Generalization Process for Loose Topological Relations of Regions

BRAHIM LEJDEL [1,2], OKBA KAZAR [2]

[1] Smart Computer Science Laboratory, University of Biskra, Algeria .

[2] Computer science department, University Hamma Lekhder of EL-oued, Algeria .

[1]Lejdel.brahim@gmail.com, [2]kazarokba@yahoo.fr

Abstract:-In Geographical Information System (GIS), we use principally three features, such as PolyLine, Point and loose region to represent the geographic object. The loose region is used to represent the area objects where they have a loose shape as building or green area. Loose regions can be seen as an extension of simple regions described in Egenhofer model. In this paper, we treat the mutation of the loose topological relations into others relations, when we need to change the scale the map. A new topological model is presented based on simple regions which are defined in *Egenhofer* model and also on the assertions of mutation of the topological relationships according certain criteria.

Keywords:-Region, generalization, visual acuity, loose topological relations.

1. Introduction

To represent exactly the reality in GIS, we have to use the loose region to represent the area object. We can define loose region as the extension of simple region represented in the *Egenhofer model*. In this new configuration, the topological relationships between the regions can be different.

In different situations, we need to change the scale of certain detailed representation because the demanded representation doesn't exist in the geographic Database. When applying this process, various changes have been held in the representation contents; as geometry, topology, etc. the loose region can be mutate to a simple region, to point or will be disappear , also the topological relationships can be mutate into others relationships according to certain measurements and thresholds. So, we will develop the mathematical assertions which guide these mutations.

This paper will be organized as follows. First, definitions and a state of the art review for generalization process will be given (Section 2). Also, definitions and a state of the art review for topological relations will be given (Section 3). Then, loose regions and topology model will be defined (Section 4). Finally, we present a conclusion and future work (Section 5).

2. Geographic Object Generalization

The generalization process can be defined as a process of abstraction of represented information subject to the change of the scale of a map. The purpose of generalization is to produce a good-looking map, balancing the requirements of accuracy, information content and legibility [1]. It encompasses the modification of the information in such a way that it can be represented on smaller surfaces, retaining the geometric and descriptive characteristics. The essence of the original map should be maintained at all smaller scales. Fig 1 gives an example of generalization of a map at the same place: shapes are simplified, some points disappear, etc.

Fig 1. Generalization process (left: 1:100,000, middle: 1:200,000, right: 1:500,000), Source: Swisstopo.

2.1. Definition

Many definitions have been given for the generalization process, The International Cartographic Association [2] has defined it as "the selection and simplified representation of detail appropriate to scale and/or the purpose of a map". The geographic object generalization is a very complex process. In order to reduce its complexity, the overall process is often decomposed into individual sub-processes, called operators [3], such as simplification, displacement,...etc. Each operator defines a transformation that can be applied to a single spatial object, or to a group of spatial objects.

2.2. State of the art for geographic object generalization

Historically speaking, the first algorithm for generalizing polylines was published in [4]. Then, several variants were published, essentially to improve the results of the initial algorithm. However, this algorithm does not take into account many aspects, such as the topological relationships between objects.

Now, several methods and concepts proposed to model and implement the generalization process but a framework for their combination into a comprehensive generalization process is still missing [5].

The first generalization process appeared in the early of 1990s [6]. It involved only a few geographic areas.

Ruas and Plazanet [7] proposed a framework controlled by a set of constraints. The dynamic generalization model is based on avoiding constraint violations and on the local qualification of a set of objects, represented by means of an object situation. A situation is described by the geographical objects involved, their relationships, and the constraint violations. Ruas and Plazanet concentrated only on constraints related to objects and not the constraints between objects, such as the topological constraints [7].

Many other works use the least squares adjustment theory to solve the generalization problems, such as [8], [9], [10], whose works aim to globally reduce all spatial conflicts. The idea is to solve spatial conflicts by modeling different constraints using mathematical expressions. Moreover Harrie [8] proposed to formulate the geometric and topological constraints as linear functions of the object coordinates. The least squares adjustment seems to be an interesting technique but these constraints are difficult to express by a linear equation.

In the same context and for reducing the spatial conflicts in the map, many interesting methods were proposed in [11] and [12]. In those approaches, a cost function (fitness) must be defined for validating statements. However, it is questionable whether it is realistic to define such a function that integrates all the constraints of generalization, such as the topological constraints.

Then several works model the spatial objects by agents such as the works of [13], [14] and [15]. In the agent-based model, the spatial objects are modeled by the decisional entities in the generalization system. These entities are software agents the goal of which is to satisfy their cartographic constraints the more as possible. The constraints are subdivided into four types, metric, topological, structural and procedural constraints [14]. The topological constraints ensure that any topological relationship between objects is maintained or modified consistently, for example, self-intersections of an object or any intersection between two objects must be avoided.

