# A whale optimization algorithm with inertia weight

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*Abstract:* - Whale Optimization Algorithm (WOA) is a novel nature-inspired meta-heuristic optimization algorithm proposed by Seyedali Mirjalili and Andrew Lewis in 2016, which mimics the social behavior of humpback whales. A new control parameter, inertia weight, is introduced to tune the influence on the current best solution, and an improved whale optimization algorithm(IWOA) is obtained. IWOA is tested with 31 high-dimensional continuous benchmark functions. The numerical results demonstrate that the proposed IWOA is a powerful search algorithm. Optimization results prove that the proposed IWOA not only significantly improves the basic whale optimization algorithm but also performs much better than both the artificial bee colony algorithm(ABC) and the fruit fly optimization algorithm(FOA).

*Key-Words:* - whale optimization algorithm;artificial bee colony algorithm;fruit fly optimization algorithm; inertia weight; benchmark functions

## 1 Introduction

The fruit fly optimization algorithm(FOA) first proposed by Pan[1] in 2012, who provided an easy and powerful approach to handle the complex optimization problems, simulates the intelligent foraging behavior of fruit flies or vinegar flies in finding food. More and more researchers improve FOA and apply FOA to different regions[2-4].

As a relatively new optimization method inspired by swarm intelligence, artificial bee colony algorithm (ABC) proposed by Karaboga [5] in 2005 imitates the foraging behavior of honeybees, which consists of three kinds of honey bees:employed bees, onlooker bees and scout bees. Since 2005, researchers devote themselves to the search methods and applications of ABC[6-10].

Besides the above two swarm intelligence algorithms, there are other swarm intelligence algorithms such as the ant colony optimization(ACO) [11-12],genetic algorithm(GA) [13-14] that simulates the Darwinian evolution, particle swarm optimization algorithm (PSO) [15-17,26-27], Evolution Strategy(ES) [18-20], differential evolution algorithm(DE)[21-22].

In 2016, Seyedali Mirjalili and Andrew Lewis first propose a new meta-heuristic optimization algorithm(namely, Whale Optimization Algorithm, WOA) mimicking the hunting behavior of humpback whales[23]. The following is the knowledge of whale in brief.

Whales are considered as the biggest mammals in the world. An adult whale can grow up to 30m long and 180t weight. Whales are mostly considered as predators, who never sleep because they have to breathe from the surface of oceans, and half of whose brain only sleeps. According to Hof and Van Der Gucht [24] , whales have common cells in certain areas of their brains similar to those of human called spindle cells. These cells are responsible for judgment, emotions, and social behaviors in humans. It has been proven that whale can think, learn, judge, communicate, and become even emotional as a human does, but obviously with a much lower level of smartness. Fig. 1 shows their special hunting method of the humpback whales. This foraging behavior is called bubble-net feeding method[25]. Humpback whales prefer to hunt school of krill or small fishes close to the surface,whose foraging is done by creating distinctive bubbles along a circle or '9'-shaped path as shown in Fig. 1 .

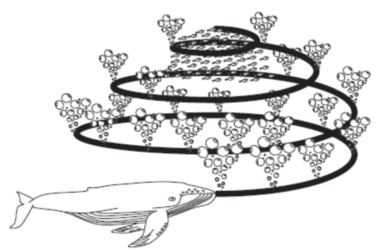


Fig.1 Bubble-net feeding behavior of humpback whales

It is worth mentioning here that bubble-net feeding is a unique behavior that can only be observed in humpback whales.

The structure of the rest of the paper is as follows. In Section 2, basic whale optimization algorithm is described. In Section 3, the inertia weight is introduced into WOA and improved whale optimization algorithm(IWOA) is proposed. In section 4, 31 benchmark functions are introduced. In Section 5, we compare IWOA with FOA, ABC and WOA. Section 6 summarizes the main findings of this study and suggests directions for future research.

# 2 Basic whale optimization algorithm

## 2.1 Encircling prey

Humpback whales can recognize the location of prey and encircle them. For the unknown position of the optimal design in the search space, the current best candidate solution is the target prey or is close to the optimum in the WOA algorithm. Once the best search agent is defined, the other search agents will hence try to update their positions towards the best search agent. The updated method is represented by the following equations:

$$D = |C.X^*(t) - X(t)| \quad (1)$$

$$X(t+1) = X^*(t) - A.D \quad (2)$$

where the meanings of t,A,C,X*,X,| | and . are shown in table 1.

