix. Hence, using global uniform ultimate boundedness (GUUB) stability [25], V_2 is exponential convergence, and the estimated tracking error $\hat{e}(t)$ can converge to a closed ball domain

$$\Omega_s = \left\{\hat{e}(t)\,|\,\|\hat{e}(t)\|^2 \leqslant \frac{\varphi(t)}{\lambda_1 \cdot \lambda_{\min}(P_1)}\right\} \tag{24}$$

Using the results $\lim_{\zeta\to\infty}\|O(\cdot)\| = 0$ and (24), we can obtained that $\lim_{\zeta\to\infty}\hat{e}(t) = 0$ when $t\to\infty$. so from Theorem 1, the result $\lim_{\zeta\to\infty} e(t) = 0$ can be obtained at $t\to\infty$.

Remak 5: We know, in traditional active FTC framework for uncertain systems, robust fault estimation and robust controller must be considered and designed. However, in the paper, the fault information and resulting disturbances are hidden in the auxiliary system, without the need for accurate fault information. And the reconfigurable controller is designed using dynamic model of the auxiliary system. So, the robustness problems of the reconfigurable controller and fault estimation are effectively addressed through the introduction of the auxiliary systems.

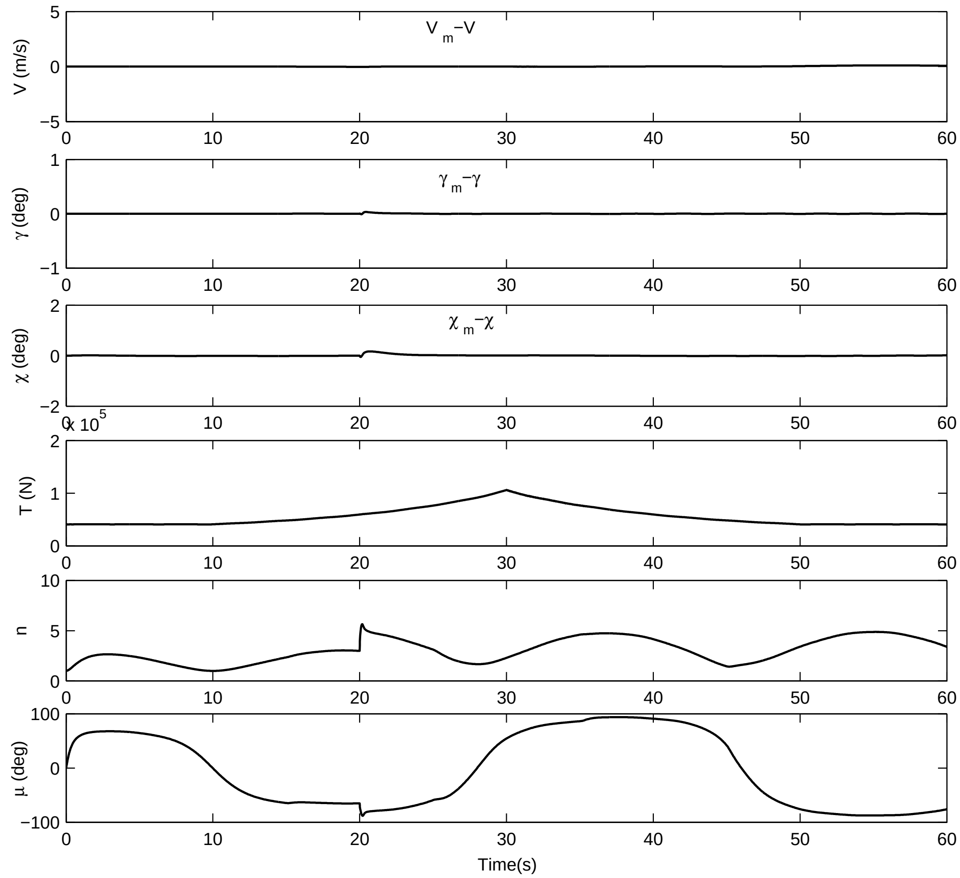

Figure 3: System response with FTC.