Also to improve the map generalization process, another approach was proposed in [16] which are based on a new concept called SGO (Self-generalizing object). An SGO is able to generalize a cartographic object automatically using one or more geometric patterns, simple generalization algorithms and spatial integrity constraints, but this approach does not define a pattern for topological constraints.

In the EuroSDR project, cartographic experts of four NMAs (National Mapping Agencies) were called to evaluate the results of the automation generalization process according to certain constraints [17]. The objective of this project consists to illustrate the state of the art of automated generalization in practice, exchange of knowledge between research community, NMAs and software vendor and to contribute to development of constraint specification. Four test cases that were selected, provided by the participated NMAs. NMAs defined their map specifications for automated generalization in template which were developed by the EuroSDR team [17]. These map specifications were formalized as a set of cartographic constraints to be followed. They distinguished between two main categories of constraints: legibility constraints and preservation constraints. After the analysis of constraints composition, the EuroSDR project team derived a list of generic and specific cartographic constraints which must respected in generalization process.

Lejdel and Kazar [18] proposed an approach for optimizing the automatic generalization process by satisfying cartographic constraints. This approach consists in providing agents geographic genetic properties to enable them to choose the optimal actions, so giving the concept of genetic agent. Each geographic agent is equipped with an optimizer, and each one executes a genetic algorithm to determine the optimal action to be executed according to its current state, in order to satisfy cartographic constraints the most possible. The genetic algorithm follows the classical steps as selection, crossover and mutation. The solution is refined gradually over the iterations until to reach convergence to a solution that approaches the optimal solution and a certain degree of imperfection is acceptable. The solution here is a set of algorithms with adapted parameters which minimize conflicts. The model of the topological constraints of this approach is not addressed in this paper.

In the recent paper, Lejdel et al. [19] define a mathematical model of the generalization of topological relationships between two rectangular ribbons or between ribbons and simple regions. Thus, two rectangular ribbons or two simple regions can be disjoint or intersect. The disjunction is defined by a distance separating the two ribbons. The intersection between two simple regions can be Point (0D), Line (1D) or area (2D) according to certain criteria. In this work, they get formally the mathematical description for each topological relationship between objects when we use thresholds and metric measurements; as area, distance, etc. These topological relationships can be: disjoint, meet, merge and crossing. When downscaling, these topological relations can be mutated into other topological relations according to certain criteria. In this paper, we developed a topological model to define a good generalization process. It based on the Egenhofer model which describes the topological relationships between simple regions.

3. Topological Relations

Topological relationships describe relationships between all objects in space, the points, lines and areas for all possible kinds of deformation.

3.1. Definition

Topology is defined as mathematical study of the properties that are preserved through deformations, twistings, and stretchings of objects. Topology is foremost a branch of mathematics, but some concepts are of importance in cartographic generalization, such as topological relationships [8]. Several researchers have defined topological relationships in the context of geographic information [4] ,[20] and [21].

3.2. State of the art for topological relations

From an historical point of view, different topological models were proposed. First, Max Egenhofer with his colleagues proposed the first topological model for two-dimensional objects [22] and then a second model family named RCC was proposed. Let us examine them rapidly.

3.2.1. Egenhofer topological relationship

To define a model of topological relationships, Egenhofer and Herring [8] proposed a spatial data model based on topological algebra. The algebra topological model is based on geometric primitives called cells that are defined for different spatial dimensions 0-D, 1-D, and 2-D. A variety of topological properties between two cells can be expressed in terms of the 9-intersection model [23]. The 9-intersection model between two cells A and B is based on the combination of six topological primitives that are interiors, boundaries, and exteriors of A $(A^\circ, \partial A, A^-)$ and $B\,(B^\circ, \partial B, B^-)$.

These six topological primitives can be combined to form nine possible combinations representing the topological relationships between these two cells. These 9-intersections are represented as one 3×3 matrix [24], see Fig.2.

$$R(A,B)=\begin{pmatrix} A^\circ \cap B^\circ & A^\circ \cap \partial B & A^\circ \cap B^- \\ \partial A \cap B^\circ & \partial A \cap \partial B & \partial A \cap B^- \\ A^- \cap B^\circ & A^- \cap \partial B & A^- \cap B^- \end{pmatrix}$$