The vectors $A$ and $C$ are calculated as follows:

$$A = 2ar - a \quad (3)$$

$$C = 2r \quad (4)$$

where $a$ is linearly decreased from 2 to 0 over the course of iterations (in both exploration and exploitation phases) and $r$ is a random vector in [0,1].

Table 1.the meanings of *t*,*A*,*C*,*X**,*X*,| | and .

| Symbol | Meaning | Symbol | Meaning |
|---|---|---|---|
| $t$ | the current iteration | $X$ | the position vector |
| $A$ | coefficient vectors | \| \| | the absolute value |
| $C$ | coefficient vectors | . | an element-by-element multiplication |
| $X^*$ | the position vector of the best solution obtained so far | | |

## 2.2 Bubble-net attacking method (exploitation phase)

In order to mathematically model the bubble-net behavior of humpback whales, two improved approaches are designed as follows:

*1. Shrinking encircling mechanism*: This behavior is achieved by decreasing the value of a in the Eq. (3). Note that the fluctuation range of $A$ is also decreased by $a$. In other words $A$ is a random value in the interval [ $-a$ , $a$ ] where a is decreased from 2 to 0 over the course of iterations. Setting random values for $A$ in [ −1,1], the new position of a search agent can be defined anywhere in between the original position of the agent and the position of the current best agent.

*2. Spiral updating position:* A spiral equation is then created between the position of whale and prey to mimic the helix-shaped movement of humpback whales as follows:

$$X(t+1) = D'.e^{bl}.\cos(2\pi l) + X^*(t) \quad (5)$$

where $D' = |X^*(t) - X(t)|$ and indicates the distance of the $i^{th}$ whale to the prey (best solution obtained so far), $b$ is a constant for defining the shape of the logarithmic spiral, $l$ is a random number in [ −1,1], and . is an element-by-element multiplication. Note that humpback whales swim around the prey within a shrinking circle and along a spiral-shaped path simultaneously. To model this simultaneous behavior, we assume that there is a probability of 50% to choose between either the shrinking encircling mechanism or the spiral model to update the position of whales during optimization. The mathematical model is as follows:

$$X(t+1) = \begin{cases} X^*(t) - A.D & if\ p < 0.5 \\ D'.e^{bl}.\cos(2\pi l) + X^{*(t)} & if\ p \geq 0.5 \end{cases} \quad (6)$$

where $p$ is a random number in [0,1]. In addition to the bubble-net method, the humpback whales search for prey randomly.

## 2.3 Search for prey (exploration phase)

The same approach based on the variation of the A vector can be utilized to search for prey (exploration). In fact, humpback whales search randomly according to the position of each other. Therefore, we use A with the random values greater than 1 or less than −1 to force search agent to move far away from a reference whale. In contrast to the exploitation phase, we update the position of a search agent in the exploration phase according to a randomly chosen search agent instead of the best search agent found so far. This mechanism and | A | > 1 emphasize exploration and allow the WOA algorithm to perform a global search. The mathematical model is as follows:

$$D = |C.X_{rand} - X| \quad (7)$$

$$X(t+1) = X_{rand} - A.D \quad (8)$$

where X rand is a random position vector (a random whale) chosen from the current population.

The IWOA algorithm starts with a set of random solutions. At each iteration, search agents update their positions with respect to either a randomly chosen search agent or the best solution obtained so far. The a parameter is decreased from 2 to 0 in order to provide exploration and exploitation, respectively. A random search agent is chosen when |A| > 1, while the best solution is selected when |A | < 1 for updating the position of the search agents. Depending on the value of *p* , WOA is able to switch between either a spiral or circular movement. Finally, the WOA algorithm is terminated by the satisfaction of a termination criterion.

## 3 Improved whale optimization algorithm

In WOA, the updated solution is mostly depended on the the current best candidate solution. Similar to PSO algorithm,an inertia weight $\omega \in [0,1]$ is introduced into WOA to obtain the improved whale optimization algorithm(IWOA).

In Encircling prey, the updated method is represented by the following equations:

$$D = |C.\omega X^*(t) - X(t)| \tag{9}$$

$$X(t+1) = \omega X^*(t) - A.D \tag{10}$$

where the meanings of $t,A,C,X^*,X,|\ |$ and . are shown in table 1.