4 Simulation Results

In this section the intention is to evaluate the performance of the novel robust adaptive fault-tolerant tracking control. The evaluation is carried out on the 3-DOF model of UAV dynamics that can be found in [19,27]. The differential equations governing the point-mass UAV dynamics are given by

$$\begin{aligned} \dot{V} &= g\left(\frac{T-D}{W} - \sin\gamma\right) \\ \dot{\gamma} &= \frac{g}{V}\left(n\cos\mu - \cos\gamma\right) \\ \dot{\chi} &= \frac{gn\sin\mu}{V\cos\gamma} \end{aligned} \tag{25}$$

The state variables are airspeed V, flight path angle γ, flight path heading angle χ, and the control variables are thrust T, load factor n, and bank angle μ. The drag force D is represented by a simple drag polar model as

$$D = 0.5\rho V^2 S C_{D_0} + \frac{2kn^2W^2}{\rho V^2 S}$$

Detailed UAV model parameters are summarized in Table 1.

Table 1: UAV model parameters

Description	Value
density, ρ	$1.2251\ \mathrm{kg/m^3}$
weight, W	$14,515\ \mathrm{kg}$
reference area, S	$37.16\ \mathrm{m^2}$
Maximum thrust, $T_{\max}$	$113,868\ \mathrm{N}$
Maximum lift coefficient, $C_{L_{\max}}$	2.0
Maximum load factor, $n_{\max}$	7
induced drag coefficient, k	0.1
parasite drag coefficient, C_{D_0}	0.02

Let $x = [V, \gamma, \chi]^T$, $u = [T, n, \mu]^T$, and $\sigma = [\sigma_1, \sigma_2, \sigma_3]^T$. Assume external disturbance vector $\omega = [\omega_1, \omega_2, \omega_3]^T = [0.2\cos(2t), 0.0002\sin(t), 0.0002\cos(t)]^T$. Then the 3-DOF dynamics model (25) with actuator LOE

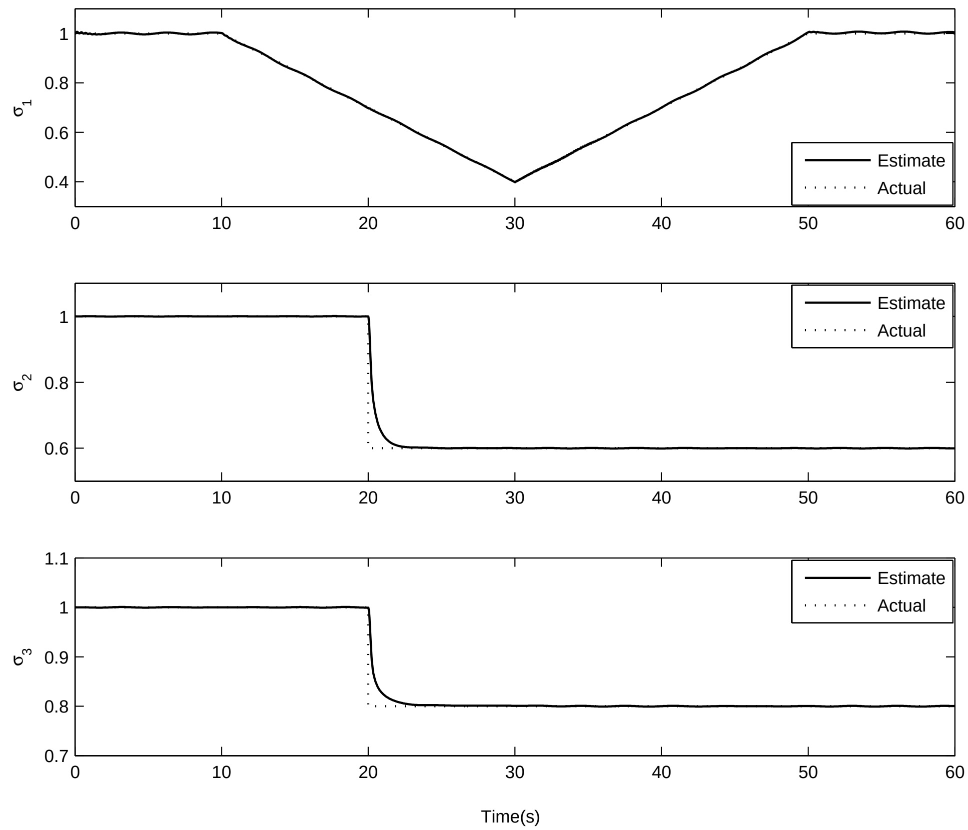

Figure 4: Response of the actuator LOE factor estimates.

faults becomes

$$\dot{x}_1 = c_{11}x_1^2 + \frac{c_{12}\sigma_2^2 u_2^2}{x_1^2} + c_{13}\sin(x_2) + c_{14}\sigma_1 u_1 + \omega_1$$
$$\dot{x}_2 = \frac{1}{x_1}(c_{21}\cos(x_2) + c_{22}\sigma_2 u_2 \cos(\sigma_3 u_3)) + \omega_2$$
$$\dot{x}_3 = \frac{c_{31}\sigma_2 u_2 \sin(\sigma_3 u_3)}{x_1\cos(x_2)} + \omega_3 \tag{26}$$

where $c_{11} = -0.5\rho g S C_{D_0}/W$, $c_{12} = -2kgW/(\rho S)$, $c_{13} = -g$, $c_{14} = g/W$, $c_{21} = -g$, $c_{22} = g$ and $c_{31} = g$.