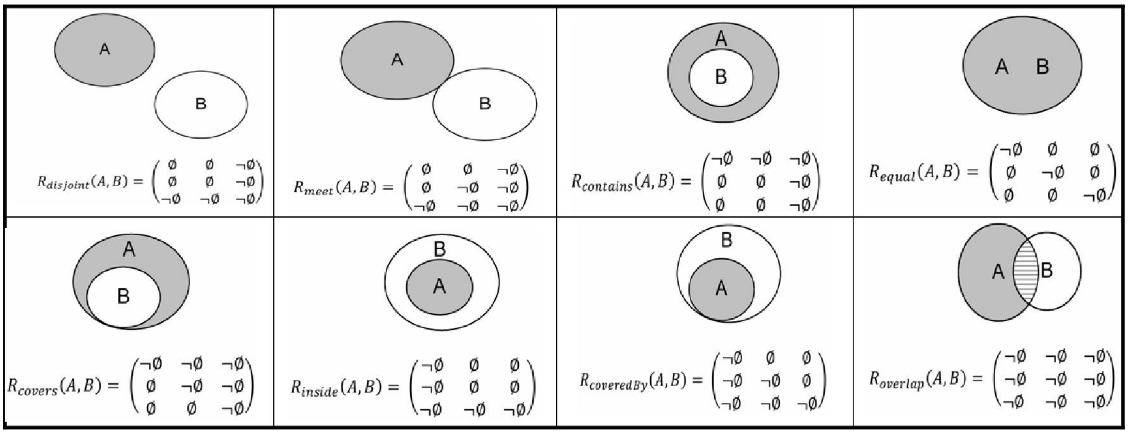


Fig 2. The eight topological relations between two regions A and B.

3.2.2. Other models RCC

Independently developed, the RCC (Region Connection Calculus) is an alternative topological approach to qualitative spatial representation and reasoning where spatial regions are subsets of topological space [25]. The RCC model distinguishes eight topological relationships between two simple regions, which are in fact exactly the same as those identified by 9-intersection (see Fig 2).The RCC-8 uses a set of eight pairwise disjoint and mutually exhaustive relations, called base relation denoted as EQ, DC, EC, PO, TPP, NTPP, TPP-1, NTPP-1, with the meaning of EQual, DisConnected, Externally Connected, Partial Overlap, Tangential Proper Part, Non-Tangential Proper Part, and their converses.

In this model, sometimes one does not want to distinguish between DC and EC, and between TPP and NTPP. So, a set of five relations are derived, called RCC-5. Other refinements have been developed, taking into account the convex hulls of region, so twenty-three topological relationships are obtained, called RCC-23. Also, [26] presents a statistical model for quantitative assessment of uncertain topological relations between two imprecise regions. This model based on a morphologic distance function to determine the type of topological relations.

3.3. Mutation of topological Region-Region relations

In this section, the Egenhofer's relations [23] are treated mainly. After generalization, the object geometries are adapted to the perceptual limits imposed by the new (smaller) scale [27]. We present in following the different mutations of topological relationships between loose regions.

3.3.1. Mutation Disjoint-to-Meet

The relation "Disjoint" mutates to relation "Meet" (see Fig 3), according the following assertions.

$$\forall\, \mathrm{O}^1, \mathrm{O}^2 \in \mathrm{GeObject}, (\forall \sigma \in \mathrm{Scale}) \wedge (O^1_\sigma = 2Dmap(O^1, \sigma))$$
$$(O^2_\sigma = 2Dmap(O^2, \sigma)) \wedge (Disjoint(\mathrm{O}^1, \mathrm{O}^2)) \wedge$$
$$(Dist(\mathrm{O}^1, \mathrm{O}^2) < \varepsilon)$$
$$\Rightarrow Meet(O^1_\sigma, O^2_\sigma).$$

But a smaller object can disappear or be eliminated if its area is too small to be well visible. So in this case, the initial relation does not hold anymore.

$$\forall O \in \mathrm{GeObject}, \forall \sigma \in \mathrm{Scale} \wedge (O_\sigma = 2Dmap(O, \sigma)) \wedge$$
$$(Area(O_\sigma) < (\varepsilon_{lp})^2)$$
$$\Rightarrow O_\sigma = \phi.$$

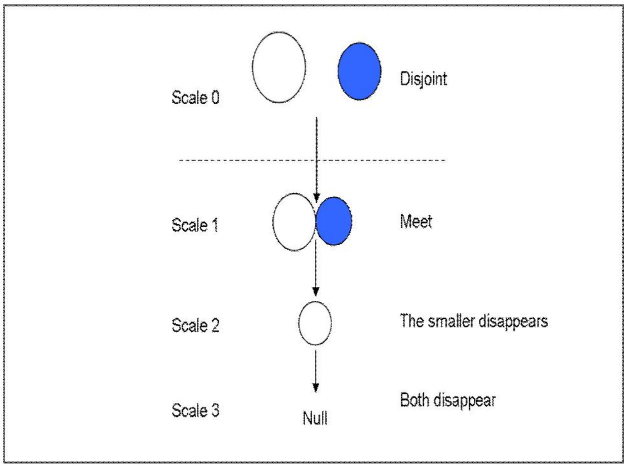


Fig 3. The mutation of disjoint to meet.

