Pole-placement and LQ Hybrid Adaptive Control Applied Isothermal Continuous Stirred Tank Reactor

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Abstract: - The paper deals with simulation experiments on the nonlinear system represented by the isothermal continuous stirred tank reactor. At first, the mathematical model derived from the material balances inside the reactor will be introduced and then the steady-state and the dynamic analyses were performed on this model. As a result of these studies, the optimal working point and the choice of the external linear model for the identification will be obtained. The spectral factorization with pole-placement method and linear-quadratic approach were employed in the controller design and computation. Both types of adaptive controllers have parameters for tuning of the output response. Moreover, controllers have satisfied basic control requirements such as the stability, the reference signal tracking and the disturbance attenuation.

Key-Words: - steady-state, dynamic analysis, polynomial approach, Pole-placement method, LQ, recursive identification, adaptive control

1 Introduction

It is known that the great majority of system has nonlinear behavior. The control of these processes with the conventional controllers with fixed parameters could lead to the unstable, inaccurate or unwanted output response when the state of the system change or the disturbance occurs. The adaptive control [1] is one way how we can solve these problems. This control method uses idea from the nature where plants or animals "adapt" their behavior to the actual state or environmental conditions. The adaptive controller adapts parameters or the structure to parameters of the controlled plant according to he selected criterion [2].

Other very problematic feature from the control point of view is time delay. Although this paper does not deal with it, the problem of time delay is described nicely in [3], [4], [5], [6] and [7]. Other solving methods and examples could be found in [8] and [9]. Agent based learning is described in [10].

The adaptive approach here is based on the choice of the External Linear Model (ELM) as a linear approximation of the originally nonlinear system, parameters of which are identified recursively and parameters of the controller are recomputed according to identified ones. The choice and the order of the ELM come from the dynamic analysis. The δ -models [11] used here are special type of discrete-time (DT) models parameters of which are related to the sampling period. It was proofed, that parameters of the δ-model approach to parameters of the continuous-time (CT) model for the small sampling period [12].

The polynomial synthesis [13] together with the spectral factorization, the Pole-placement method and the Linear-Quadratic (LQ) approach [14] were used for designing of the controller. The product of this synthesis is the continuous-time controller which satisfies basic control requirements such as the stability, the reference signal tracking and the disturbance attenuation. The resulted controller is called "hybrid" because it works in continuous-time but its parameters are recomputed in discrete time intervals together with the δ -ELM identification.

There are several types of chemical reactors. The main groups are tank reactors and tubular reactors. The continuous stirred tank reactor (CSTR) is ideal from the control point of view – it could have variety of quantities which can affect the production.

There are two ways how we can observe the behavior of the system – by experiments on the real system or its smaller real model [14]. This method produce more realistic results but it could be dangerous or time and money demanding. The other approach uses modeling techniques for creating of a mathematical model as an abstract representation of the system. The mathematical model in the form of the set of Ordinary Differential Equations (ODE) is then subjected to simulations which show the static and the dynamic behavior of the system. The role of the simulation grows nowadays with the increasing speed and the decreasing price of computers.

The contribution is divided into six main parts. The second part after this introduction will describe simulated nonlinear system which is the isothermal Continuous Stirred-Tank Reactor (CSTR) the mathematical model of which is described by the set of five ordinary differential equations [16]. The third part will present the results of the steady-state and dynamic analyses, the fourth part gives an overview and the theoretical background to used hybrid adaptive control while the fifth part presents results of the control. The last, the sixth, part is conclusion.

All results shown in this contribution come from the simulation on the mathematical model and they were done on the mathematical simulation software Matlab. Detailed simulation of similar chemical reactor can be found for example in [17].

2 Nonlinear System

The nonlinear system here is represented by the Isothermal Continuous Stirred Tank Reactor with complex reaction inside [18]. This reaction could be described by the scheme:

$$\begin{aligned} A + B &\xrightarrow{k_1} X \\ B + X &\xrightarrow{k_2} Y \\ B + Y &\xrightarrow{k_3} Z \end{aligned} \tag{1}$$

and the schematic representation of this reactor is in Fig. 1.

The full mathematical description of the system is of course very complex. Introduction of the assumptions usually reduce this complexity. We assume that the reactant inside is perfectly mixed and the volume is constant during experiments. As the reactor is isothermal, the temperature of the reactant is not taken into the account.

The mathematical model comes from material balances inside the reactor. In this case, as we have five state variables (concentrations of the compounds A, B, X, Y and Z), the mathematical model is represented by the set of five ordinary differential equations (ODE):

$$\begin{aligned} \frac{dc_A}{dt} &= \frac{q}{V}\left(c_{A0} - c_A\right) - k_1 \cdot c_A \cdot c_B \\ \frac{dc_B}{dt} &= \frac{q}{V}\left(c_{B0} - c_B\right) - k_1 \cdot c_A \cdot c_B - \\ &\quad - k_2 \cdot c_B \cdot c_X - k_3 \cdot c_B \cdot c_Y \\ \frac{dc_X}{dt} &= \frac{q}{V}\left(c_{X0} - c_X\right) + k_1 \cdot c_A \cdot c_B - k_2 \cdot c_B \cdot c_X \\ \frac{dc_Y}{dt} &= \frac{q}{V}\left(c_{Y0} - c_Y\right) + k_2 \cdot c_B \cdot c_X - k_3 \cdot c_B \cdot c_Y \\ \frac{dc_Z}{dt} &= \frac{q}{V}\left(c_{Z0} - c_Z\right) + k_3 \cdot c_B \cdot c_Y \end{aligned} \tag{2}$$

where q denotes volumetric flow rate, V is used for volume of the reactant, c_A, c_B, c_X, c_Y and c_Z are concentrations, $k_{1\text{-}3}$ are rate constants and t is time.