In all simulations, it is assumed that the objective is to assure that the forward velocity $V(t) = x_1(t)$ is regulated around their desired values 300 m/s. The flight-path angle $\gamma(t) = x_2(t)$ and heading angle $\chi(t) = x_3(t)$ respectively follow outputs of two reference models

$$\dot{x}_{m1} = x_{m2}$$
$$\dot{x}_{m2} = -9x_{m1} - 6x_{m2} + 9r_\gamma(t)$$

and $\dot{x}_m = -x_m + r_\chi(t)$. Where $r_\gamma(t)$ is a $5-\text{deg}$ heading angle doublet

$$r_\gamma(t) = \begin{cases} 0 & t \leqslant 15 \\ 0.5(t-15) & 15 < t \leqslant 25 \\ 5 & 25 < t \leqslant 35 \\ 0.5(45-t) & 35 < t \leqslant 45 \\ 0 & t > 45 \end{cases}$$

and $r_\chi(t) = 30\sin(\pi t/18)$.

In the simulations, both constant and time-varying actuator faults σ are created as follows:

$$\sigma_1 = \begin{cases} 1.5(2.6/3 - 0.02t) & 10 < t \leqslant 30 \\ 1.5(0.02t - 1/3) & 30 < t \leqslant 50 \\ 1 & \text{others} \end{cases}$$
$$\sigma_2 = \begin{cases} 1 & t \leqslant 20 \\ 0.6 & 20 < t \end{cases}$$
$$\sigma_3 = \begin{cases} 1 & t \leqslant 20 \\ 0.8 & 20 < t \end{cases}$$

The initial flight condition are chosen as $V(0) = 300$ m/s, $\gamma(0) = 0$ deg, and $\chi(0) = 0$ deg. The initial state values of observer (8) are chosen as same as initial flight condition. Other initial values of observer are $m(0) = 0.15$, and $\sigma(0) = [1, 1, 1]^T$. The observer and controller parameters are selected as $\gamma_1 = 2$, $G = 1000$, $\delta_0 = 5$, $\zeta = 50$, $A = \text{diag}(-2, -2, -2)$, $K = \text{diag}(1, 1, 1)$, and

$$P = \begin{bmatrix} 0.3 & 0 & 0 \\ 0 & 1800 & 0 \\ 0 & 0 & 2000 \end{bmatrix}$$

4.1 Simulation 1: Without FTC

The simulation is focused on the case when the system is controlled by non-affine nonlinear controller, but the parameters of actuator LOE factor are unknown. The control law is now chosen as

$$\dot{u}_n = -\zeta u_n + \zeta u$$
$$u = -F_d^{-1}[F_n|_{\hat{\sigma}=1} + Ke - A_m x_m - B_m r]$$

The response of the flight control system in this case is shown in Figure 1. The superscript m denotes the outputs of the reference model. It is seen that the parameter variations lead to large errors.

4.2 Simulation 2: With FTC

The system responses of the UAV with the adaptive FTC is shown in Figure 2. It is seen that the state response is substantially improved compared to the case of without FTC. Figure 3 shows estimates of actuator LOE factor σ. From Figure 3, it is seen that fault estimates $\hat{\sigma}$ converge close to true values σ.

5 Conclusion

To rely on adaptive and sliding mode techniques, we propose fault tolerant tracking control scheme for the plants that are non-affine in the control input vector against actuator faults. In the developed FTC, two main problems are solved, there are 1): Robust actuator LOE factor estimates that are non-affine, and the knowledge of upper bound of uncertainty or external disturbance is not needed 2): RFTC for nonlinear models which are non-affine in the control input. The RFTC is different from other existing methods. Stability analysis is given for the closed-loop fault tolerant tracking system. The proposed FTC approach is tested using a 3-DOF UAV point mass model and is shown to result in substantially improved system response.

The proposed method can also been applied in the area of flight control where the corresponding nonlinear models are characterized by the control input variables appearing in a non-affine fashion and where model parameters are generally unknown, time-varying, and non-affine.

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