3.3.2. Mutation Overlap-to-Meet

The relation "overlap" can mutate to "meet" relation according to the following condition (see Fig 4):

$$\forall\, \mathrm{O}^1, \mathrm{O}^2 \in \mathrm{GeObject}, (\forall \sigma \in \mathrm{Scale}) \wedge (O^1_\sigma = 2Dmap(O^1, \sigma)) \wedge$$
$$(O^2_\sigma = 2Dmap(O^2, \sigma)) \wedge (Overlap(\mathrm{O}^1, \mathrm{O}^2)) \wedge$$
$$(Area(\mathrm{O}^1 \cap \mathrm{O}^2) < Area(\neg(\mathrm{O}^1 \cap \mathrm{O}^2)))$$
$$\Rightarrow Meet(O^1_\sigma, O^2_\sigma).$$

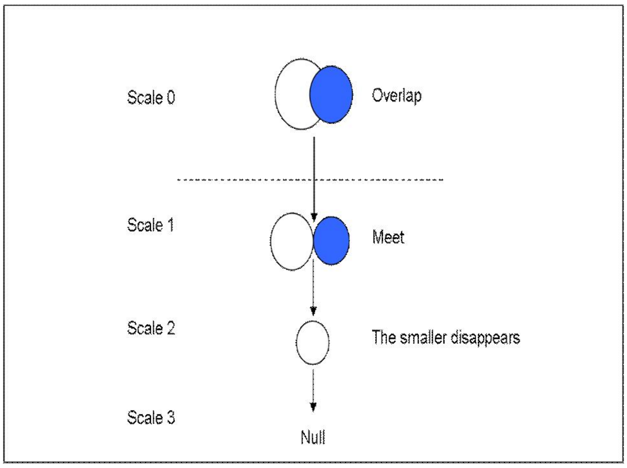


Fig 4. The mutation of Overlap to Meet.

In addition, similarly, the smaller object can disappear.

3.3.3. Mutation Overlap to Cover

Also, the relation "overlap" may be mutate to relation "cover", to formulate this mutation, one use the following assertion, see Fig 5:

$\forall$ O[1], O[2] $\in$ GeObject , ($\forall \sigma \in$ Scale) $\wedge$
$(O^1_\sigma = 2Dmap\ (O^1, \sigma)) \wedge (O^2_\sigma = 2Dmap\ (O^2, \sigma)) \wedge$
$(Overlap\ (O^1, O^2)) \wedge (Area\ (O^1 \cap O^2) > Area\ (\neg(O^1 \cap O^2)))$
$\Rightarrow Cover\ (O^1_\sigma, O^2_\sigma).$

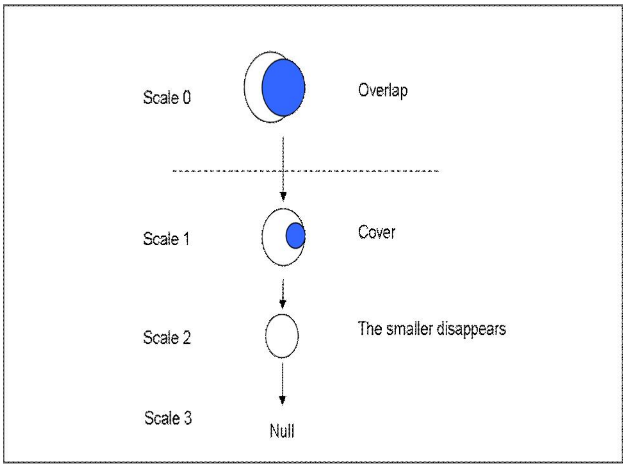


Fig 5. The mutation of Overlap to Cover.

3.3.4. Mutation Contains to Cover

The mutation of relation "contains" to "Cover", was expressed by the following assertion (see Fig 6):

$\forall O^1, O^2 \in \text{GeObject}, (\forall \sigma \in \text{Scale}) \wedge (O^1_\sigma = 2Dmap(O^1, \sigma)) \wedge$
$(O^2_\sigma = 2Dmap(O^2, \sigma)) \wedge (Contains(O^1, O^2)) \wedge$
$(Dist(O^1, O^2) < \varepsilon_1) \Rightarrow Cover(O^1_\sigma, O^2_\sigma).$

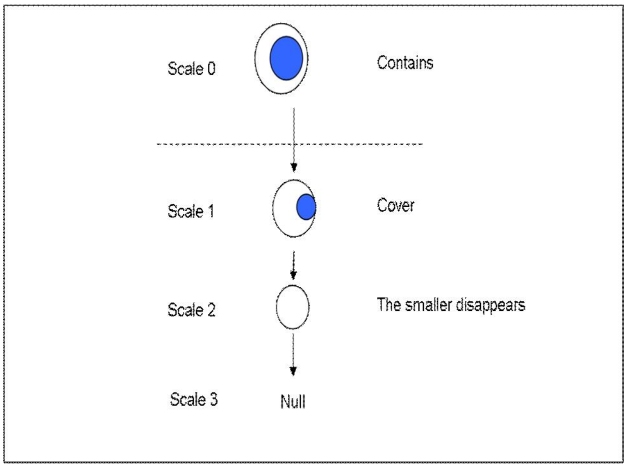


Fig 6. The mutation of Contains to Cover.