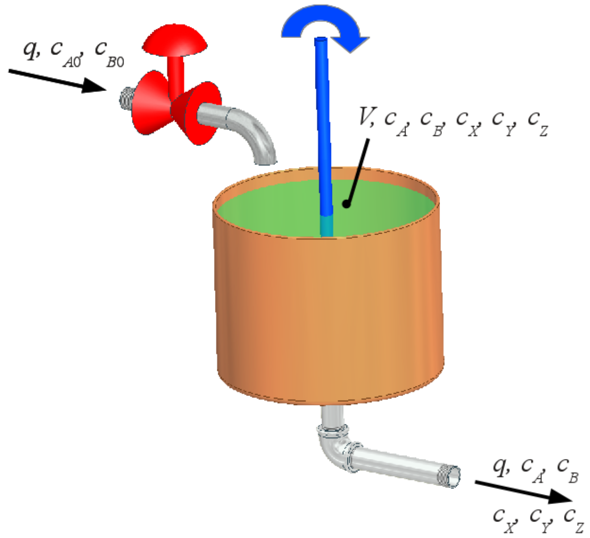

Fig. 1 Scheme of the isothermic Continuous Stirred Tank Reactor

We can say, that the system is nonlinear mainly because of the multiplication of the state variables. The fixed parameters of this reactor are shown in Table 1.

3 Simulation Analyses

Once we have the mathematical model, we can subject it to simulation analyses. The steady-state and the dynamic analyses were used in our case. Because of the length of this contribution, the concentrations of the products X, Y and Z are observed in the simulation studies

3.1 Steady-state analysis

The goal of the steady-state analysis is to observe the behavior of the system in the steady-state, e.g. for time t → ∞.

The mathematical solution lays is relatively simple – this condition means that all derivatives with respect to time in equations (2) are equal to zero, i.e.

$$\frac{d(\cdot)}{dt} = 0 \quad (3)$$

The set of ODE (2) is then simplified to the set of nonlinear algebraic equations which could be solved for example by the Simple iteration method.

Name of the parameter	**Symbol and value of the parameter**
Volume of the reactant	$V = 1\ m^3$
Rate constant of the reaction 1	$k_1 = 5\times10^{-4}\ m^3.kmol^{-1}.s^{-1}$
Rate constant of the reaction 2	$k_2 = 5\times10^{-2}\ m^3.kmol^{-1}.s^{-1}$
Rate constant of the reaction 3	$k_3 = 2\times10^{-2}\ m^3.kmol^{-1}.s^{-1}$
Input concentration of the concentration c_A	$c_{A0} = 0.4\ kmol.m^{-3}$
Input concentration of the concentration c_B	$c_{B0} = 0.6\ kmol.m^{-3}$
Input concentration of the concentration c_X	$c_{X0} = 0\ kmol.m^{-3}$
Input concentration of the concentration c_Y	$c_{Y0} = 0\ kmol.m^{-3}$
Input concentration of the concentration c_Z	$c_{Z0} = 0\ kmol.m^{-3}$

Table 1 Fixed parameters of the reactor

There could be theoretically six input variables to the system – volumetric flow rate of the reactant q and five initial concentrations c_{A0}, c_{B0}, c_{X0}, c_{Y0} and c_{Z0}. On the other hand, practical experiences have shown, that only volumetric flow rate of the reactant q could be used.

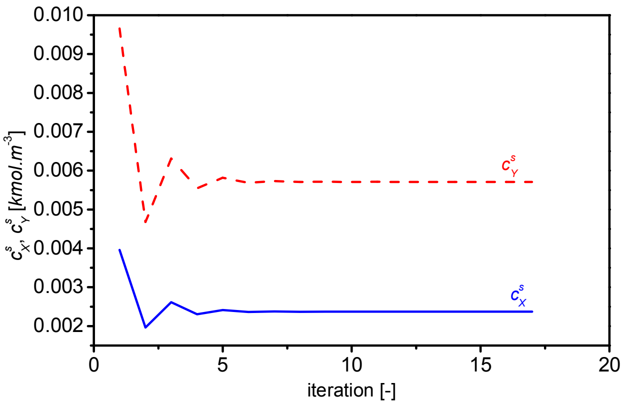

Fig. 2 Computation of the steady-state values of concentrations c_X and c_Y through the iterations

The results of the first steady-state analysis in Fig. 2 and Fig. 3 show the iterations in computations. It can be clearly seen, that the set of nonlinear algebraic equations converges to the accurate results relatively quickly, after 15-17 iterations.

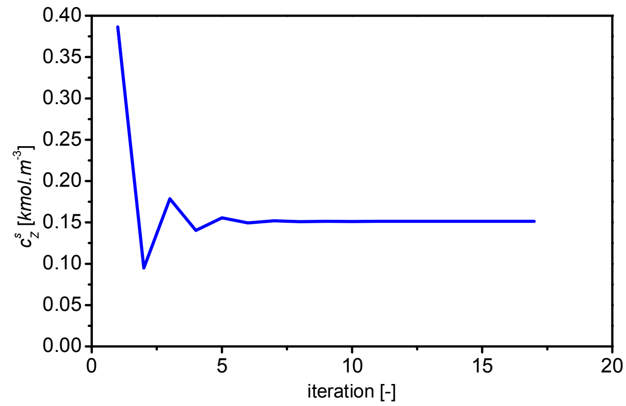

Fig. 3 Computation of the steady-state values of the concentration c_Z through the iterations

The second steady-state analysis computes steady-state values of the state variable for different values of the volumetric flow rate $q = <0; 0.01>\ m^3.s^{-1}$ as an input variable.