In *Spiral updating position,a* spiral equation is then created between the position of whale and prey to mimic the helix-shaped movement of humpback whales as follows:

$$X(t+1) = D'.e^{bl}.\cos(2\pi l) + \omega X^*(t) \tag{11}$$

where $D' = |\omega X^*(t) - X(t)|$ and indicates the distance of the i^th whale to the prey (best solution obtained so far), *b* is a constant for defining the shape of the logarithmic spiral, *l* is a random number in [−1,1], and . is an element-by-element multiplication. Note that humpback whales swim around the prey within a shrinking circle and along a spiral-shaped path simultaneously. To model this simultaneous behavior, we assume that there is a probability of 50% to choose between either the shrinking encircling mechanism or the spiral model to update the position of whales during optimization. The mathematical model is as follows:

$$X(t+1) = \begin{cases} \omega X^*(t) - A.D & if\ p < 0.5 \\ D'.e^{bl}.\cos(2\pi l) + \omega X^{*(t)} & if\ p \geq 0.5 \end{cases} \tag{12}$$

where *p* is a random number in [0,1]. In addition to the bubble-net method, the humpback whales search for prey randomly.

The concrete steps of the IWOA are the following:

*Step1*. Initialize the whales population $X_i (i = 1,2,\cdots,n)$ and Maxgen(maximum number of iterations). Let $t = 1$.

*Step2*. Calculate the fitness of $X_i (i = 1,2,\cdots,n)$, and find the best search solution $X^*$.

*Step3*. Repeat the following:

For every $X_i (i = 1,2,\cdots,n)$,update $a, A, C, l, p$.

If $p < 0.5$,then if $|A| < 1$,update the position of the current search agent by the Eq.(9) and if $|A| \geq 1$,select a random search solution $X_{rand}$ and update the position of the current search agent by the Eq.(8).

If $p \geq 0.5$,update the position of the current search by the Eq.(11)

Check if any search agent goes beyond the search and amend it.Calculate the fitness of $X_i (i = 1,2,\cdots,n)$,and if there is a better solution,find the best search solution $X^*$.

Let $t = t + 1$.

Until $t$ reaches Maxgen, the algorithm is finished.

*Step 4*. Return the best optimization solution $X^*$ and the best optimization value of fitness values.

## 4 Test functions

In order to test the performance of the IWOA, 31 benchmark functions commonly used in the literature [2-3,23] are taken, which consist of 18 unimodal functions and 13 multimodal functions. There are 18 functions with *n*-dimension and 3 functions with 2-dimension.31 benchmark functions where $f_4 = f_4' + f_4''$ and $f_8 = f_8' + f_8''$ are concrete in the following:

(1) $f_1 = \sum_{i=1}^n x_i^2$ ,where $-100 \leq x_i \leq 100$ . The minimum value of $f_1$ is 0.

(2) $f_2 = \sum_{i=1}^n |x_i| + \prod_{i=1}^n |x_i|$ ,where $-10 \leq x_i \leq 10$ . The minimum value of $f_2$ is 0.

(3) $f_3 = \max_i\{|x_i|, 1 \leq i \leq n\}$ , where $-100 \leq x_i \leq 100$ . The minimum value of $f_3$ is 0.

(4) $f_4' = \sum_{i=1}^{n-1} 100(x_{i+1} - x_i^2)^2$ ,where $-30 \leq x_i \leq 30$ . The minimum value of $f_4'$ is 0.

(5) $f_4'' = \sum_{i=1}^{n-1} (x_i - 1)^2$ ,where $-30 \leq x_i \leq 30$ . The minimum value of $f_4''$ is 0.

(6) $f_4 = \sum_{i=1}^{n-1} [100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2]$ ,where $-30 \leq x_i \leq 30$ . The minimum value of $f_4$ is 0.

(7) $f_5 = \sum_{i=1}^{n} (\lfloor x_i + 0.5 \rfloor)^2$ ,where $-100 \le x_i \le 100$ . The minimum value of $f_5$ is 0.

(8) $f_6 = \sum_{i=1}^{n} i x_i^4 + rand()$ ,where $-1.28 \le x_i \le 1.28$ . The minimum value of $f_6$ is 0.

(9) $f_7 = \sum_{i=2}^{n} i x_i^2$ ,where $-5.12 \le x_i \le 5.12$ . The minimum value of $f_7$ is 0.

(10) $f_8' = \sum_{i=2}^{n} i(2x_i^2 - x_{i-1})^2$ ,where $-10 \le x_i \le 10$ . The minimum value of $f_8'$ is 0.

(11) $f_8'' = (x_1 - 1)^2$ ,where $-10 \le x_i \le 10$ . The minimum value of $f_8''$ is 0.