4. Scaling Mutation of Topological Relations

As previously told according to scales, geographic objects can mutate according to two rules. As scale diminishes: Loose region will mutate to simple region, to a point and then will disappear.

4.1. General Process

The generalization process is very complex. As we treat in this context, the generalization of the topological relations during downscaling, we would simplify it. So, the complete process can be modeled as follows:

- Step 0: original geographic features only modeled as loose region,
- Step 1: as scale diminishes, loose regions will be generalized and possibly can mutated into simple region,
- Step 2: as scale continues to diminish, small simple region mutate to point,
- Step 3: as scale continues to diminish, points can be disappear.

Let us call this process "generalization-reduction-disappearance" (GRD process).

4.2. Visual acuity applied to geographic objects

It is well known that "*Cartographic representation is linked to visual acuity*". Thresholds must be defined. In classical cartography, the limit ranges from 1 mm to 0.1 mm. If one takes a road and a certain scale and if the transformation gives a width more that 1 mm, this road is an area, between 1 mm and 0.1mm, then a line and if less that 0.1mm the road disappears. The same reasoning is valid for cities or small countries such as Andorra, Liechtenstein, Monaco, etc. In these cases, the "holes" in Italy or in France disappear cartographically. With the thresholds previously defined, we can formally get (in which *2Dmap* is a function transforming a geographic object to some scale possibly with generalization, in the 2-dimension domain) [19]:

a/ Disappearance of a geographic object (O) at scale σ:

$\forall O \in \text{GeObject}, \forall \sigma \in \text{Scale} \wedge O_\sigma = 2Dmap(O, \sigma) \wedge$
$Area(O_\sigma) < (\varepsilon_{lp})^2 \Rightarrow O_\sigma = \phi.$

b/ Mutation of an loose region into a point (for instance the centroid of the concerned object, for instance taken as the center of the minimum bounding rectangle):

$\forall O \in \text{GeObject}, \forall \sigma \in \text{Scale} \wedge O_\sigma = 2Dmap(O, \sigma) \wedge$
$(\varepsilon_i)^2 > Area(O_\sigma) > (\varepsilon_{lp})^2$
$\Rightarrow O_\sigma = Centroid(O).$

4.3. Granularity of interest

The previous remarks are not only valid for cartography, but also for any type of reasoning. Beyond thresholds of visual acuity which is a fundamental concept in cartography, let us define "granularity of interest": this is the minimum level of interest for a geographic user. For instance a nationwide politician will be interested at state level whereas an urban will be concerned only at the level of the city for which he works.

In the sequel to simplify the presentation, we will continue to use the thresholds for visual acuity instead of granularity of interest.

5. Generalizing Loose Topological Relations of Region

The generalization of spatial data implied the generalization of the topological relations according to certain accurate rules. We considered here the GRD process described in section IV.A. The objective of this section is to formulate the list of these rules, between loose regions.

5.1. Mutation of loose topological relations

Often, due to measurement errors and independent processing or generalizations, geographic objects do not coincide exactly. Eghenhofer (2009) investigate the possible connections between the topological relationships and metrics [28].

When one wants to evaluate the topological relations between them, he needs to take this aspect into account. Within the context of granularity of interest, when downscaling, this characteristic will disappear. Let define loose topological relations when dealing in such cases. By considering the conventional topological relations, let us immediately say that the disjoint relation is not concerned, except when the regions are very close (see Fig 7).

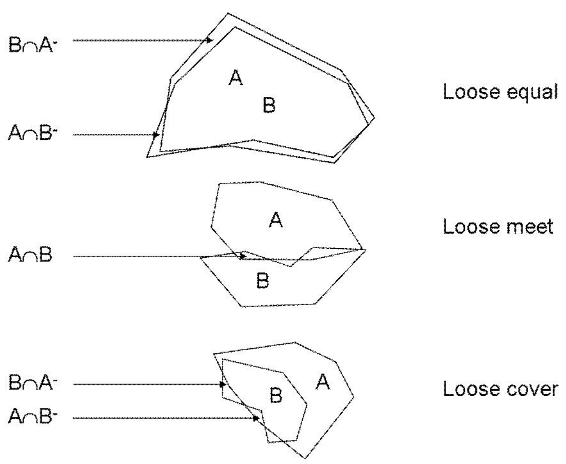


Fig 7. Loose topological relations.