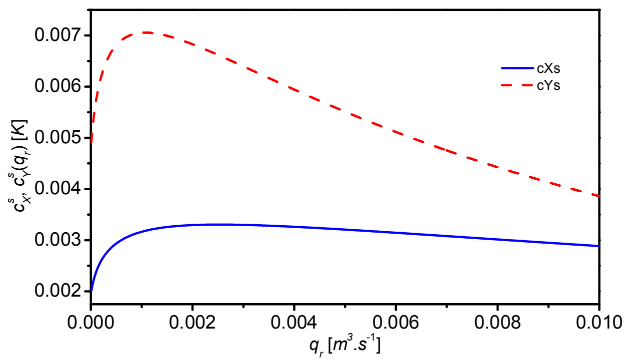

Fig. 4 The steady-state characteristics of concentrations c_X and c_Y for various volumetric flow rate q

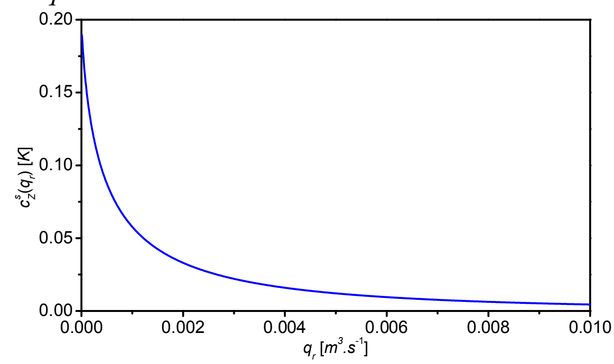

Fig. 5 The steady-state characteristics of concentrations c_Z for various volumetric flow rate q

Fig. 4 and Fig. 5 have confirmed what we have expected – the system has nonlinear behavior. One output from the steady-state analysis is also an optimal working point of the system. In our case, the volumetric flow rate of the reactant $q = 1\times10^{-4}$ was used as a working point. The steady-state values of all state variables in this point are then

$$c_A^s = 0.2407; \quad c_B^s = 0.1324; \\ c_X^s = 0.0024; c_Y^s = 0.0057; c_Z^s = 0.1513; \tag{4}$$

3.2 Dynamic Analysis

The dynamic analysis observes behavior of the system after the step change of the input variable, in our case again volumetric flow rate of the reactant. The mathematical meaning of this analysis is numerical solution of the set of ODE (2).

There are several numerical methods which can be used. In our case, the Runge-Kutta's standard method was used for several reasons. At first, it is old method with big theoretical background, accurate enough and at last but not the least it is easily programmable or even more it is build-in function in various mathematical software such as Matlab, Mathematica etc.

The six step chages of the input $q = \pm 30$, ± 60 and ± 100 % were performed and the results are shown in Fig. 6 - Fig. 7.

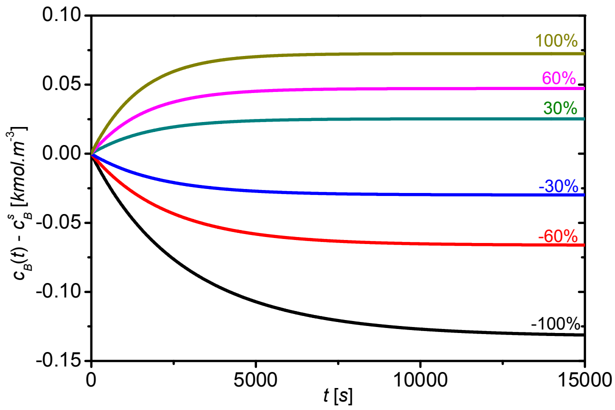

Fig. 6 The dynamic characteristics of the concentration c_B after step changes of input volumetric flow rate q

Note that all outputs in Fig. 6 – Fig. 9 represents the difference from its actual value and steady-state value of which is also an input condition to the dynamic study. As a result, all outputs starts from zero. This was done for better understanding of the system's gain in the controller design.

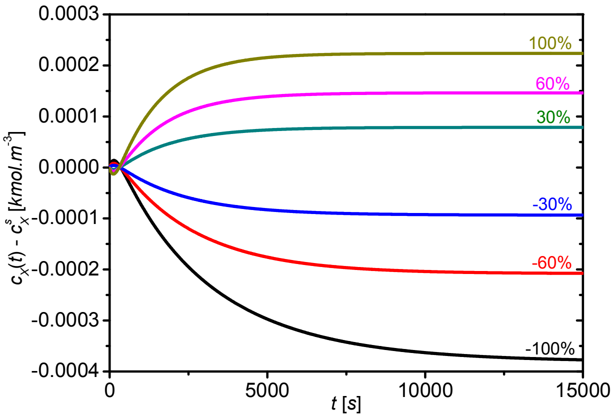

Fig. 7 The dynamic characteristics of the concentration c_X after step changes of input volumetric flow rate q

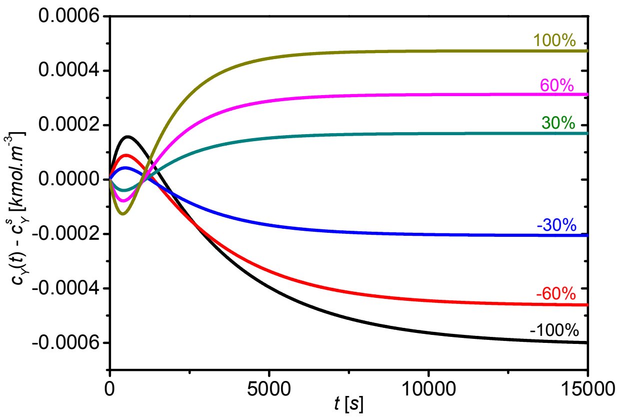

Fig. 8 The dynamic characteristics of the concentration c_Y after step changes of input volumetric flow rate q