(12) $f_8 = \sum_{i=2}^{n} i(2x_i^2 - x_{i-1})^2 + (x_1 - 1)^2$ ,where $-10 \le x_i \le 10$. The minimum value of $f_8$ is 0.

(13) $f_9 = -\exp(-0.5 \sum_{i=1}^{n} x_i^2)$ ,where $-1 \le x_i \le 1$ . The minimum value of $f_9$ is -1.

(14) $f_{10} = \sum_{i=1}^{n} (10^6)^{\frac{i-1}{n-1}} x_i^2$ ,where $-100 \le x_i \le 100$ . The minimum value of $f_{10}$ is 0.

(15) $f_{11} = \sum_{i=1}^{n} \left( \sum_{j=1}^{i} x_j \right)^2$ ,where $-100 \le x_i \le 100$ . The minimum value of $f_{11}$ is 0.

(16) $f_{12} = \sum_{i=1}^{n} | x_i |^{i+1}$ ,where $-1 \le x_i \le 1$ . The minimum value of $f_{12}$ is 0.

(17) $f_{13} = -20 \exp\left( -0.2 \sqrt{\frac{1}{n} \sum_{i=1}^{n} x_i^2} \right) - \exp\left( \frac{1}{n} \sum_{i=1}^{n} \cos(2\pi x_i) \right) + 20 + e$, where $-32 \le x_i \le 32$. The minimum value of $f_{13}$ is 0.

(18) $f_{14} = \sum_{i=1}^{n} | x_i \sin(x_i) + 0.1 x_i |$ ,where $-10 \le x_i \le 10$. The minimum value of $f_{14}$ is 0.

(19) $f_{15} = f_s(x_1, x_2) + f_s(x_2, x_3) + \cdots + f_s(x_n, x_1)$ , where

$f_s(x, y) = 0.5 + \frac{\sin^2(\sqrt{x^2 + y^2}) - 0.5}{(1 + 0.001(x^2 + y^2))^2}$ , $-100 \le x_i \le 100$ . The minimum value of $f_{15}$ is 0.

(20) $f_{16} = f_{10}(x_1, x_2) + \cdots + f_{10}(x_{n-1}, x_n) + f_{10}(x_n, x_1)$ where $f_{10}(x, y) = (x^2 + y^2)^{0.25} [\sin^2(50(x^2 + y^2)^{0.1}) + 1]$, $-100 \le x_i \le 100$. The minimum value of $f_{16}$ is 0.

(21) $f_{17} = \frac{1}{4000} \sum_{i=1}^{n} x_i^2 - \prod_{i=1}^{n} \cos\left( \frac{x_i}{\sqrt{i}} \right) + 1$ ,where $-600 \le x_i \le 600$. The minimum value of $f_{17}$ is 0.

(22) $f_{18} = -\sum_{i=1}^{n-1} \left( \exp\left( -\frac{x_i^2 + x_{i+1}^2 + 0.5 x_i x_{i+1}}{8} \right) \cos\left( 4\sqrt{x_i^2 + x_{i+1}^2 + 0.5 x_i x_{i+1}} \right) \right)$

where $-5 \le x_i \le 5$. The minimum value of $f_{18}$ is $1 - n$.

(23) $f_{19} = \sum_{i=1}^{n-1} \left( 0.5 + \frac{\sin^2(\sqrt{100 x_i^2 + x_{i+1}^2}) - 0.5}{1 + 0.001(x_i^2 - 2 x_i x_{i+1} + x_{i+1}^2)^2} \right)^2$ ,where $-100 \le x_i \le 100$. The minimum value of $f_{19}$ is 0.

(24) $f_{20} = \sum_{i=1}^{n} (x_i^2 - 10\cos(2\pi x_i) + 10)$ ,where $-5.12 \le x_i \le 5.12$. The minimum value of $f_{20}$ is 0.

(25) $f_{21} = 1 - \cos\left( 2\pi \sqrt{\sum_{i=1}^{n} x_i^2} \right) + 0.1 \sqrt{\sum_{i=1}^{n} x_i^2}$ ,where $-100 \le x_i \le 100$. The minimum value of $f_{21}$ is 0.

(26) $f_{22} = -\sum_{i=1}^{n} x_i \sin\sqrt{| x_i |}$ ,where $-500 \le x_i \le 500$ . The minimum value of $f_{22}$ is -418.9829*5.

