Estimation of optimum antennas number
of a spread spectrum MIMO system under signal fading

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Abstract: - In this presented work, the problem of selecting the number of antennas for a spread spectrum multiple-input multiple-output (MIMO) cellular system is investigated, in order to maximize the Shannon's channel capacity (estimated in average sense) assigned to each system's user when operating in a Rayleigh fading environment. Then, the spectral efficiency is estimated in terms of the achievable average channel capacity per user, during the operation over a broadcast time-varying link, and leads to a simple novel-closed form expression for the optimum number of transmitting antennas value based on the maximization of the achieved average channel capacity per system's user.

Key-words: -MIMO systems, channel capacity, cellular systems, Rayleigh fading.

1 Introduction

MIMO communication techniques have been an important area of focus for next-generation wireless systems because of their potential for high capacity, increased diversity, and interference suppression. Then, the deployment of multiple antennas at both the transmitter and the receiver increases radio capacity in a wireless communication system, [1,2]. The information-theoretic analysis of a MIMO system has been thoroughly studied, [2-6] and some results how to appropriately select the number of transmit antennas are given in [5] and [7]. In contrast to previous published works, in this letter, we propose a novel approach to estimate the optimum number of transmit/receive antennas in a spread spectrum cellular MIMO system, operating in a Rayleigh fading environment, based on the maximization of the theoretically achieved average channel capacity available to each system's user. The final closed-form expression, analytically derived, to the author's best knowledge, is the first time such expression has been exposed, avoiding the need for lengthy system's simulations. However, in a future analysis a simulation process must be described in order to compare with the theoretical results derived here.

2. System's Description

We consider a spread spectrum MIMO cellular communication system that consists of K simultaneously transmitting users per cell, each of them transmitting in parallel, via N_T different transmitting antennas the same spread spectrum signal of bandwidth W_{ss}, after spreading the same signal of bandwidth W by the system's processing gain i.e. $G_p = W_{ss}/W$. Each system's user transmit a signal of bandwidth W_{ss} with average transmitted power $P_i = P$, $1 \le i \le 12K$, without splitting transmitted power among transmit antennas as in [3]. In addition, we consider the first 12 hexagon cells of the cellular MIMO system and then, only the average co-channel interference (CCI) power received from the $11K$ co-channel interfering users of the first dominant tier is considered significant. In order to simplify the mathematical description, we approximate all the 12 hexagon cells of the considered system by circular regions of radius R with the same area. The original transmitted signal is corrupted by additive white Gaussian noise, multiple-access interference (MAI) and CCI power. Furthermore, MAI and CCI are considered as Gaussian distributed interferences even for small values of the number of system's users, [8]. Without loss of generality, we consider the case where the number N_T of transmit and receive antennas N_R are equal i.e., $N_T = N_R$.

3 Operation in a non-fading awgn environment

When an appropriate power control scheme is applied, the spread signal-to-interference plus noise ratio (SINR) $\Gamma_{i,ss,t}$ received from the t-th, $1 \le t \le N_R$ (=N_T), receiving antenna of the i-th, $1 \le i \le 12K$, user in an AWGN environment, as it reaches the boundary of each cell, can readily be determined by considering the average MAI and average received CCI power resulting from the totally $11K$ co-channel interfering users: $2K$, $3K$ and $6K$ respectively located at distances R, $2R$ and $2.633R$, as following, [9]:

$$\Gamma_{i,sst} = \frac{\alpha R^{-4}}{N_0 \cdot W_{SS} + N_T \cdot [(K-1)\alpha R^{-4} + 2K\alpha R^{-4} + 3K\alpha(2R)^4 + 6K\alpha(2.633R)^4]} =$$
$$= \frac{P_i}{N_0 \cdot W_{SS} + N_T \cdot [(K-1)\alpha R^{-4} + 2K\alpha R^{-4} + 3K\alpha(2R)^4 + 6K\alpha(2.633R)^4]} =$$
$$= \frac{P_i}{N_0 \cdot W_{SS} + N_T \cdot (3.312K-1) \cdot P_i} \quad (1)$$

where P_i, $1 \le i \le 12K$, is the average received signal power from each system's user when an appropriate power control scheme is applied, α is a constant factor, [9], the suffice 't' indicates the t-th, $1 \le t \le N_T$, transmitting/receive antenna, the suffice 'ss' indicates the spread spectrum MIMO system and N_0 is the power spectral density of the additive white gaussian noise. In eq.(1), interfering transmitted power from other transmit antennas of the same user is assumed, negligible. Then, $\Gamma_{i,ss,t}$ can be equivalently expressed as:

$$\Gamma_{i,ss,t} = \frac{\Gamma_i}{G_p + N_T \cdot (3.3123K-1) \cdot \Gamma_i} \quad (2)$$

where Γ_i is the received signal-to-noise ratio (SNR) over the signal bandwidth W, in an AWGN environment, from the i-th, $1 \le i \le 12K$, user, for all the N_T transmitting antennas i.e.:

$$\Gamma_i = P_i / N_0 W \quad (3)$$

Then, in an AWGN environment, the channel capacity $C_{i,t}$ assigned to each user i, $1 \le i \le 12K$, i.e., the single user's conditional channel capacity in the Shannon sense, when arbitrarily complex coding and delay is applied, will be expressed in the form, [10]:

$$C_{i,t} = W_{ss} \cdot \log_2(1 + \Gamma_{i,ss,t}) = W_{ss} \cdot \log_2\left(1 + \frac{P_i}{N_0 W_{ss} + N_T(3.3123K-1)P_i}\right) \quad (4)$$