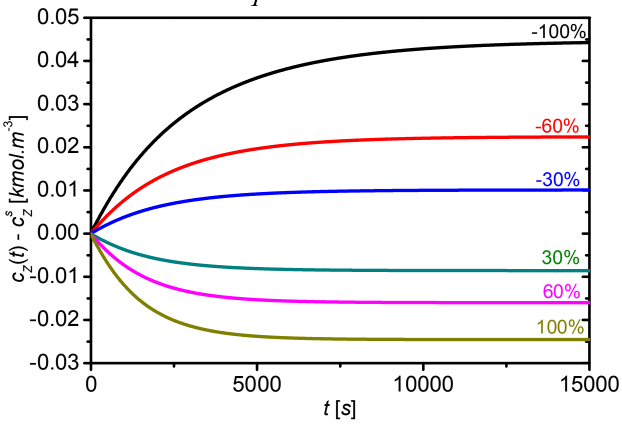

Fig. 9 The dynamic characteristics of the concentration c_Z after step changes of input volumetric flow rate q

Some of the outputs have shown negative properties from the control point of view – see Fig. 7 and Fig. 8 such as non-minimum phase behavior, nonlinearity etc. On the other hand, outputs c_B and c_Z in Fig. 6 and Fig. 9 could be described by first or second order transfer functions:

$$G(s)=\frac{b(s)}{a(s)}=\frac{b_0}{a_1 s+a_0} \qquad (5)$$

$$G(s)=\frac{b(s)}{a(s)}=\frac{b_1 s+b_0}{a_2 s^2+a_1 s+a_0} \qquad (6)$$

Interesting thing can be also find in the last graph, where positive change of the input produces negative course of the output concentration c_Z and reverse.

4 Control of the Plant

The control strategy here is based on the Adaptive control. As there are a lot of adaptive approaches, the Adaptive control with External Linear Model (ELM) of the originally nonlinear system parameters of which are identified recursively. This approach satisfies that the controller could react immediately to the changes inside the system caused by the changes of the system's state, disturbance influence etc.

4.1 External Linear Model

The choice of the ELM is tightly connected with the dynamic analysis. Let us suppose, that the ELM of the controlled output obtained from the dynamic analysis could be described by the second order transfer function with relative order one in the *s*-plain, equation (6)

Parameters of polynomials $a(s)$ and $b(s)$ are commensurable polynomials and the feasibility condition is fulfilled for $\deg a(s) \geq \deg b(s)$.

The transfer function is relation of the output from the system to the input which mathematically means that this continuous-time (CT) model (6) could be rewritten to:

$$a(\sigma)y(t)=b(\sigma)u(t) \qquad (7)$$

where $a(\sigma)$ and $b(\sigma)$ are polynomials from (6) and σ is the differentiation operator. The identification of the CT model is not very simple. On the other hand, discrete-time (DT) identification could be inaccurate. Compromise between these two methods can be found in the use of so called Delta (δ-) models. This model uses a new complex variable γ defined generally as [19]:

$$\gamma=\frac{z-1}{\beta \cdot T_v \cdot z+(1-\beta)\cdot T_v} \qquad (8)$$

The optional parameter β from the interval $0 \leq \beta \leq 1$ then produces infinite number of the models. Parameter T_v denotes the sampling period. A forward δ-model was used in this work. The γ operator is then

$$\beta=0 \Rightarrow \gamma=\frac{z-1}{T_v} \qquad (9)$$

and the continuous model (7) could be then rewritten to

$$a^{\delta}(\delta)y(t')=b^{\delta}(\delta)u(t') \qquad (10)$$

where polynomials $a^{\delta}(\delta)$ and $b^{\delta}(\delta)$ are discrete polynomials and their coefficients are different from those of the CT model $a(\sigma)$ and $b(\sigma)$. Time t' is the discrete time and with the new substitution $t' = k - n$ for $k \geq n$ the δ-model for this concrete transfer function would be:

$$\begin{aligned}\delta^2 y(k-n) &= b_1^{\delta}\delta u(k-n)+b_0^{\delta}u(k-n)- \\ &-a_1^{\delta}\delta y(k-n)-a_0^{\delta}y(k-n)\end{aligned} \qquad (11)$$

The equation (11) produces both the regression vector $\boldsymbol{\varphi}_{\delta}$ and the vector of parameters

$$\begin{aligned}\boldsymbol{\varphi}_{\delta}(k-1) &= \left[\left[-y_{\delta}(k-1),-y_{\delta}(k-2),\ldots\right.\right. \\ &\left.\ldots,u_{\delta}(k-1),u_{\delta}(k-2)\right]^T \\ \boldsymbol{\theta}_{\delta}(k) &= \left[a_1^{\delta},a_0^{\delta},b_1^{\delta},b_0^{\delta}\right]^T\end{aligned} \qquad (12)$$

where y_{δ} and u_{δ} denotes the recomputed output and input variables to the δ-model and

$$\begin{aligned}y_{\delta}(k) &= \frac{y(k)-2y(k-1)+y(k-2)}{T_v^2} \\ y_{\delta}(k-1) &= \frac{y(k-1)-y(k-2)}{T_v} \\ y_{\delta}(k-2) &= y(k-2) \\ u_{\delta}(k-1) &= \frac{u(k-1)-u(k-2)}{T_v} \\ u_{\delta}(k-2) &= u(k-2)\end{aligned} \qquad (13)$$

The differential equation (11) has then the vector form:

$$y_{\delta}(k)=\boldsymbol{\theta}_{\delta}^T(k)\cdot\boldsymbol{\varphi}_{\delta}(k-1)+e(k) \qquad (14)$$

where $e(k)$ is a general random immeasurable error.