(27) $f_{23} = 4x_1^2 - 2.1 x_1^4 + \frac{1}{3} x_1^6 + x_1 x_2 - 4x_2^2 + 4x_2^4$ , where $-5 \le x_i \le 5$. The minimum value of $f_{23}$ is -1.0316.

(28) $f_{24} = (x_2 - \frac{5.1}{4\pi^2} x_1^2 + \frac{5}{\pi} x_1 - 6)^2 + 10(1 - \frac{1}{8\pi}) \cos x_1 + 10$, where $-5 \le x_i \le 5$. The minimum value of $f_{24}$ is 0.398.

(29) $f_{25} = [1 + (x_1 + x_2 + 1)^2 (19 - 14x_1 + 3x_1^2 - 14x_2 + 6x_1 x_2 + 3x_2^2] \times [30 + (2x_1 - 3x_2)^2 (18 - 32x_1 + 12x_1^2 + 48x_2 - 36 x_1 x_2 + 27 x_2^2]$, where $-2 \le x_i \le 2$. The minimum value of $f_{25}$ is 3.

(30) $f_{26} = \sum_{i=1}^{n} (y_i^2 - 10\cos(2\pi y_i) + 10)$ ,where $y_i = \begin{cases} x_i & | x_i | < 1/2 \\ round(2x_i)/2 & | x_i | \ge 1/2 \end{cases}$ ,where $-5.12 \le x_i \le 5.12$ . The minimum value of $f_{26}$ is 0.

(31) $f_{27} = \sum_{i=1}^{n} i x_i^2$ ,where $-10 \le x_i \le 10$ . The minimum value of $f_{27}$ is 0.

## 5 Computational results

### 5.1 (IWOA and WOA) vs. (ABC and FOA)

In this section, we compare the presented IWOA with WOA, ABC algorithm and FOA based on 31 benchmark functions. For all the algorithms, a population size and maximum iteration number equal to 30 and 1000 have been utilized. We run 30 replications for 31 benchmark functions shown in Section 4. The dimension of 28 benchmark functions in addition to is fixed at 30. In order to compare IWOA and WOA with ABC and FOA, we take in IWOA shown in table 2. From table 2, it is found that the IWOA performs steadily well in term of mean values and performs significantly better than ABC and FOA.The concrete comparison is in the following.