5.1.1. Loose meet

The criterion to define a loose meet is based on the area of the intersection of two regions, A and B. For instance, given a threshold ε_{LM}:

$$\frac{Area(A\bigcap B)}{Area(A\bigcup B)} < \varepsilon_{LM} \Rightarrow Lmeet(A,B)$$

When downscaling from σ_1 to σ_2, this mutation

Lmeet-to-meet can be defined:

$$Lmeet(A_{\sigma_1}, B_{\sigma_1}) \wedge (\sigma_2 < \sigma_1) \wedge (A_{\sigma_1} = 2Dmap(A,\sigma_1)) \wedge$$
$$(A_{\sigma_2} = 2Dmap(A,\sigma_2)) \wedge (\frac{Area(A_\sigma \bigcap B_\sigma)}{Area(A_\sigma \bigcup B_\sigma)} < \varepsilon_2{}^2)$$
$$\Rightarrow Meet(A_{\sigma_2}, B_{\sigma_2})$$

5.1.2. Loose cover

Here one has to evaluate the area of the sliver polygons. This area is composed of two parts, $A\bigcap B^-$ and $A^-\bigcap B$. In other terms, this is a symmetric difference defined as follows: $A\oplus B = (A\bigcap B^-)\bigcup(A^-\bigcap B)$. Therefore by defining another threshold, the corresponding criterion can be:

$$\frac{Area(A\oplus B)}{Area(A\bigcup B)} < \varepsilon_{LC} \Rightarrow \mathrm{Lcover}(A,B)$$

So, the mutation Lcover-to-cover when

downscaling:

$$Lcvr(A_{\sigma_1}, B_{\sigma_1}) \wedge (\sigma_2 < \sigma_1) \wedge (A_{\sigma_1} = 2Dmap(A,\sigma_1)) \wedge$$
$$(A_{\sigma_2} = 2Dmap(A,\sigma_2)) \wedge$$
$$(\frac{Area((A\bigcap B^-)\cup(A^-\bigcap B))}{Area(A\bigcup B)} < (\varepsilon_4)^2)$$
$$\Rightarrow \mathrm{Cover}((A_{\sigma_2}, B_{\sigma_2})).$$

5.1.3. Loose equal

The loose-equal relation can be defined from the loose-cover relation, but the area in the intersection must not be far from the union. So two criteria must be used with another threshold:

$$\frac{Area(A\oplus B)}{Area(A\bigcup B)} < \varepsilon_{LC} \wedge \frac{Area(A\bigcap B)}{Area(A\bigcup B)} < 1-\varepsilon_{LE} \Rightarrow Lequal(A,B)$$

Similarly, this relation can mutate to an Equal

relation when downscaling:

$$Lequal\ (A_{\sigma_1}, B_{\sigma_1}) \wedge (\sigma_2 < \sigma_1) \wedge (A_{\sigma_1} = 2Dmap\ (A, \sigma_1)) \wedge$$
$$(A_{\sigma_2} = 2Dmap\ (A, \sigma_2)) \wedge$$
$$(\frac{Area\ ((A \cap B^-) \cup (A^- \cap B))}{Area\ (A \cup B)} < (\varepsilon_5)^2)$$
$$\Rightarrow Equal\ ((A_{\sigma_2}, B_{\sigma_2}))$$

5.2. Holding topological constraints

Since certain topological relations must be persistent, regardless the scale of representation, those relations must hold. See for instance in Fig 8, the Mediterranean Coast in the South of France: as the coast is generalized (the coast mutate into a polyline), some harbors will be in the middle of the sea such as Nice, whereas others will be inside the country such as Marseilles and Montpellier; in addition, the confluent of the Rhone river will be badly positioned in the middle of the land. The constraints are as follows:

Covers (France, Nice)

Covers (France, Marseilles)

Covers (France, Montpellier)

Covers (France, Rhone).

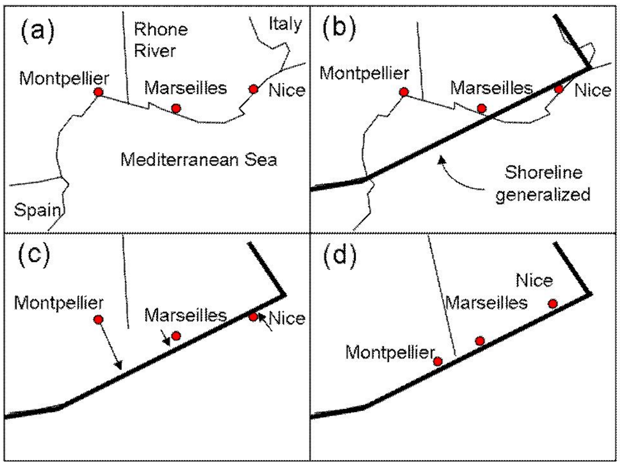


Fig 8. Holding topological constraints for harbors in the Mediterranean Sea. (a) Before generalization. (b) Only the coastline is generalized. (c) Harbors must move. (d) After generalization.

Another example of topological constraint when generalizing the Eastern French border is the case of Geneva which must hold outside France (see Fig 9).

The constraint is as follows:

Meet (France, Geneva).

To avoid such topological errors due to generalization process, one must use more advanced geometric algorithms. An algorithm could be proposed which will hold this type of topological relation.