4.2 On-line Identification

The unknown parameter from the differential equation (14) is the vector of parameters $\boldsymbol{\theta}_{\delta}$. The regression vector $\boldsymbol{\varphi}_{\delta}$ is constructed from the previous values of the measured inputs u and outputs y. One of the controller's tasks is compute this vector on-line during the control.

The Recursive Least-Squares (RLS) method was used for this challenge. This method is widely used for this on-line identification because it has big theoretical background and it is easily programmable. It might be modified with exponential or directional forgetting [20] because parameters of the identified system can vary during the control which is typical for nonlinear systems. The use of some forgetting factor could result in better output response.

The RLS method with the changing exponential forgetting used here is described by the set of equations:

$$\begin{aligned}
&\varepsilon(k) = y(k) - \boldsymbol{\varphi}_\delta^T(k)\cdot\hat{\boldsymbol{\theta}}_\delta(k-1)\\
&\xi(k) = \left[1+\boldsymbol{\varphi}_\delta^T(k)\cdot\mathbf{P}(k-1)\cdot\boldsymbol{\varphi}_\delta(k)\right]^{-1}\\
&\boldsymbol{L}(k) = \xi(k)\cdot\mathbf{P}(k-1)\cdot\boldsymbol{\varphi}_\delta^T(k)\\
&\mathbf{P}(k) = \frac{1}{\lambda_1(k-1)}\Bigg[\mathbf{P}(k-1) - \ldots\\
&\ldots - \frac{\mathbf{P}(k-1)\cdot\boldsymbol{\varphi}_\delta(k)\cdot\boldsymbol{\varphi}_\delta^T(k)\cdot\mathbf{P}(k-1)}{\lambda_1(k-1)+\boldsymbol{\varphi}_\delta^T(k)\cdot\mathbf{P}(k-1)\cdot\boldsymbol{\varphi}_\delta(k)}\Bigg]\\
&\hat{\boldsymbol{\theta}}_\delta(k) = \hat{\boldsymbol{\theta}}_\delta(k-1) + \boldsymbol{L}(k)\varepsilon(k)
\end{aligned} \tag{15}$$

where optional the changing forgetting factor λ_1 is computed from the equation

$$\lambda_1(k) = 1 - K\cdot\xi(k)\cdot\varepsilon^2(k) \tag{16}$$

and K is small number, in our case $K = 0.001$.

4.3 Design of the Controller

The control configuration is displayed in Fig. 10.

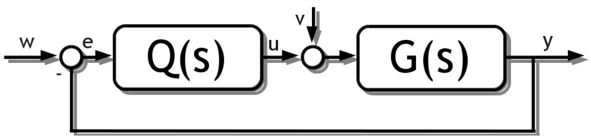

Fig. 10 Control system configuration

where $G(s)$ represents the transfer function (5) of the controlled output and $Q(s)$ denotes the transfer function of the controller in the continuous-time, generally:

$$Q(s) = \frac{q(s)}{p(s)} \tag{17}$$

Polynomials $q(s)$ and $p(s)$ are similarly to system's polynomials $a(s)$ and $b(s)$ commensurable polynomials with the properness condition $\deg p(s) \geq \deg q(s)$.

The Laplace transform of the transfer function $G(s)$ in (6) is generally:

$$G(s) = \frac{Y(s)}{U(s)} \Rightarrow Y(s) = G(s)\cdot U(s) \tag{18}$$

where Laplace transform of the input signal u is from Fig. 10

$$\begin{aligned}
U(s) &= Q(s)\cdot E(s) + V(s) = \ldots\\
\ldots &= Q(s)\cdot\left[W(s) - Y(s)\right] + V(s)
\end{aligned} \tag{19}$$

If we put polynomials $a(s)$, $b(s)$, $p(s)$ and $q(s)$ into (19) instead of Laplace transforms $G(s)$ and $Q(s)$, the equation (18) has form

$$\begin{aligned}
Y(s) &= \frac{b(s)q(s)}{a(s)p(s)+b(s)q(s)}\cdot W(s) + \ldots\\
&\ldots + \frac{a(s)p(s)}{a(s)p(s)+b(s)q(s)}\cdot V(s)
\end{aligned} \tag{20}$$

Both fractions has the same denominators which is called a characteristic polynomial of the closed loop and this polynomial, in this case

$$a(s)\cdot p(s) + b(s)\cdot q(s) = d(s) \tag{21}$$

where $d(s)$ is a stable optional polynomial and the whole equation (21) is called Diophantine equation [13]. The stability of the control system is fulfilled if the stable polynomial $d(s)$ on the left side of the Diophantine equation (21) is also stable polynomial.

The basic control requirements such as an asymptotic tracking of the reference signal and the disturbance attenuation is attained if the polynomial $p(s)$ includes the least common divisor of denominators of transfer functions of the reference w and disturbance v:

$$p(s) = f(s)\cdot\tilde{p}(s) \tag{22}$$

If we expect both these signals from the range of the step functions, the polynomial $f(s) = s$. The Diophantine equation (21) is then

$$a(s)\cdot s\cdot\tilde{p}(s) + b(s)\cdot q(s) = d(s) \tag{23}$$

and the transfer function of the feedback controller is

$$\tilde{Q}(s) = \frac{q(s)}{s\cdot\tilde{p}(s)} \tag{24}$$

As it is written above, the polynomial $d(s)$ on the right side of the Diophantine equation (23) is the stable optional polynomial.