Table 2 Comparison among IWOA,WOA,ABC and FOA

| | | $f_1$ | $f_2$ | $f_3$ | $f_4'$ | $f_4''$ |
|---|---|---|---|---|---|---|
| IWOA | Mean | **0** | **0** | **0** | **0** | 29 |
| | Std | 0 | 0 | 0 | 0 | 0 |
| WOA | Mean | 3.2578e-46 | 2.6884e-29 | 7.2469e-08 | 2.8365 | 0.1006 |
| | Std | 8.1220e-46 | 3.0757e-29 | 7.0724e-08 | 15.3980 | 0.3047 |
| ABC | Mean | 5.09087e-09 | 2.9195e-06 | 59.6811 | 1.37669 | 1.50381e-04 |
| | Std | 6.04513e-09 | 1.04875e-06 | 4.40445 | 0.845752 | 2.46618e-04 |
| FOA | Mean | 2.5043e-09 | 0.0027 | 9.1444e-06 | 2.6828e-06 | 5.7032e-11 |
| | Std | 2.3612e-11 | 1.3886e-05 | 4.9704e-08 | 3.1448e-08 | 4.59296e-11 |
| | | $f_4$ | $f_5$ | $f_6$ | $f_7$ | $f_8'$ |
| IWOA | Mean | 29 | **0** | 0.0262 | **0** | **0** |
| | Std | 0 | 0 | 0.0224 | 0 | 0 |
| WOA | Mean | 38.3357 | **0** | 0.0360 | 3.4951e-49 | 0.0961 |
| | Std | 28.5100 | 0 | 0.0338 | 7.5910e-49 | 0.3426 |
| ABC | Mean | 4.93219 | 0.866667 | 0.176713 | 1.42539e-11 | 0.0549641 |
| | Std | 4.27956 | 0.819307 | 0.0450133 | 1.47294e-11 | 0.0352682 |
| FOA | Mean | 28.708 | **0** | **0.0018** | 1.4722e-05 | 3.8599e-06 |
| | Std | 50.0046 | 0 | 5.4855e-04 | 1.3870e-07 | 3.9300e-08 |
| | | $f_8''$ | $f_8$ | $f_9$ | $f_{10}$ | $f_{11}$ |
| IWOA | Mean | 1 | 1 | **-1** | **0** | **0** |
| | Std | 0 | 0 | 0 | 0 | 0 |
| WOA | Mean | **3.5769e-12** | 0.6752 | **-1** | 9.1530e-43 | 5.1209e-10 |
| | Std | 4.6204e-12 | 0.0332 | 0 | 2.5285e-42 | 9.9814e-10 |
| ABC | Mean | 2.35606e-05 | **0.0545354** | **-1** | 0.0421119 | 12602.8 |
| | Std | 4.77473e-05 | 0.0358611 | 1.10061e-15 | 0.0777918 | 2772.45 |
| FOA | Mean | 1.7871e-06 | 0.9977 | -1.0000 | 2.1976e-04 | 7.8518e-07 |
| | Std | 2.7400e-06 | 2.6897e-05 | 1.3272e-07 | 2.3050e-06 | 6.3274e-09 |
| | | $f_{12}$ | $f_{13}$ | $f_{14}$ | $f_{15}$ | $f_{16}$ |
| IWOA | Mean | **0** | **8.8818e-16** | **0** | **0** | **0** |
| | Std | 0 | 0 | 0 | 0 | 0 |
| WOA | Mean | 1.6631e-147 | 1.6165e-14 | 0.1427 | 2.4553 | 0.1293 |
| | Std | 8.5546e-147 | 7.9796e-15 | 0.1956 | 1.0059 | 0.1701 |
| ABC | Mean | 2.77347e-10 | 1.79011e-05 | 5.44277e-04 | 2.44705 | 7.71574 |
| | Std | 4.51079e-10 | 1.04539e-05 | 8.35577e-04 | 0.468454 | 2.46721 |
| FOA | Mean | 8.3473e-07 | 1.1423e-04 | 2.7373e-04 | 5.0041e-09 | 2.6190 |
| | Std | 7.9834e-09 | 5.9602e-07 | 1.4516e-06 | 4.5851e-11 | 0.7872 |
| | | $f_{17}$ | $f_{18}$ | $f_{19}$ | $f_{20}$ | $f_{21}$ |
| IWOA | Mean | **0** | **-29** | **0** | **0** | **0** |
| | Std | 0 | 0 | 0 | 0 | 0 |
| WOA | Mean | 0.0153 | -21.7477 | 2.5176 | 12.1148 | 0.1132 |
| | Std | 0.0187 | 1.1289 | 0.9295 | 3.4941 | 0.0346 |
| ABC | Mean | 0.0862962 | -23.7643 | 2.39 | 0.133351 | 3.58919 |
| | Std | 0.0286 | 0.952259 | 0.208155 | 0.34383 | 0.415772 |
| FOA | Mean | 4.6265e-12 | -29.0000 | 5.9007 | 1.8920e-04 | 5.0473e-06 |
| | Std | 4.3520e-14 | 1.5676e-07 | 0.2223 | 1.9121e-06 | 2.7015e-08 |
| | | $f_{22}$ | $f_{23}$ | $f_{24}$ | $f_{25}$ | $f_{26}$ |
| IWOA | Mean | -9.1089e-228 | 0 | 55.6021 | 600 | **0** |
| | Std | 0 | 0 | 2.8908e-14 | 0 | 0 |
| WOA | Mean | -6.0114e+03 | **-1.0316** | **0.3979** | **3.0000** | 7.2333 |
| | Std | 800.5744 | 1.3030e-09 | 5.5628e-08 | 1.1135e-07 | 2.2542 |
| ABC | Mean | -12213.8 | **-1.0316** | **0.3979** | 3.00057 | 3.2905 |
| | Std | 128.981 | 5.13342e-16 | 0 | 0.00173543 | 2.70384 |
| FOA | Mean | **-1.8415** | -0.7708 | 5.3635 | 185.8300 | 1.8951e-04 |
| | Std | 1.0030 | 0.1390 | 4.8876 | 171.4576 | 1.5110e-06 |
| | | $f_{27}$ | | | | |
| IWOA | Mean | **0** | | | | |
| | Std | 0 | | | | |
| WOA | Mean | 4.9584e-47 | | | | |
| | Std | 1.1710e-46 | | | | |
| ABC | Mean | 2.85732e-10 | | | | |
| | Std | 3.10916e-10 | | | | |
| FOA | Mean | 1.4780e-05 | | | | |
| | Std | 1.3781e-07 | | | | |