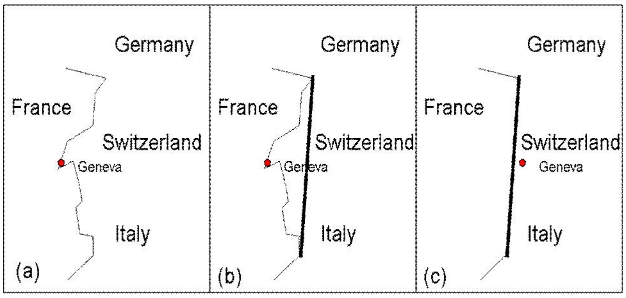


Fig 9. Holding topological constraints for outside border cities.

Therefore one has to move the object (harbor) in order to follow the constraints according to a distance Dist. Three cases must be considered, in which H denotes the harbor and C the country (see Fig 10).

- In case #1 : the harbor (H) is located inside the country: Dist min (C, H)
- In case #2 , the harbor at the exterior from the country: Distmax (C, H).
- In case #3 , a part of the harbor at the exterior of the country: Dist(P(x,y), C)/ P(x,y) = Intersect(C',H).

For the Meet constraints, cases are reciprocal.

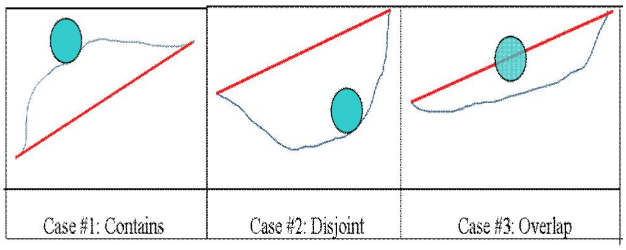


Fig 10. The three cases of movement.

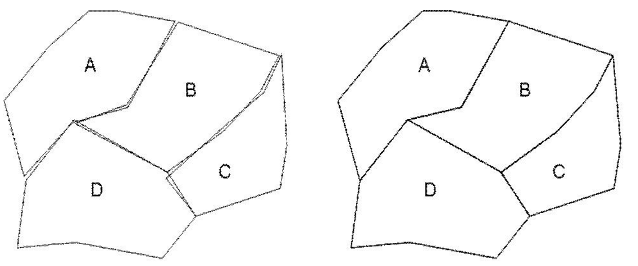
5.3. Generalized loose tessellations when downscaling

By irregular tessellation (or tessellation), one means the total coverage of an area by subareas. For instance the conterminous States in the USA form a tessellation to cover the whole country. Generally speaking administrative subdivisions form tessellations, sometimes as hierarchical tessellations. Let us consider a domain D and several polygons Pi; they form a tessellation iff (see Fig 11b):

- For any point pk, if pk belongs to D then there exists Pj, so that pk belongs to Pj

- For any pk belonging to Pj, then pk belongs to D.

A tessellation can be also described by Egenhofer relations applied to Pi and D, but in practical cases, due to measurement errors, this definition must be relaxed in order to include sliver polygons (see Fig 11a). Those errors are often very small, sometimes a few centimeters at scale 1. In other words, one has a tessellation from an administrative point of view, but not from a mathematical point of view. Let's call them "loose tessellation". When downscaling, those errors will be rapidly less than the threshold εlp so that the initial slivered or loose tessellation will become a good-standing tessellation.



a/ A slivered tessellation b/ A good standing tessellation

Fig 11. A tessellation with sliver polygons and a good standing tessellation.

6. Conclusion and Future Works

In this paper, we develop a topological model of loose regions. This Model principally based on the Eghenhofer model. Also, we treat the mutation of the topological relations which can hold between the loose regions into others relations, when downscaling. To assure the topological consistency, topological conditions are used to mutate the relationships in terms of meeting, overlapping, disjunction, and merging between map objects into others relationships.

This work can open various future works, such as:

- Integration of this topological model in the automatic generalization process or on-the-fly web map generation.
- Use these basic topological relations to model the other relations which can be between the complex regions.

References

[1] Cecconi, A., Integration of Cartographic Generalization and Multi-scale databases for Enhanced Web Mapping", PhD Thesis, University of Zurich, 2003.
[2] ICA, International Cartographic Association: Multilingual Dictionary of Technical Terms in Cartography, Wiesbaden, F. Steiner, 1973.
[3] McMaster, B., and Shea, S., Generalization in Digital Cartography, Association of American Geographers, Wahington D.C, 1992.
[4] Douglas D., Peuker T., Algorithms for the Reduction of the Number of Points Required to Represent a Digitized Line or its Caricature, The Canadian Cartographer, 1973, 10(2): 112-122.
[5] Galanda, M. , Weibel R., An Agent-Based Framework for Polygonal Subdivision Generalization", in Richardson D. and van Oosterom P. (eds.), Advances in Spatial Data Handling. 10th International Symposium on Spatial Data Handling, Berlin Heidelberg: Springer, 2002, 121–136.
[6] Müller, J., and Wang, Z., Area-Patch Generalization: A Competitive Approach. The Cartographic Journal, 1992, 29(2) : 137–144.
[7] Ruas, A, Plazanet, C.,Strategies for Automated Generalization, Proceedings of the 7thInternational Symposium on Spatial Data Handling, Advances in GIS Research II, 1996, 1-6.
[8] Harrie, L., the Constraint Method for Solving Spatial Conflicts in Cartographic Generalization", Cartography and Geographic Information Science, 1999, 26(1), 55-69.
[9] Sarjakoski, T. , Kilpelainen, T.,Holistic Cartographic Generalization by Least Squares Adjustment for Large Data Sets, Proceedings of 19th International Cartographic Conference, Ottawa, Canada,1999,1091–1098.
[10] Sester, M., Generalization Based on Least Squares Adjustment". International