There are several ways how we can construct this polynomial. The simples one is the based on pole-placement method where $d(s)$ is divided into one or more parts with double, triple, etc. roots, e.g.

$$\begin{aligned} d(s) &= (s+\alpha)^m \\ d(s) &= (s+\alpha_1)^{m/2}\cdot(s+\alpha_2)^{m/2} \\ &\dots \end{aligned} \tag{25}$$

where $\alpha_i > 0$ is optional position of the root. This variable affects the results of the control as you will see in the results.

The disadvantage of the Pole-placement method can be found in the uncertainty. There is no general rule which can help us with the choice of roots which is, of course, different for different controlled processes. One way how we can overcome this unpleasant feature is to use spectral factorization. Big advantage of this method is that it can make stable roots from every polynomial, even if it is unstable. The polynomial $d(s)$ is in this case

$$d(s)=n(s)\cdot m(s)\ d(s)=n(s)\cdot g(s) \tag{26}$$

where parameters of the polynomial $n(s)$ are computed from the spectral factorization of the polynomial $a(s)$ in the denominator of (6), i.e.

$$n^*(s)\cdot n(s)=a^*(s)\cdot a(s) \tag{27}$$

and the second polynomial $m(s)$ is constructed with the use of Pole-placement method. The degree of polynomials $\tilde{p}(s)$, $q(s)$ and $d(s)$ are in this case:

$$\begin{aligned} &\deg\tilde{p}(s)\ge\deg a(s)-1\Rightarrow\deg\tilde{p}(s)=1 \\ &\deg q(s)=\deg a(s)=2 \\ &\deg d(s)=\deg a(s)+\deg\tilde{p}(s)+1=4 \end{aligned} \tag{28}$$

The polynomial $m(s)$ is then of the second degree because

$$\begin{aligned} &\deg m(s)=\deg d(s)-\deg a(s)=2 \\ &m(s)=(s+\alpha_i)^2 \end{aligned} \tag{29}$$

and we obtain one double root a_i.

The second approach which can be used here employ spectral factorization and LQ approach. The polynomial d(s) is divided again into two parts:

$$d(s)=n(s)\cdot g(s) \tag{30}$$

Where $n(s)$ comes from the spectral factorization explained above and the polynomial $g(s)$, is computed with the use of the Linear Quadratic (LQ) tracking [14] which is based on the minimizing of the cost function in the complex domain

$$J_{LQ}=\int_0^\infty\left\{\mu_{LQ}\cdot e^2(t)+\varphi_{LQ}\cdot\dot{u}^2(t)\right\}dt \tag{31}$$

where $\varphi_{LQ} > 0$ and $\mu_{LQ} \geq 0$ are weighting coefficients, $e(t)$ is the control error and $\dot{u}(t)$ denotes the difference of the input variable. It practically means, that parameters of the polynomial $g(s)$ are computed from the spectral factorization

$$\begin{aligned} &(a(s)\cdot f(s))^*\cdot\varphi_{LQ}\cdot a(s)\cdot f(s)+\dots \\ &\dots b^*(s)\cdot\mu_{LQ}\cdot b(s)=g^*(s)\cdot g(s) \end{aligned} \tag{32}$$

Degrees of unknown polynomials $\tilde{p}(s)$, $q(s)$ and $d(s)$ are for the fulfilled properness condition generally in this case:

$$\begin{aligned} &\deg\tilde{p}(s)\ge\deg a(s)-1\Rightarrow\deg\tilde{p}(s)=1 \\ &\deg q(s)=\deg a(s)+\deg f(s)-1=2 \\ &\deg d(s)=2\deg a+1=5 \\ &\deg n(s)=\deg a(s)=2 \\ &\deg g(s)=\deg d(s)-\deg n(s)=3 \end{aligned} \tag{33}$$

Parameters of the unknown polynomials polynomials $\tilde{p}(s)$, $q(s)$ and $d(s)$ are computed from the Diophantine equation (23) by the method of uncertain coefficients for both strategies. Polynomials $a(s)$ and $b(s)$ of the ELM are known from the recursive identification described in part 4.2.

5 Simulation results

The control strategy described above is called "hybrid" because the computation of the control input is defined in the continuous time but the identification of the ELM runs in the discrete time with the use of δ-models.

The simulation parameters for all studies are displayed in Table 2.

6.1 Pole-placement Method

The first simulation study was done for various values of the α_i as a position of the root in the Pole-placement method.

The transfer function of the controller Q(s) is then for the degrees of the polynomials computed in (28) is then

$$\tilde{Q}(s)=\frac{q(s)}{s\cdot\tilde{p}(s)}=\frac{q_2s^2+q_1s+q_0}{s\cdot(s+p_0)} \tag{34}$$

Three values of were tested – e.g. α_i = 0.001, 0.002 and 0.005.

Name of the parameter	Symbol and value of the parameter
Control input	$u(t)=\frac{q(t)-q^s}{q^s}\cdot 100\,[\%]$
Controlled output	$y(t)=c_Z(t)-c_Z^s\,\left[kmol.m^{-3}\right]$
Limitation of the input	$u(t)$ = <-100%; 100%>
Sampling period	$T_v = 10\ s$
Simulation time	$T_f = 60\ 000\ s$
Number of step changes of $w(t)$	6
Initial vector of parameters	$\boldsymbol{\theta}_\delta(0)=\left[0.1, 0.1, 0.1, 0.1\right]^T$