The total (overall) channel capacity C available to all $12K$ simultaneously transmitting users and for all the N_T transmitting antennas, will be equal to the sum of the corresponding individual rates, i.e., [10],:

$$C = \sum_{t=1}^{N_T} \sum_{i=1}^{12K} C_{i,t} = 12K \cdot \sum_{t=1}^{N_T} W_{ss} \cdot \log_2(1 + \Gamma_{i,ss,t}) =$$
$$= 12K \cdot N_T \cdot W_{ss} \cdot \log_2\left(1 + \frac{\Gamma_i}{G_p + N_T \cdot (3.3123K-1) \cdot \Gamma_i}\right) =$$
$$= N_T W_{ss} \cdot \log_2\left(1 + \frac{12K \cdot \Gamma_i}{G_p + N_T \cdot (3.3123K-1) \cdot \Gamma_i}\right) \quad (5)$$

due to the fact that, in practice, $\Gamma_{i,ss,t}$, $i=[1,..,12K]$, $t=[1,..,N_T]$, is assumed well below unity (in linear scale), [8].

4 Optimal antennas number in a rayleigh fading environment

Although, recent information theory results show that the uncertainties on the fading channel tap amplitudes and delays is what hinders channel capacity in the spread spectrum regime, [6], in this work, it is assumed that the urban Rayleigh fading channel assigned to each transmitting user is perfectly known and modeled as a tapped delay line, [11]. In addition, in general, the multipath-intensity profile (MIP) in a Rayleigh fading environment is exponential, but here, MIP is assumed discrete and constant, so that the "resolvable" path model can be considered to have equal path strengths on the average.

In the spread spectrum MIMO cellular system under consideration, we assume that all the associated spread spectrum signals are received by an optimum maximal-ratio combining (MRC) RAKE receiver. In particular, each user's MRC RAKE receiver has M_C taps corresponding to M_C resolvable signal paths, on the condition that the transmitted signal bandwidth W_{ss} is much greater than the coherence bandwidth W_{coh} of the Rayleigh fading channel, [11], given by:

$$M_C = [W_{ss} \cdot T_m] + 1 \quad (6)$$

where T_m is the total multipath delay spread of the urban Rayleigh fading channel and [.] returns the largest integer less than, or equal to, its argument. Assuming that for all $12K$ users, all the M_C branches are selected and combined, the average received SIR $<\gamma_{i,l,ss}>$ in the l-th, $l=1,\dots M_C$, branch of the corresponding i-th, $1 \le i \le 12K$, MRC RAKE receiver, resulting from the signals of all $12K$ simultaneously transmitting users and all N_T antennas, can be written as:

$$<\gamma_{i,l,ss}> = \sum_{t=1}^{N_T} E\left[\sum_{i=1}^{12K} \gamma_{l,i,ss}\right]_t = \sum_{t=1}^{N_T} E[\gamma_{l,1,ss} + \dots + \gamma_{l,12K,ss}]_t \quad (7)$$

where $\langle . \rangle = E[\ .]$ indicates the average value. Since the $12K$ received signals are independent identically distributed (i.i.d.) random variables, each of which corresponds to a SINR that follows a chi-square distribution with two degrees of freedom, we can rewrite eq.(7) as:

$$<\gamma_{i,l,ss}> = 12K \cdot N_T \Gamma_{i,ss,t} = \frac{12KN_T\Gamma_i}{G_p + N_T(3.3123K-1)\Gamma_i} \quad (8)$$

Then, the probability density function (p.d.f.) of the combined SINR $\gamma_{i,l,ss}$ at each MRC RAKE receiver's output will be given by:

$$p(\gamma_{i,l,ss}) = \frac{1}{(M_C-1)!} \cdot \frac{(\gamma_{i,l,ss})^{M_C-1}}{\langle \gamma_{i,l,ss} \rangle^{M_C}} \cdot \exp\left(-\frac{\gamma_{i,l,ss}}{\langle \gamma_{i,l,ss} \rangle}\right) \quad (9)$$

where $<\gamma_{i,l,ss}>$ is given by eq.(8).