Archives of Photogrammetry and Remote Sensing, 2000, 33 (B4/3) : 931-938.

[11] Ware, M., Thomas, N., Jones, C., Resolving Graphic Conflicts in Scale Reduced Maps: Refining, the Simulated Annealing Technique, Proceedings of 11th Annual GIS Research UK Conference (GISRUK) , City University, 2003, 244-248.

[12] Wilson, D., Ware, M., Ware, A., Reducing Graphic Conflict in Scale Reduced Maps Using a Genetic Algorithm. ICA Map Generalization Workshop, Paris, 2003, 01-11.

[13] Duchêne, C., Automated Map Generalization using Communicating Agents". Proceedings of the 21st International Conference of Cartography (ACI/ICA), Durban, Afrique du Sud , 2003, 160-169.

[14] Ruas, A., Modèle de généralisation de Données Géographiques à Base de Contraintes et d'Autonomie, PhD thesis, Université de Marne-la-Vallée, 1999.

[15] Ruas, A. , Duchêne C., A Prototype Generalisation System Based on the Multi-Agent System Paradigm. In W.A.Mackaness, A.Ruas and L.T. Sarjakoski, editor: Generalisation of Geographic Information: Cartographic Modelling and Applications, Elsevier Ltd, chapter 14, 2007, 269-284.

[16] Sabo, N., Généralisation et des patrons géométriques pour la création des objets auto-generalisants (SGO) afin d'améliorer la généralisation cartographique à la volée, PhD thesis, Université de laval, Quebec, 2007.

[17] Stoter, J., D. Burghardt, C. Duchêne, B. Baella, N. Bakker, C. Blok, M. Pla, N. Regnauld, G. Touya, S. Schmid. , Methodology for Evaluating Automated Map Generalization in Commercial Software, Computers, Environment and Urban Systems, 2009, 33 (5) : 311-324.

[18] Lejdel, B., Kazar, O., Genetic Agent for Optimizing Generalization Process of Spatial Data, International Journal of Digital Information and Wireless Communications (IJDIWC),2012, 1(3) : 729-737.

[19] Lejdel, B., Kazar, O., Laurini R., Mathematical Framework for Topological Relationships between Ribbons and Regions", Journal of Visual Languages and Computing, 2015, 26(1):66-81.

[20] Egenhofer, M., and Franzosa, R., Point-Set Topological Spatial Relations. International Journal of Geographical Information Systems, 1991, 5(2): 161-174.

[21] Winter, S., Frank A., Topology in Raster and Vector Representation, GeoInformatica, 2000, 4(1) :35-65.

[22] Egenhofer, M.,Herring J., A Mathematical Framework for the Definition of Topological Relationships". In Proceedings of the 4th International Symposium on Spatial Data Handling, 1990, 803-813.

[23] Egenhofer, M., Topology and Reasoning: Reasoning about Binary Topological Relations". In Second Symposium on Large Spatial Databases, In O. Gunther and H.-J. Schek (Eds.), LNCS in Advances in Spatial Databases, Springer-Verlag, 1991, 141-160.

[24] Clementini, E., Sharma J., Egenhofer M. Modeling Topological Spatial Relations: Strategies for Query Processing, In Journal of Computers and Graphics, 1994, 18 (6): 815-822.

[25] Randell, D., Cui Z., Cohn A., A Spatial Logic based on Regions and Connection. In: 3rd International Conference on Principles of Knowledge Representation and Reasoning, Morgan Kaufmann Publisher, 1992, 165–176.

[26] Winter, S., Uncertain Topological Relations between Imprecise Regions, International Journal of Geographical Information Systems, 2000, 14 (5) : 411- 430.

[27] Laurini, R., A Conceptual Framework for Geographic Knowledge Engineering", Journal of Visual Languages and Computing, 2014, 25(1):2-19.

[28] Egenhofer M., and Dube M., Topological Relations from Metric Refinements, ACM SIGSPATIAL GIS, 17TH ACM SIGSPATIAL International Conference on Advances in Geographic Information Systems, Seattle, WA D. Agrawal, W. Aref, C. Lu, M. Mokbel, P. Scheuermann, C. Shahabi and O. Wolfson (eds.), 2009, 158-167.