**Note**: The best values are written in bold.

The mean values of IWOA and WOA are far less than those of ABC and FOA for functions $f_1, f_2, f_3, f_7, f_{10}, f_{11}, f_{12}, f_{13}, f_{16}, f_{27}$.The mean values of IWOA are far less than those of ABC and FOA for functions $f_4', f_8', f_{14}, f_{15}, f_{17}, f_{18}, f_{19}, f_{20}, f_{21}, f_{26}$. And the mean values of IWOA, WOA and FOA equal to those of ABC for function $f_5$. The mean value of IWOA,WOA,ABC and FOA equal to the optimal value for function $f_9$. The mean values of IWOA,WOA and ABC are more than that of FOA for function $f_6$.The mean values of WOA and ABC arrive at the optimal values far less than those of IWOA and FOA for functions $f_{23}, f_{24}, f_{25}$.

The mean values of ABC for functions $f_4'', f_4$ are less than those of IWOA,WOA and FOA,but do not arrive the optimal value because of the seldom iterations.But the mean values of IWOA,WOA,ABC and FOA is far away from the optimal values for function $f_{22}$ and it is shown that four algorithms is unfit for function $f_{22}$. The mean value of WOA for function $f_8''$ is less than that of IWOA, ABC and FOA,while the mean value of ABC for function $f_8$ is less than that of WOA,IWOA and FOA.

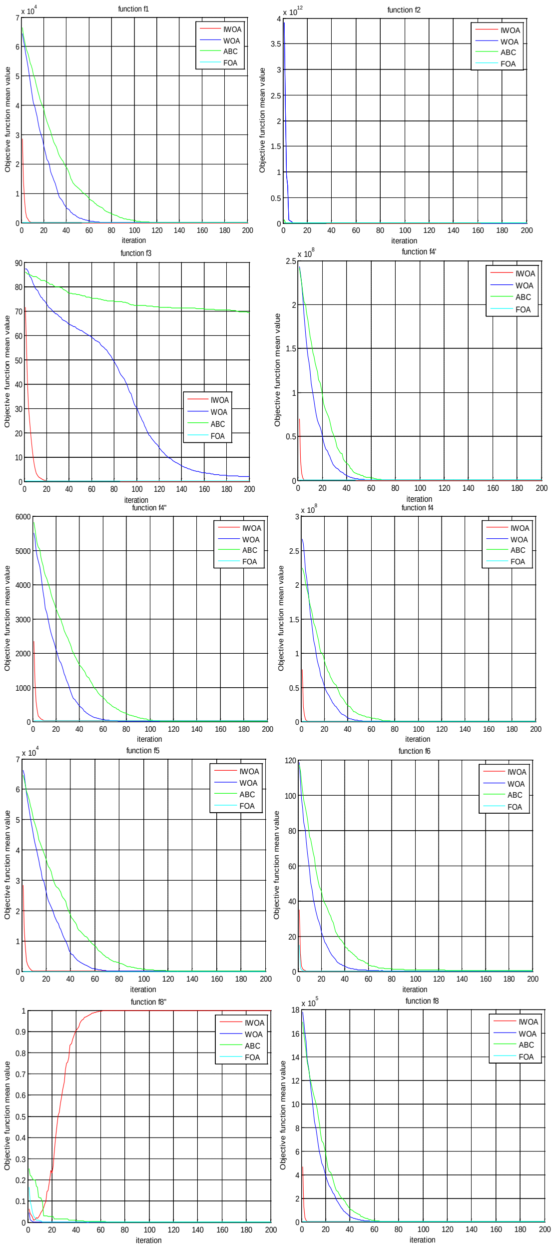
General speaking, IWOA is superior to ABC and FOA.Hence, it can be concluded that as a whole the proposed IWOA significantly improves the basic WOA.