We now estimate the average total channel capacity $<C>$ available to all $12K$ active users, averaged over the p.d.f. of the combined SIR $\gamma_{i,l,ss}$ at the MRC RAKE receiver output, so that:

$$\langle C \rangle = N_T \cdot W_{SS} \cdot \int_0^\infty \log_2(1+\gamma_{i,l,ss}) \cdot p(\gamma_{i,l,ss}) \, d\gamma_{i,l,ss} \quad (10)$$

and, taking into account eq.(9):

$$\langle C \rangle = N_T W_{ss} \int_0^\infty \log(1+\gamma_{i,l,ss}) \frac{1}{(M_C-1)!\langle\gamma_{i,l,ss}\rangle^{M_C}} \gamma_{i,l,ss}{}^{M_C-1} \exp\left(-\frac{\gamma_{i,l,ss}}{\langle\gamma_{i,l,ss}\rangle}\right) d\gamma_{i,l,ss} \quad (11)$$

Clearly, this capacity estimation indicates the average channel capacity that appears at the MRC RAKE receiver output in the form of the average best recovered data rate from all the $12K$ simultaneously transmitting (active) users. Therefore, we can write that:

$$\langle C_i \rangle = \frac{\langle C \rangle}{12K} = \frac{N_T \cdot W_{ss}}{12K} \cdot \int_0^\infty \log_2(1+\gamma_{i,l,ss}) \cdot \frac{1}{(M_C-1)!\cdot\langle\gamma_{i,l,ss}\rangle^{M_C}} \cdot$$
$$\cdot \gamma_{i,l,ss}{}^{M_C-1} \cdot \exp\left(-\frac{\gamma_{i,l,ss}}{\langle\gamma_{i,l,ss}\rangle}\right) d\gamma_{i,l,ss} \quad (12)$$

from which, the average channel capacity per system's user, $<C_{i,t}>$, normalized over the system's bandwidth W_{ss}, will be given by:

$$\frac{\langle C_i \rangle}{W_{ss}} = \frac{\langle C \rangle}{W_{ss}\cdot(12K)} = \frac{N_T}{12K}\int_0^\infty \log(1+\gamma_{i,l,ss}) \frac{1}{(M_C-1)!\cdot\langle\gamma_{i,l,ss}\rangle^{M_C}} \gamma_{i,l,ss}{}^{M_C-1}\exp\left(-\frac{\gamma_{i,l,ss}}{\langle\gamma_{i,l,ss}\rangle}\right) d\gamma_{i,l,s} \quad (13)$$

When path-diversity reception, provided by each system's user MRC RAKE receiver, is applied and assuming that MIP has equal path strengths on the average, the SIR after path-diversity, $<\gamma_{i,l,ss,pt}>$, will be given by, [9]:

$$<\gamma_{i,l,ss}> = M_C \cdot \frac{12KN_T\Gamma_i}{G_p + N_T(3.3123K-1)\Gamma_i} \quad (14)$$

The problem of finding the optimal number of antennas $N_{T,op}$, (the new suffice 'op' refers to the optimal number) that maximizes the estimated normalized average channel capacity per system's user, given by eq.(13), can then be stated as follows:

$$\max\int_0^\infty \log_2(1+\gamma_{i,l,ss}) \cdot \frac{(\gamma_{i,l,ss})^{M_C-1}}{(M_C-1)!\cdot\langle\gamma_{i,l,ss}\rangle^{M_C}} \cdot \exp\left(-\frac{\gamma_{i,l,ss}}{\langle\gamma_{i,l,ss}\rangle}\right) \cdot d\gamma_{i,l,ss} \quad (15)$$

The combined average spread SIR after path-diversity reception i.e., $<\gamma_{i,l,ss,pt}>\cdot M_C$, that maximizes the normalized average channel capacity per user, equals to 6 dB, [12,13]. Therefore, using directly eq.(14), we can write that:

$$<\gamma_{i,l,ss}>\cdot M_C = M_C^2 \cdot \frac{12KN_T\Gamma_i}{G_p + N_T(3.3123K-1)\Gamma_i} = 10^{0.6} \quad (16)$$

Finally, the optimum number $N_{T,op}$ of antennas in a spread spectrum MIMO cellular system operating in a urban Rayleigh fading environment, can be found directly from eq.(16), as following:

$$N_{T,op} = \frac{3.9 \cdot G_p}{12K\Gamma_i M_C^2 - 3.9\Gamma_i(3.3123K-1)} \quad (17)$$

5 Conclusion

In this work, the channel capacity of a MIMO cellular system, when path-diversity reception is applied in a urban Rayleigh fading environment, is examined. Then, a novel closed-form theoretical expression for the optimum number of transmitting antennas with respect to the maximization of the achieved average channel capacity (in the Shannon sense) available to each system's active

user, is obtained, without applying a simulation process. Finally, the expression derived provides a simple and analytical tool for initial estimation of antennas number, in a spread spectrum MIMO cellular system under fading operation.

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