Table 2 Simulation parameters

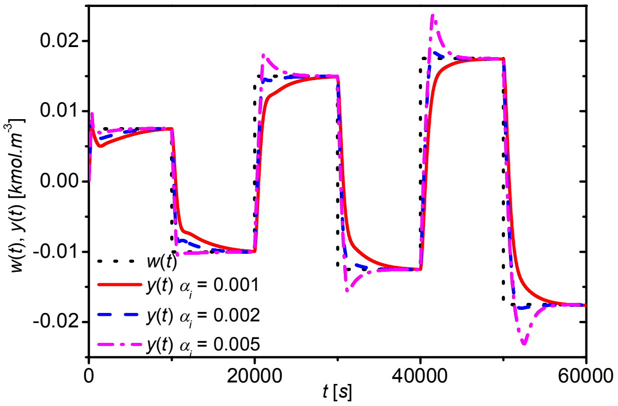

Fig. 11 The course of the output variable, $y(t)$, and the reference signal, $w(t)$, for the control with Pole-placement method

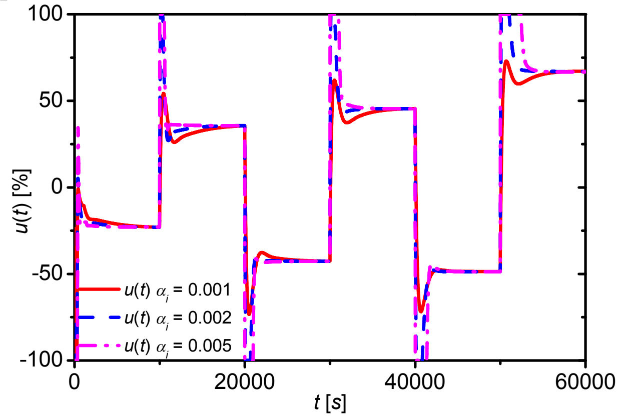

Fig. 12 The course of the input variable, $u(t)$, for the control with Pole-placement method

All courses presented in Fig. 11 and Fig. 12 has acceptable results except the very beginning of the control because the identification need some time to "adapt" as it starts from the general point $\boldsymbol{\theta}_\delta(0)$. The value of the optional parameter α_i affects mainly the speed of the control and overshoots – decreasing value of this parameter produces smoother course of both input and output variables but without the overshoot.

The controller has small problems at the very beginning of the control because of the recursive identification which starts from the general point $\boldsymbol{\theta}_\delta(0)$ in Table 2. But after some initialization time, the recursive identification runs very smoothly – see Fig. 13 and Fig. 14 which present the course of the identified parameters for the α_i =0.002.

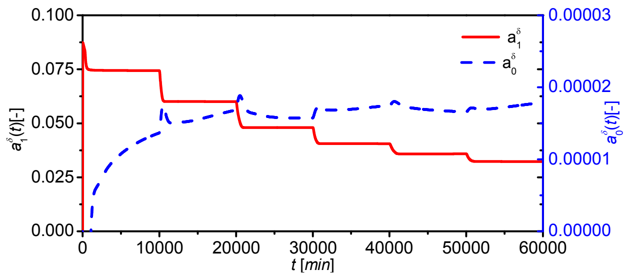

Fig. 13 The course of the identified parameters a_1^δ and a_0^δ for control with Pole-placement method

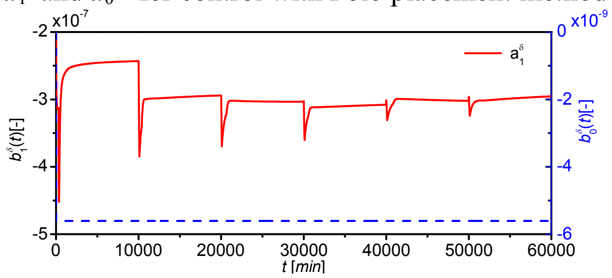

Fig. 14 The course of the identified parameters b_1^δ and b_0^δ for control with Pole-placement method

6.2 LQ Approach

The LQ strategy has two tuning parameters – weighting factors ϕ_{LQ} and μ_{LQ}.
The transfer function of the feedback controller according degrees computed in (33) is in this case:

$$\tilde{Q}(s)=\frac{q(s)}{s\cdot\tilde{p}(s)}=\frac{q_2 s^2+q_1 s+q_0}{s\cdot\left(s^2+p_1 s+p_0\right)} \qquad (35)$$

Experiments were shown, that effect of these factors is similar. As a result, we fixed factor μ_{LQ} to $\mu_{LQ} = 2$ and change only ϕ_{LQ} = 0.0025, 0.05 and 0.5. The results are shown in Fig. 15 and Fig. 16.

We have found values of ϕ_{LQ} which have similar results as in previous simulation study in purpose. In this case, decreasing value of the weighting factor ϕ_{LQ} produces quicker output response with overshoots.

Fig. 17 and Fig. 18 presents the course of the identified parameters for this simulation study. The weighting factor ϕ_{LQ} is 0.05 and μ_{LQ} is 2. The results are very similar to the previous study – the identification has problem only at the very beginning and it is relatively smooth after some initial time.

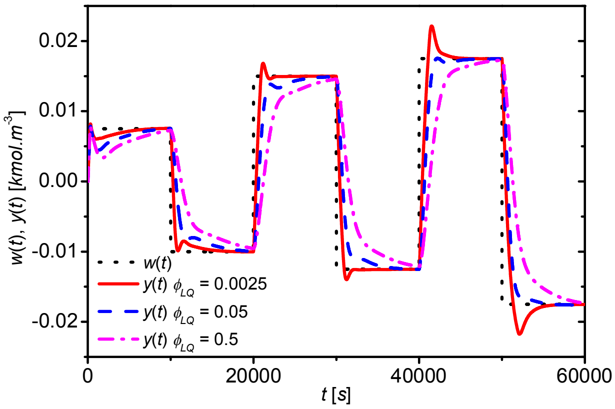

Fig. 15 The course of the output variable, $y(t)$, and the reference signal, $w(t)$, for the control with LQ method