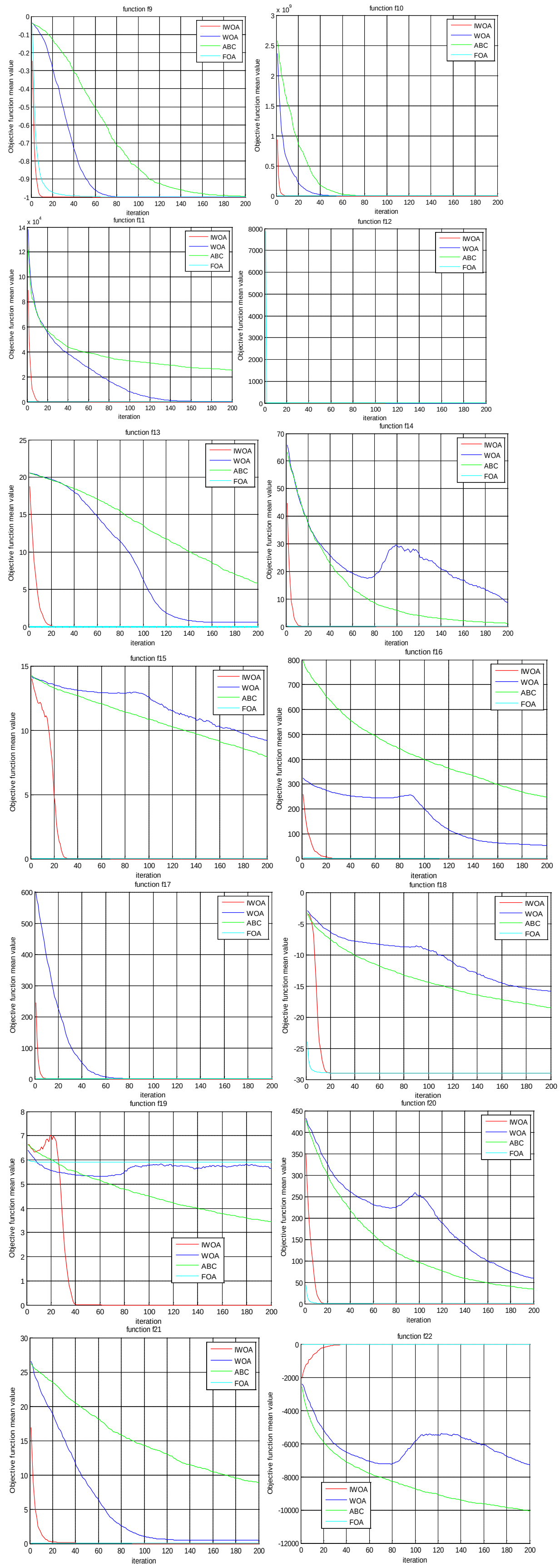
### 5.2 Convergence analysis

We report the function value graph along the iterations of the compared algorithms in Fig.4 and observe that overall the convergence curves of the IWOA descend much faster and reach the low level value than that of the WOA,ABC and FOA. In fig.4, the conclusions on 31 benchmark functions using IWOA,WOA,ABC and FOA are the same as those in section 4. Thus, it can be shown that the IWOA significantly outperforms the optimization algorithms for function optimization in comparison.

## 6 Conclusion

The whale optimization algorithm(WOA) is a new swarm intelligence approach.This paper introduces the inertia weight into WOA to obtain an improve whale optimization algorithm(IWOA) for high-dimensional continuous function optimization problems. To our best knowledge, this is the first improved WOA.In order to illustrate the IWOA,we take the inertia weight varying step length 0.1 from 0 to 1 and the results are the IWOA is better than the WOA. The numerical results demonstrate that the proposed IWOA is a powerful search algorithm, and as a whole it performs significantly better than ABC and FOA. Our future work will develop more improved whale optimization algorithms to solve more and more optimization problems and apply them to different regions.



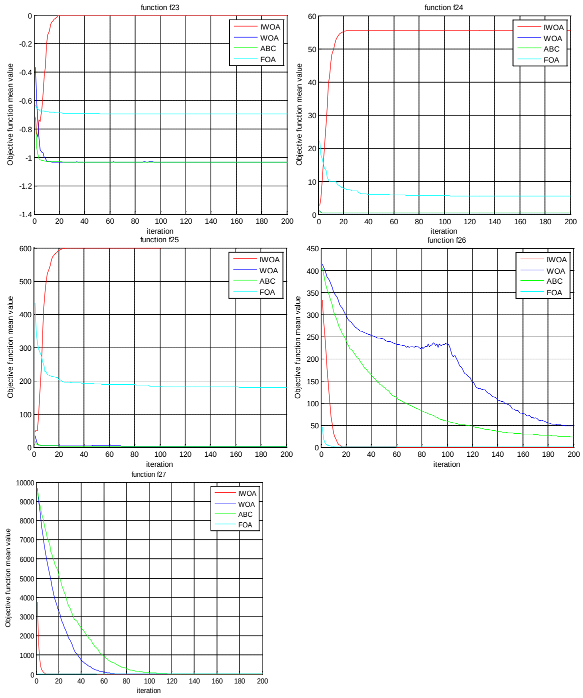



Fig.4. Comparison of convergence curves of IWOA,WOA,ABC and FOA

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