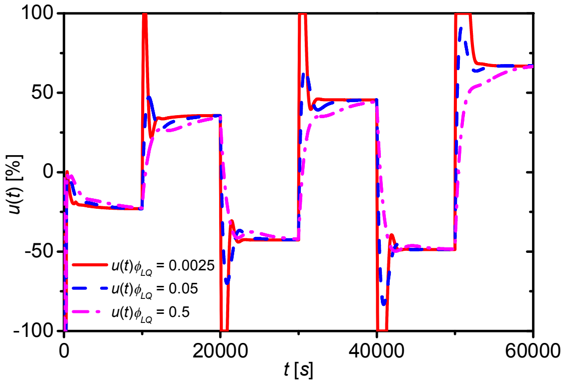

Fig. 16 The course of the input variable, $u(t)$, for the control with LQ method

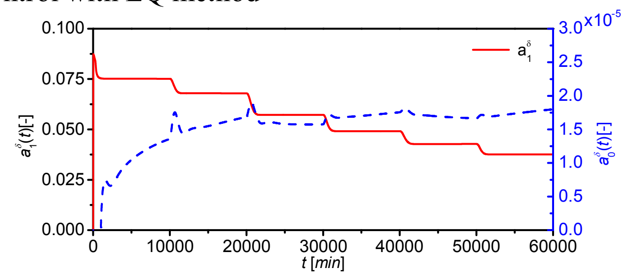

Fig. 17 The course of the identified parameters a_1^{δ} and a_0^{δ} for control with LQ approach

6.3 Disturbance Attenuation

Previous graphs have shown that first two control requirements for the stability and the reference signal tracking were accomplished. The disturbance attenuation as a last requirement was tested for two disturbances – the disturbance $v_1(t)$ on the input concentration c_{B0} and the disturbance $v_2(t)$ on the output concentration c_Z from the system.

Only one step change of the reference signal was performed during the simulation time $T_f = 30\ 000$ *s*. The first disturbance $v_1(t) = -15\%\ c_{B0}$ was injected to the system during the time $t = <10\ 000;\ 30\ 000>$ *s* and the second one $v_1(t) = 20\%\ c_Z$ through time $t = <22\ 000;\ 30\ 000>$ *s*. The factor α_i was in the Pole-placement approach set to $\alpha_i = 0.002$ and the weighting parameters in the LQ approach were $\mu_{LQ} = 2$ and $\phi_{LQ} = 0.0002$.

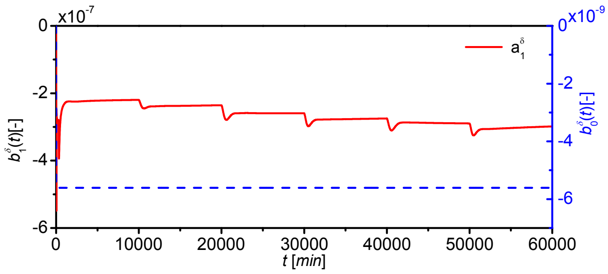

Fig. 18 The course of the identified parameters b_1^{δ} and b_0^{δ} for control with LQ approach

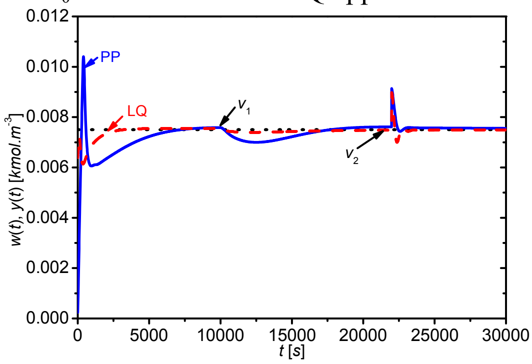

Fig. 19 The course of the output variable, $y(t)$, and the reference signal, $w(t)$, for pole-placement (PP) and LQ approaches in the disturbance attenuation

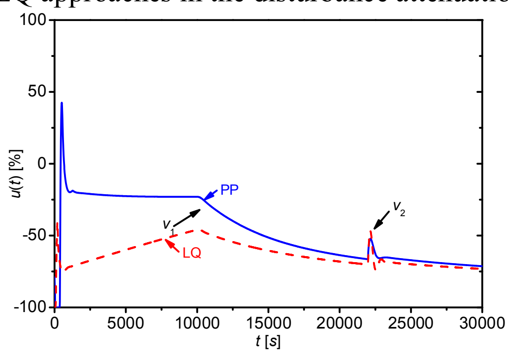

Fig. 20 The course of the input variable, $u(t)$, for pole-placement (PP) and LQ approaches in the disturbance attenuation

Results in Fig. 19 and Fig. 20 shows the usability of both methods for controlling of systems where disturbances could occur. This hybrid adaptive controller deals with the disturbances in the input and output and it is worth to note that both disturbances affects the system from time $t = <22\ 000;\ 30\ 000>$ *s* and with no big problem to the output response.

6 Conclusion

The contribution shows two control approaches to the adaptive control of a nonlinear system represented by the isothermal CSTR reactor. The steady-state and the dynamic analyses have shown that the system has nonlinear behavior the concentration of the product Z, c_Z, could be described by the second order transfer function with relative order one. This transfer function was used as an ELM of the system for the on-line identification in the control part. Both adaptive approaches with Pole-placement LQ include tuning parameters(α_i, ϕ_{LQ} and μ_{LQ}) which could affect the course of the output. Presented results have shown usability of these strategies for the nonlinear systems and they both satisfies basic control requirements including the disturbance attenuation. The future work will lead up to applicability of these methods to the real systems.

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